

Parameterization of Bernard's Mathematical Model to Estimate Intensity-Duration-Frequency Curves in the Curitiba and São José dos Pinhais Sub-basins Belonging to the Upper Iguaçu River Basin, Brazil

Alvaro Javier Esteban Villota Mora^{1*}, Yeimy Ordoñez Muñoz¹, Adalberto Matoski¹,
Ronaldo Luis dos Santos Izzo¹ and Fernando Hermes Passig²

¹Department of Civil Construction, Federal Univ. of Technology Paraná, Street Deputado Heitor Alencar Furtado, 5000, Campus Curitiba, Ecoville, Paraná CEP: 81280-340, Brazil.

²Department of Chemistry and Biology, Federal Univ. of Technology Paraná, Street Deputado Heitor Alencar Furtado, 5000, Campus Curitiba, Ecoville, Paraná CEP: 81280-340, Brazil.

E-mail: aljaesvimo@gmail.com*; yeordonez29@gmail.com; adalberto@utfpr.edu.br; fhpassig@professores.utfpr.edu.br; izzo@utfpr.edu.br

Received: 19 March 2023 / Revised form Accepted: 12 September 2023

© 2024 Geological Society of India, Bengaluru, India

ABSTRACT

The main objective of this research was to generate the parameters of a mathematical model for predicting precipitation behavior in the Curitiba and São José dos Pinhais sub-basins, which belong to the upper Iguaçu basin. Initially, a time series of pluviographic records from five meteorological stations was acquired. These data underwent goodness-of-fit tests and probability distribution function analyses, revealing that the Gumbel distribution provided the best fit for the data from all five analyzed stations. Subsequently, the Gumbel distribution and the polynomial regression method determined the relationship between precipitation, intensity, duration, and frequency (IDF). IDF curves for different return periods were plotted using this established model as a function of precipitation event duration. The results showed that the Curitiba station recorded the highest rainfall (146.2 mm) within 24 hours in February 1990. Statistical analysis based on the Mann-Kendall test indicated that the rainfall values from the five stations analyzed exhibited a homogeneous distribution. These findings reveal that the future IDF curves will rise as time goes by, and a possible climate change may occur, which could be due to the various factors that affect the climate. In conclusion, this research successfully derived the parameters necessary for calculating the intensity of future precipitation events across different durations and return periods for the five stations analyzed.

Keywords: IDF curves; Gumbel distribution; precipitation; IDF curves; return periods.

INTRODUCTION

Flash floods, caused by short and intense rainstorms, are one of the biggest weather-related disasters. These events rank among the most common hazards on earth and can lead to material and human loss (Willner et al., 2018; Hosseinzadehtalaei et al., 2020a, 2020b). The execution and construction of flood control structures and water

infrastructure projects, such as dams, roads, sewage, and stormwater drainage systems, are generally based on statistics of extreme precipitation properties and events, represented in intensity-duration-frequency (IDF) curves. IDF curves quantify the frequency of a given rainfall intensity over a range of time scales or temporal durations (Boukhelifa et al., 2018; Yan et al., 2018; Hosseinzadehtalaei et al., 2020a).

IDF curves must be determined from rainfall data over a long and representative period of extreme local events. According to the World Meteorological Organization (2018), a 30-year period is suitable for calculating extreme events for future projections.

Rainfall data can be compiled for statistical analysis in partial or annual series. The partial series comprises data values exceeding a predetermined base value, regardless of annual occurrence. Conversely, the annual series includes the highest value within each hydrological year and is the most commonly used method, as it does not require a predetermined base value (Beguería, 2005; Sansigolo, 2008).

Hosseinzadehtalaei et al. (2020a) evaluated the impact of climate change on short-term extreme precipitation and IDF curves in Europe for medium and long-term periods. The authors concluded that incorporating IDF curves is essential for the future design of urban water systems. Furthermore, the authors found that uncertainty is more significant for more extreme precipitations, to a lesser extent dependent on duration, and particularly pronounced for shorter durations.

On the other hand, Teixeira et al. (2011) evaluated the IDF curves obtained from annual and partial duration records in the locality of Pelotas, Brazil, using Student's t-test statistics. They considered a base value of 55 mm for the partial duration records, totaling 279 values. However, the authors did not find new data regarding the IDF curves that emerged from the analysis of the partial duration series with the recommended base value of precipitation compared to the historical records.

Using daily rainfall data, Cardoso et al. (2014) analyzed the frequency distribution and temporal variability of rainfall intensity in

Lages, Santa Catarina, Brazil. They found that the predominant rainfall temporal distribution pattern in Lages was advanced or convective, with higher rainfall occurring in the first half of its duration (Cardoso et al. 2014).

There are few scientific studies in Brazil on the determination of IDF curves. It may be due to the scarcity of rainfall records, low density of rainfall data, and short observation periods.

Da Silva and Silva evaluated the rainfall trend and seasonal concentration of precipitation rainfall in the Moxotó river basin in

Pernambuco-Brazil. From applying the Mann-Kendall test, the authors concluded that the analyzed basin does not have a homogeneous distribution, noting variations in the rainfall trend. In addition, the authors found that the precipitation concentration occurs mainly in January and July, with peaks from February to May (da Silva and Silva 2017).

Studies by Teixeira et al. (2011), Cardoso et al. (2014), and da Silva and Silva (2017) have investigated the IDF curves in different regions of Brazil. However, a mathematical model has not been

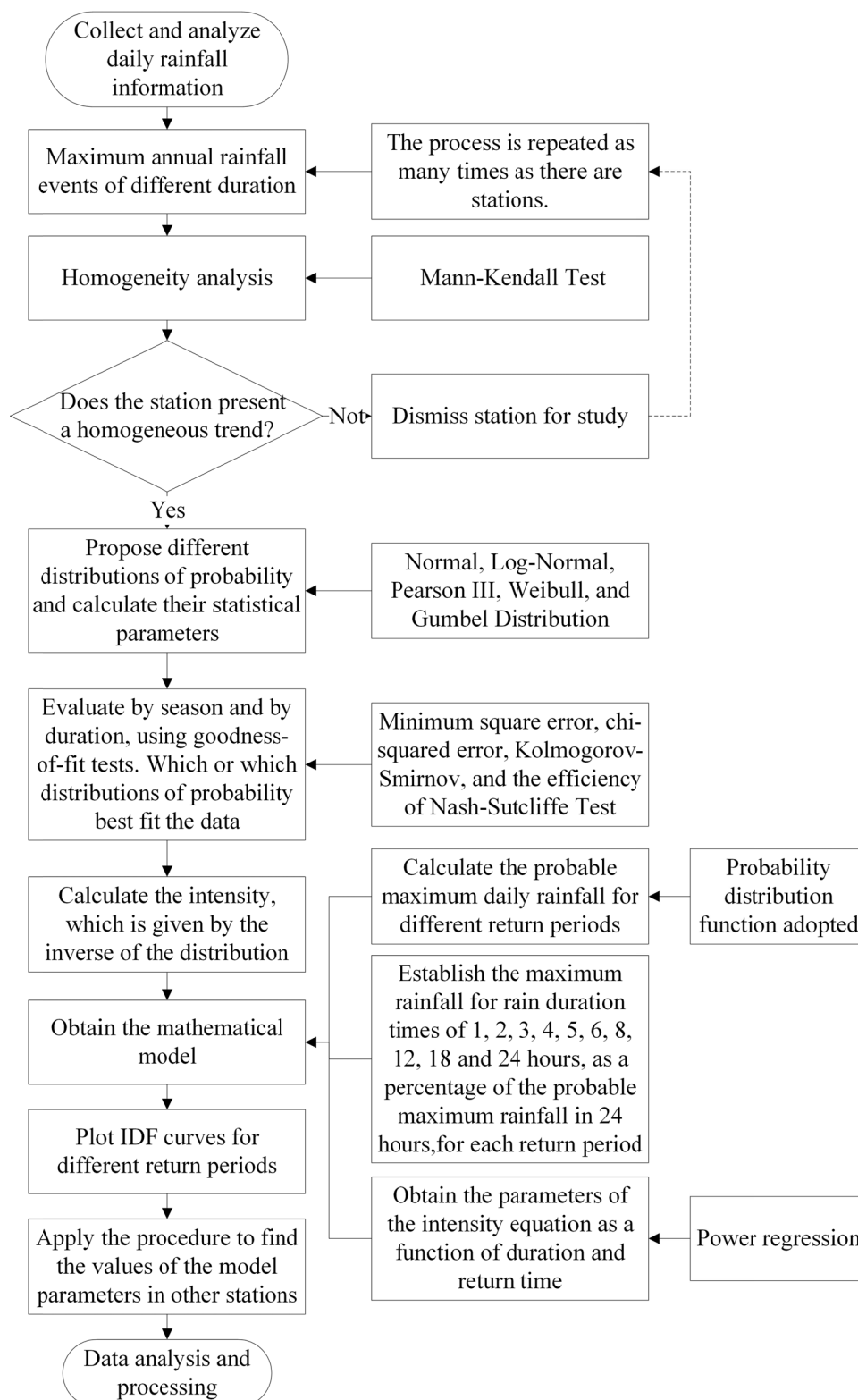


Fig. 1. Flowchart of the methodological process

parameterized to generate IDF curves for the Curitiba and São Jose dos Pinhais sub-basins.

In this context, this research aims to generate the parameters for calculating intensities under different return periods and event durations, following the mathematical model proposed by Bernard (1932) for the Curitiba and São Jose dos Pinhais sub-basins. Hydrological information for these sub-basins is crucial for developing drainage systems in the cities of Curitiba and São José dos Pinhais and their metropolitan regions. This study also contributes to characterizing the upper Iguaçu basin, which is a key factor for the civil construction, agricultural, and industrial sectors in the involved cities and Paraná.

MATERIALS AND METHODS

The methodological process carried out in this study primarily consisted of seven phases, as depicted in figure 1.

The first phase involved evaluating the data corresponding to the maximum daily rainfall of the Curitiba, Prado Velho, Campo Comprido, Ilha do Rio Claro, and Fazendinha stations, all of which are part of the Instituto de Águas do Paraná (Águas-Paraná).

This study utilized data on maximum precipitation and the frequency of occurrence of precipitation intensity from pluviographic records for the Curitiba and São José dos Pinhais sub-basins in Paraná, Brazil. Table 1 provides the location and analyzed period of the five meteorological stations used to obtain daily data on maximum precipitation.

Table 1. Localization of meteorological stations

Station – Municipality	Latitude	Longitude	Elevation m.s.n.m	Period
Curitiba - Curitiba	25°26'47"	49°13'51"	929	1990-2021
Prado Velho PUC - Curitiba	25°27'00"	49°14'56"	884	1990-2014
Campo Comprido Se Copel - Curitiba	25°25'49"	49°21'50"	965	1992-2018
Ilha do Rio Claro - São José dos Pinhais	25°48'46"	48°55'25"	237	1990-2021
Fazendinha - São José dos Pinhais	25°31'09"	49°08'48"	910	1990-2021

The reading of rainfall data and the calculations in this study were conducted using the free and open-source software *RStudio*, which provides an integrated programming environment for solving statistical problems.

In the second phase, the intensity (mm/h) was calculated for each series and then divided by the duration in hours. In the third phase, rainfall intensity values were assessed using probability distributions. The fourth phase involved determining the probability distributions that best described the historical data series by applying goodness-of-fit tests.

In the fifth phase, the parameters of the mathematical model were obtained. This model comprises the following steps: calculating the maximum probable daily rainfall for different return periods based on the adopted probability distribution, defining the maximum rainfall for each return period and for rain durations of 1, 2, 3, 4, 5, 6, 8, 12, 18, and 24 hours; and calculating the parameters of the intensity equation as a function of duration and return time (2, 5, 10, 25, 50, 75, 100, and 500 years) through potential regression.

IDF curves for different return periods were represented and analyzed in the sixth phase. These curves were obtained through simulations using the previous stage's mathematical model. Finally, in the seventh phase, IDF curves were calculated and plotted to best fit the previously received data.

Test of Homogeneity

To determine whether the data show trends or changes in variance, the nonparametric Mann-Kendall (M.K.) test was applied. This statistical test has often been used to quantify the significance of trends in meteorological time series (Subash et al., 2011; Da Silva et al., 2013, 2015; Duhan and Pandey, 2013; da Silva and Silva, 2017; Andrades et al., 2019; Carvajal and Grajales, 2019).

For a time series of observations x_1, x_2, x_n , Mann (1945) and Kendall (1975) mention that the null hypothesis (H_0) is when all the data in the time series are random, independent, and equally distributed variables (forming a homogeneous series).

On the other hand, the alternative hypothesis (H_a) is true if the time series data follow a monotonic trend (non-homogeneous). The development of this test was carried out in three phases (Mann, 1945):

1. The values of the variables are ordered ($x_1, x_2, \dots, x_n$)
2. The sign of the difference of each pair of values is determined by comparing their magnitudes ($x_j - x_k$) where $j > k$ according to the following: ($x_j - x_k$) = 1, if ($x_j - x_k$) > 0; (0), if ($x_j - x_k$) = 0; (-1), if ($x_j - x_k$) < 0.
3. Obtaining Mann Kendall's "S" statistic using the following Equation (Kendall, 1975):

$$S = \sum_{k=1}^{n-1} \sum_{j=k+1}^n \text{sgm}(x_j - x_k) \quad (1)$$

Where:

$$\text{sgm}(x) = \begin{cases} +1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

The variable S should present an approximately normal distribution with average zero and variance (Var (S)) (Equation (2)):

$$\text{Var} (S) = [(n(n-1)(2n+5))/18] \quad (2)$$

The result of S indicates the possibility of attending tendencies when the value S is significantly different from zero. The statistical value of the test was described by Z, represented in the following Equation:

$$\begin{aligned} Z &= (S - 1) / (\text{Var} (S))^{1/2} \quad \text{if } S > 0 \\ Z &= 0 \quad \text{if } S = 0 \\ Z &= (S + 1) / (\text{Var} (S))^{1/2} \quad \text{if } S < 0 \end{aligned} \quad (3)$$

A positive value of Z indicates an increasing trend and a negative value shows a decrease in the data trend. Therefore, the homogeneity checks for a significance level of $\alpha=10\%$ reject the null hypothesis when the absolute value of Z is higher than the critical Z_{critic} obtained from the critical values table for the Man-Kendall test.

Probability Distribution Functions

This research analyzed the precipitation intensity values using the following probability distribution functions: Normal, Log-Normal, Pearson III, Weibull, and Gumbel. Hydrology commonly uses these distributions (Florentin et al., 1966; Suárez-Aguilar et al., 2020). They were applied to select the mathematical model best fitting the analyzed data. The normal distribution was calculated using Equation (4) (Kite, 1988):

$$F(x)_N = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-0.5\left(\frac{t-\mu}{\sigma}\right)^2} dt \quad (4)$$

Where μ is the average, and σ is the standard deviation.

On the other hand, the log-normal distribution is represented by Equation (5).

$$F(x)_{LN} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x \frac{1}{t\beta} e^{-0.5\left(\frac{\ln t - \alpha}{\beta}\right)^2} dt. \quad (5)$$

Where α and β are parameters determined by the natural logarithm of the average and standard deviation of the random variable. The three-parameter Pearson III or Gamma distribution is defined by Equation (6), where α_1 , β_1 , and σ_1 are statistical parameters.

$$F(x)_P = \frac{1}{\alpha_1 \Gamma(\beta_1)} \int_{-\infty}^x \left(\frac{t - \sigma_1}{\alpha_1}\right)^{\beta_1 - 1} e^{-\left(\frac{t - \sigma_1}{\alpha_1}\right)} dt. \quad (6)$$

The Weibull distribution assumes that all data are uniformly distributed, yielding probability values from 0 to 100%. Thus, the Weibull distribution was calculated using the Equation described in Equation (7).

$$F(x)_W = 1 - e^{-\left(\frac{x}{\beta}\right)^\alpha} \quad (7)$$

The Gumbel distribution belongs to the double exponential type defined by Equation (8) (Gumbel, 2004).

$$F(x) = e^{-e^{-\left(\frac{x-\beta}{\alpha}\right)}}, \alpha = \frac{\sqrt{6}}{\pi} * \sigma, \beta = \bar{x} - 0.5772 * \alpha \quad (8)$$

Where β and α represent the statistical parameters, while σ and $\bar{x}$ are the standard deviation and average, respectively.

Goodness-of-Fit Tests

The goodness-of-fit of a probability distribution can be assessed by comparing the theoretical and sample values of relative frequency or accumulated frequency functions (Grajales-Cardona and Carvajal-Serna, 2019).

In this study, the adjustment of the probability distribution functions was evaluated through the following tests: minimum square error, chi-squared error (χ^2), Kolmogorov-Smirnov, and the efficiency of Nash-Sutcliffe.

Equation (9) defines the minimum square error with the observed data x_{oi} and the estimated x_{ei} for each distribution function.

$$D = \text{Min} \left\{ \|x_o - x_e\| = \left(\sum_{i=1}^n (x_{oi} - x_{ei})^2 \right)^{\frac{1}{2}} \right\} \quad (9)$$

For the chi-square test χ^2 , the rainfall data was divided into seven class intervals, where θ_i represents the observed data in interval i . Subsequently, the expected number of events in the same interval, ϵ_i , was calculated according to the following Equation:

$$D = \epsilon_i = n[F(S_i) - F(L_i)], i = 1, 2, \dots, k. \quad (10)$$

Where n is the number of data, $F(S_i)$ and $F(L_i)$ represent the probability distribution function evaluated at the upper and lower limits of interval i , respectively.

The Kolmogorov-Smirnov test compares the absolute value of the difference D_k between the observed distribution function $F_o(x_m)$ according to Equation (11) (Simard and L'Ecuyer, 2011).

$$D_k = \max |F_o(x_m) - F(x_m)| \quad (11)$$

In this case, the critical value “d” depends on the number of data and the significance level used. If $D_k < d$, H_0 is accepted, indicating recognition of the probability distribution function. In this research, the observed probability distribution function was calculated using the following Equation:

$$F_o(x_m) = 1 - (m / (n + 1)) \quad (12)$$

Where m is the order of the data, x_m is the data arranged in

descending order (highest to lowest), and n is the total amount of data.

The Nash-Sutcliffe efficiency test evaluates the variability of observations using the probability function and is defined by the following Equation:

$$E = 1 - \frac{\sum_{i=1}^n (x_{oi} - x_{ei})^2}{\sum_{i=1}^n (x_{ei} - \bar{x}_e)^2} \quad (13)$$

Finally, the Nash-Sutcliffe efficiency (E) was evaluated using the values recommended by Nash and Sutcliffe (1970) (see Table 2).

Table 2. Reference values of the Nash-Sutcliffe criterion

E	Adjustment
< 0.2	Insufficient
0.2 - 0.4	Satisfactory
0.4 - 0.6	Good
0.6 - 0.8	Very good
> 0.8	Excellent

Source: Molnar (2011)

IDF Curves Calculation

The IDF curves were calculated through the probable maximum precipitation (X.T.) derived from the yearly maximum monthly precipitation. Subsequently, the probability of occurrence for the precipitation event was estimated using the Gumbel equation (Equation 8).

Subsequently, this probability of occurrence was corrected according to the recommendation of Weiss (1964). Weiss suggests that probabilistic analysis using the maximum annual rainfall observed within a single fixed interval should be increased by 13%. This correction provides more accurate estimates than those obtained in the analysis based solely on natural maximum rainfall. Therefore, for this study, the representative value adopted for the basin was multiplied by 1.13 to account for a fixed and uniform observation interval. Then, these corrected values were further adjusted using the maximum probability duration coefficients recommended by Campos (1978) (Table 3). These coefficients were obtained as a percentage of the probable maximum precipitation for 24 hours, corresponding to each return period based on the selected rainfall duration times.

Table 3. Values for relationships to rain lasting 24 hours

Duration in hours									
1	2	3	4	5	6	8	12	18	24
0.30	0.39	0.46	0.52	0.57	0.61	0.68	0.80	0.91	1.00

Source: Campos (1978)

Finally, the rainfall intensities were calculated for different duration times according to the return period following. Finally, the rainfall intensities for various duration times were calculated based on the return period using the Equation proposed by Bernard (1932) in Equation (14).

$$I = (a * T^b) / t^c \quad (14)$$

Where I represents the intensity (mm/hr), t is the duration of the rain event (min), T represents the return period (years), and a , b , and c are adjustment parameters.

RESULTS AND DISCUSSION

Maximum Monthly Rainfall

The maximum values of monthly rainfall were obtained from the

records of daily rainfall expressed in millimeters (mm) at the two meteorological stations analyzed in this study, issued by the *Instituto AguasParaná*.

Table 4 presents the maximum monthly rainfall for Curitiba. Historical records indicate that the Curitiba station experienced the highest rainfall compared to the Prado Velho and Campo Comprido stations. It resulted in a notable accumulation of precipitation, reaching 146.2 mm in 24 hours in February 1990 (Curitiba), as opposed to 125.4 mm in December 2010 (Prado Velho) and 130.7 mm in January 2004 (Campo Comprido).

Table 5 displays the maximum monthly rainfall for the São José dos Pinhais sub-basin. The results show that the highest monthly rainfall recorded at the Ilha do Rio Claro station occurred in March 2011 (188.3 mm), compared to January 2007 (140.6 mm) for the Fazendinha station.

According to the National Institute of Meteorology INMET (2009), in Curitiba, precipitation is abundant throughout the year, with no distinct dry season, and can manifest as rain, hail, or snow. The annual rainfall averages approximately 1575 mm. Fronts from Argentina and Antarctica bring weather systems to Paraná, Brazil, throughout the year, resulting in tropical storms in the summer and cold winds in the winter.

Homogeneity Test

The results of the Mann-Kendall homogeneity test applied to the precipitation data analyzed in this study are shown in table 6. According to the results obtained at the Curitiba station, the negative Z value indicates a decreasing trend in the data: a reduction in rainfall

over time. Conversely, the statistical analysis does not reject the null hypothesis, as $Z_{crit} = 1.28\% > Z = 0.0811$. It indicates that the data comprising the time series are independent and equally distributed random variables, indicating a homogeneous series. The same analysis was performed for the Prado Velho, Campo Comprido, Ilha do Rio Claro, and Fazendinha stations, and it was found that the data series analyzed were homogeneous.

Salas et al. (1990) mention that evaluating the quality of the data from hydrological stations is essential to determine possible changes in variance, mean, and trend over the years. Therefore, to establish the homogeneity of hydrological data statistically, confirming that the series exhibits consistent variance and mean is imperative.

Similarly, Lopez and Frances (2014) note that a homogeneous series reflects seasonality in the precipitation data of a hydrological station, implying that the precipitation behavior will be permanent and constant through time. Conversely, when the statistically evaluated information is not homogeneous, it indicates a non-stationary trend. Therefore, assuming that random variables are identically distributed over time is invalid. Consequently, it is impossible to assume that the probability function is constant over the years.

Goodness-of-Fit tests

The goodness-of-fit test results applied to the precipitation data analyzed in this study are shown in table 7. The minimum square error and chi-square error tests indicated that the best data fit was achieved with the Pearson III distribution, yielding regression coefficients of 1.00 and 7.928, respectively. On the other hand, the Kolmogorov-Smirnov test revealed the best fit with the Gumbel

Table 4. Maximum monthly rainfall of Curitiba sub-basin (Data provided by AguasParaná)

Curitiba station			Prado Velho station			Campo Comprido Se Copel station		
Year	Month of maximum precipitation	Precipitation (mm)	Year	Month of maximum precipitation	Precipitation (mm)	Year	Month of maximum precipitation	Precipitation (mm)
1990	NOV	68.7	1981	NOV	39	1992	MAI	64.5
1991	NOV	26.7	1982	FEV	94.6	1993	MAI	102.1
1992	MAI	65.8	1983	DEZ	91.8	1994	DEZ	84.4
1993	MAI	95.3	1984	JUN	60.6	1995	JAN	96.1
1994	FEV	55.2	1985	ABR	37.2	1996	JAN	74.7
1995	JAN	121	1986	FEV	61	1997	JAN	91.1
1996	FEV	83.4	1987	FEV	78.1	1998	AGO	100.4
1997	FEV	102.5	1988	MAI	77.2	1999	JUL	70
1998	SET	93.1	1989	JAN	67.3	2000	FEV	80.1
1999	FEV	146.2	1990	NOV	63.8	2000	SET	49.1
2000	MAR	74.1	1991	FEV	63.2	2001	JUN	81.5
2001	FEV	71.8	1992	MAI	68.5	2002	SET	62.3
2002	JAN	79.7	1993	MAI	95.4	2003	DEZ	61.7
2003	JUL	93.4	1994	JAN	87.4	2004	JAN	130.7
2004	MAR	48.6	1995	JAN	113.8	2005	SET	68.7
2005	AGO	74.2	1996	FEV	85	2006	NOV	72.6
2006	NOV	48	1997	FEV	114.8	2007	DEZ	68.7
2007	JAN	69.4	1998	AGO	92.4	2008	ABR	74.1
2008	MAR	82	1999	FEV	112.8	2009	NOV	66.4
2009	NOV	74.4	2000	FEV	68.2	2010	DEZ	108.1
2010	DEZ	119.6	2001	NOV	74.6	2011	AGO	106.5
2011	AGO	103.4	2002	JAN	71	2012	JUN	97.6
2012	JUN	113.4	2003	JUL	81	2013	JUN	108.2
2013	JUN	130.8	2004	MAR	76.4	2014	JUN	75.1
2014	JUN	95.2	2005	OUT	67.8	2015	MAI	81.1
2015	DEZ	60.3	2006	MAR	57	2016	JUL	70.5
2016	FEV	86.3	2007	DEZ	73.2	2017	FEV	70
2017	JUN	72.4	2008	MAR	83	2018	MAR	95.7
2018	JAN	94.2	2009	SET	73.6			
2019	FEV	124.7	2010	DEZ	125.4			
2020	NOV	86	2011	FEV	96.4			
2021	JAN	86.3	2012	JUN	113.4			
			2013	JUN	119.4			

Table 5. Maximum monthly rainfall of São Jose dos Pinhais sub-basin (Data provided by AguasParaná)

Year	Ilha do Rio Claro station		Fazendinha station	
	Month of maximum precipitation	Precipitation (mm)	Month of maximum precipitation	Precipitation (mm)
1990	NOV	80.5	NOV	69
1991	MAR	85.8	OUT	57.4
1992	FEV	121.4	JUL	58
1993	JAN	94	MAI	88.4
1994	MAR	66.8	NOV	58.2
1995	DEZ	138	JAN	115
1996	NOV	75.5	FEV	58.2
1997	SET	93.2	MAR	71.8
1998	AGO	83.9	AGO	73
1999	JAN	100.4	FEV	69.4
2000	FEV	70	FEV	50.4
2001	FEV	107.3	JAN	66.6
2002	AGO	75.1	JAN	73.6
2003	JAN	94.1	JUL	75.8
2004	SET	146.4	JAN	82.2
2005	MAR	109.1	AGO	81.2
2006	JAN	96	FEV	70.8
2007	DEZ	73.3	JAN	140.6
2008	JAN	121	ABR	66.2
2009	SET	82	SET	71.2
2010	ABR	97.9	FEV	63.6
2011	MAR	188.3	AGO	86.8
2012	NOV	88.3	JUN	94.8
2013	JUN	94	JUN	102
2014	JAN	62.7	JUN	98.8
2015	FEV	177.7	DEZ	57.5
2016	FEV	78.2	FEV	70.2
2017	DEZ	117	DEZ	73.3
2018	DEZ	65	JAN	74.1
2019	JUN	111	FEV	80
2020	FEV	65.3	FEV	56.4
2021	MAR	89	FEV	62.4

Table 6. Results Test Mann-Kendal for detecting tendencies in Curitiba station

Variable	Station				
	Curitiba Velho	Prado Comprido	Campo Claro	Ilha do Rio	Fazendinha
I	482	528	376	495	495
T	478	528	376	495	495
S	-4	0	0	0	0
Z	-0.0811	-0.0155	-0.0198	-0.0162	-0.0162
Zcrit	-1.28	-1.28	-1.28	-1.28	-1.28
α	10%	10%	10%	10%	10%

distribution, resulting in a regression coefficient of 0.083. It is worth noting that most distributions demonstrated satisfactory fits based on the results of the Kolmogorov-Smirnov test, displaying values below the critical threshold at a significance level of 5%. However, Mormeneo and Castellví (2001) considered that the best fit corresponds to the distribution exhibiting the most significant deviation from the critical value.

Finally, the Nash-Sutcliffe test indicated the best fit with the Weibull distribution, achieving a regression coefficient of 0.999.

Consequently, from table 7, selecting a distribution function that best fits the analyzed data is impossible. Therefore, it is crucial to assess the results of the goodness-of-fit tests to determine the probability distribution function that most accurately fits the analyzed data.

The distribution functions were evaluated and assigned values from 1 to 5, where 1 indicates a better fit and 5 a worse fit. The results

Table 7. Summary results of tests of goodness of fit for the probability distribution functions

Function	Minimum squared error	Chi-squared χ^2	Kolmogorov-Smirnov	Nash-Sutcliffe
Normal	0.985	0.114	0.085	0.985
Lognormal	0.984	1.922	0.094	0.976
Pearson III	1.000	7.928	0.964	0.403
Gumbel	0.986	1.636	0.083	0.981
Weibull	0.954	0.107	0.109	0.999

Table 8. Evaluation results of the goodness-of-fit test for the probability distribution functions

Function	Minimum squared error	Chi-squared χ^2	Kolmogorov-Smirnov	Nash-Sutcliffe	Total
Normal	3	4	2	2	11
Log-normal	4	2	3	4	13
Pearson III	1	1	5	5	12
Gumbel	2	3	1	3	9
Weibull	5	5	4	1	15

suggest that Gumbel's distribution function offers the best fit for the data, followed by Normal, Pearson III, Log-Normal, and Weibull (Table 8).

Suárez-Aguilar *et al.* (2020) estimated the IDF curves of extreme rainfall in Tunja-Colombia. The authors evaluated data fit using the same tests as in this study. They found that the Gumbel distribution presented the best fit for the maximum precipitation data in Tunja. Additionally, the authors suggested that the methodology employed to determine the best fit of the data can be applied to analyze other variables influencing climate behavior.

IDF Curves Calculation

Table 9 presents the probable maximum rainfall for different return periods, calculated by applying the Gumbel function and then corrected according to the coefficient of 1.13 recommended by Weiss (1964).

Table 10 presents the expected relationship between maximum daily rainfall for different durations and return periods based on the corrected rainfall and duration coefficients recommended by Campos (1978).

Table 11 provides the calculations for equivalent rainfall intensity derived from the data presented in table 10. In this table, precipitation levels are correlated with the duration of each event and the corresponding return period, according to Bernard (1932) (Equation (8)).

The potential regression was applied using the intensities and durations of events calculated for each return period, establishing a characteristic function for each return period. Table 12 illustrates the potential regression using the Gumbel distribution method for a return period of 2 years.

The same procedure was performed for all return periods (2, 5, 10, 25, 50, 75, 100, and 500 years). In figure 2, the rainfall intensity

Table 9. Probable maximum daily rainfall for different return periods

Return Period (Years)	Reduced variable YT	Precipitation XT'(mm)	Probability of occurrence F(xT)	Corrected Precipitation
2	0.366	81.543	0.5	92.144
5	1.499	104.526	0.8	118.115
10	2.250	119.743	0.9	135.310
25	3.199	138.969	0.96	157.035
50	3.902	153.233	0.98	173.153
75	4.311	161.523	0.99	182.521
100	4.600	167.390	0.99	189.151
500	6.214	200.107	0.998	226.121

Table 10. Maximum rainfall for different rainy seasons

Duration	Quotient	Maximum rainfall (mm) vs. rainfall duration times (years)							
		2	5	10	25	50	75	100	500
24	X24 = 100%	92.144	118.115	135.310	157.035	173.153	182.521	189.151	226.121
18	X18 = 91%	83.851	107.484	123.132	142.902	157.569	166.094	172.128	205.770
12	X12 = 80%	73.715	94.492	108.248	125.628	138.522	146.017	151.321	180.897
8	X8 = 68%	62.658	80.318	92.011	106.784	117.744	124.114	128.623	153.762
6	X6 = 61%	56.208	72.050	82.539	95.792	105.623	111.338	115.382	137.934
5	X5 = 57%	52.522	67.325	77.127	89.510	98.697	104.037	107.816	128.889
4	X4 = 52%	47.915	61.420	70.361	81.658	90.039	94.911	98.359	117.583
3	X3 = 46%	42.386	54.333	62.242	72.236	79.650	83.960	87.010	104.016
2	X2 = 39%	35.936	46.065	52.771	61.244	67.530	71.183	73.769	88.187
1	X1 = 30%	27.643	35.434	40.593	47.111	51.946	54.756	56.745	67.836

Table 11. Rainfall intensity (mm/h) - Curitiba station

Duration		Rain intensity (mm/hr) vs. Return period (years)							
		2	5	10	25	50	75	100	500
24	1440	3.839	4.921	5.638	6.543	7.215	7.605	7.881	9.422
18	1080	4.658	5.971	6.841	7.939	8.754	9.227	9.563	11.432
12	720	6.143	7.874	9.021	10.469	11.544	12.168	12.610	15.075
8	480	7.832	10.040	11.501	13.348	14.718	15.514	16.078	19.220
6	360	9.368	12.008	13.756	15.965	17.604	18.556	19.230	22.989
5	300	10.504	13.465	15.425	17.902	19.739	20.807	21.563	25.778
4	240	11.979	15.355	17.590	20.415	22.510	23.728	24.590	29.396
3	180	14.129	18.111	20.747	24.079	26.550	27.987	29.003	34.672
2	120	17.968	23.032	26.385	30.622	33.765	35.592	36.884	44.094
1	60	27.643	35.434	40.593	47.111	51.946	54.756	56.745	67.836

function for a return period of 2 years is depicted. Noticeably, the potential regression equation comprises a constant and a regression coefficient. Furthermore, the correlation coefficient (R) obtained was close to 1, resulting in 0.999.

Table 12. Potential regression by the Gumbel distribution method for a return period of 2 years

No.	x	y	ln x	ln y	ln x*ln y	(lnx)^2
1	1440	3.839	7.272	1.345	9.784	52.888
2	1080	4.658	6.985	1.539	10.747	48.786
3	720	6.143	6.579	1.815	11.943	43.287
4	480	7.832	6.174	2.058	12.707	38.116
5	360	9.368	5.886	2.237	13.169	34.646
6	300	10.504	5.704	2.352	13.414	32.533
7	240	11.979	5.481	2.483	13.609	30.037
8	180	14.129	5.193	2.648	13.752	26.967
9	120	17.968	4.787	2.889	13.829	22.92
10	60	27.643	4.094	3.319	13.591	16.764

In this way, table 13 summarizes the various potential regressions applied to the intensities and durations corresponding to each return period.

Based on the data in table 13, another potential regression was conducted, establishing a relationship between the return period “T” and the regression constant “d”. It resulted in deriving the general intensity function for the Curitiba station. The summarized values can be found in Table 14, while the relationship between the constant “d” and the return period is illustrated in figure 3.

Equation 9 contains parameters that make up the mathematical model proposed by Bernard (1932). These parameters were derived from the earlier calculations and the coefficients obtained from the potential regressions. Consequently, a standardized mathematical model for the Curitiba station was established. This model enables

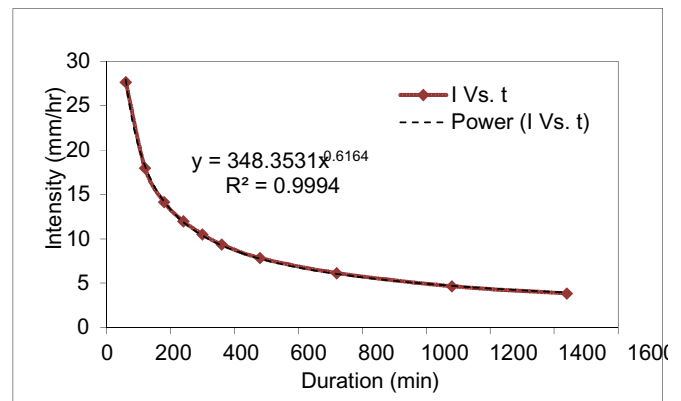
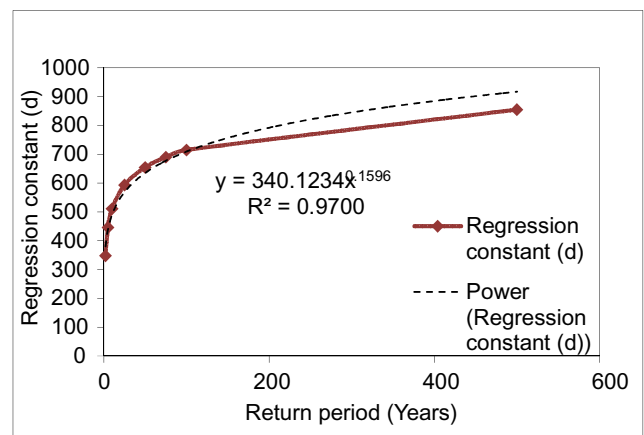
**Fig. 2.** Rainfall intensity function for the return period of 2 years**Fig. 3.** Regression constant “d” vs. Return period

Table 13. Regression Application Summary

Return period	Constant regression term d	Regression coefficient c
2	348.353	-0.616
5	446.536	-0.616
10	511.542	-0.616
25	593.676	-0.616
50	654.609	-0.616
75	690.025	-0.616
100	715.091	-0.616
500	854.856	-0.616
Average	601.836	-0.616

Table 14. Potential regression, $d=KTm$, the T return period

N	T	d	ln T	ln d	ln T*ln d	(ln T) ²
1	2	348.353	0.693	5.853	4.057	0.480
2	5	446.536	1.609	6.102	9.820	2.590
3	10	511.542	2.303	6.237	14.362	5.302
4	25	593.676	3.219	6.386	20.557	10.361
5	50	654.609	3.912	6.484	25.366	15.304
6	75	690.025	4.318	6.537	28.222	18.641
7	100	715.091	4.605	6.572	30.267	21.208
8	500	854.856	6.215	6.751	41.954	38.621

the analysis of precipitation event intensity for various durations and return periods.

Figure 4 displays the IDF curves generated from the obtained mathematical model. The intensity decreases with longer durations of the rain event. Furthermore, future IDF curves are anticipated to rise and accentuate, signifying increased precipitation.

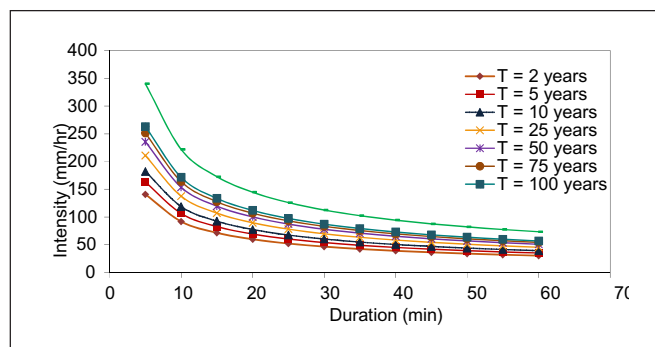
The calculations described above were also applied to the Prado Velho, Campo Comprido, Ilha do Rio Claro, and Fazendinha stations, all impacting the Iguaçu River Basin. Consequently, the values of the parameters a , T , b , t , and c of the mathematical model recommended by Bernard (1932) were determined. Table 15 provides the precipitation model for each analyzed station and the determination coefficients obtained in this study.

On the other hand, figure 5 illustrates the IDF curves generated from the mathematical model for each analyzed station.

The results of this research show a good correlation between the generated IDF curves and the determination coefficient for a control period of 30 years, indicating the possibility of reliable forecasting to establish future IDF curves, considered essential tools for urban drainage projects.

According to the results obtained, it is possible to relate the increased rainfall events over time. Zanella (2006) analyzed the intense precipitation events and impacts generated in Curitiba/PR. The author found an increase in rainfall events in recent decades. In addition, the author mentions that these records are essential for analyzing impacts such as floods in Curitiba.

Da Silva et al. (2015) evaluated the spatial and temporal variability and trends of annual rainfall (1960-2000) of the Cobres-Portugal river

**Fig. 4.** Intensity-Duration-Frequency curves for Curitiba Station

basin using statistical tools. Statistical analysis showed that rainfall is highly variable and found a decrease in the rain over time, finding signs of a significant reduction in rainfall in the basin. However, some pluviometers showed increased rainfall during the last years analyzed.

Vu et al. (2018) constructed short-term IDF curves using a dynamic-statistical approach to assess the impacts of climate change in the Bac Ninh region of Vietnam. The authors found an increase in the rainfall intensity in the future. This increment was found for the years (2071–2100) ranging from 56 to 61% for a 10- and 100-year return period for the 24-hr duration. These results strongly suggest that severe flooding is likely in the future climate of the study region.

Rodríguez et al. (2014) evaluating the influence of climate change on the IDF curves of Barcelona (Spain), authors analyzed using data from six thermo-pluviometric stations located in the Barcelona metropolitan area, finding an increase of at least 4% in the expected daily precipitation with a return period greater than 20 years.

Cardoso et al. (2014) obtained the IDF curve equation for Lages-Brazil based on the annual series for an observation period of 10 years (2000-2009). The authors observed that the predominant rainfall temporal distribution for Lages was advanced, or convective, with higher rainfall in the first half of its duration. On the other hand, the authors mention that the Equation represented an essential contribution to the region, with the offer of updated services estimations on the maximum intensity of rainfall, making possible a more rational dimensioning of work in various hydrology-related topics.

Finally, it is important to mention that monitoring the historical and future IDF curves is essential. Previously, it implied monitoring of precipitation events under climate change. That is why studies like this are relevant since they work as a basis for later studies where it will be possible to see if the events predicted in this study are related to those that will happen in the future. The above is very important in urban areas such as Curitiba and São José dos Pinhais cities, where there may be a risk of flooding due to the topography and the extensive interconnections between different urban drainage systems. That is why urban drainage infrastructure systems must be designed based on precipitation events based on analyzing the data available in each basin. In addition, the adaptation of adequate strategies allows the dimensioning of hydraulic drainage structures of the different cities to be adequately projected.

Table 15. Parameter values K, m, n from other weather stations

Station - Municipality	Precipitation Model	Determination coefficient R ²
CURITIBA - Curitiba	$i = (340.1234 * T^{0.159569}) / t^{0.61639}$	0.9695
PRADOVELHO PUC - Curitiba	$i = (323.1693 * T^{0.147036}) / t^{0.61639}$	0.9737
CAMPO COMPRIDO SE COPEL - Curitiba	$i = (329.5146 * T^{0.129115}) / t^{0.61639}$	0.9785
ILHA DO RIO CLARO - São José dos Pinhais	$i = (389.7942 * T^{0.161624}) / t^{0.61639}$	0.9695
FAZENDINHA - São José dos Pinhais	$i = (300.5898 * T^{0.139463}) / t^{0.61639}$	0.9758

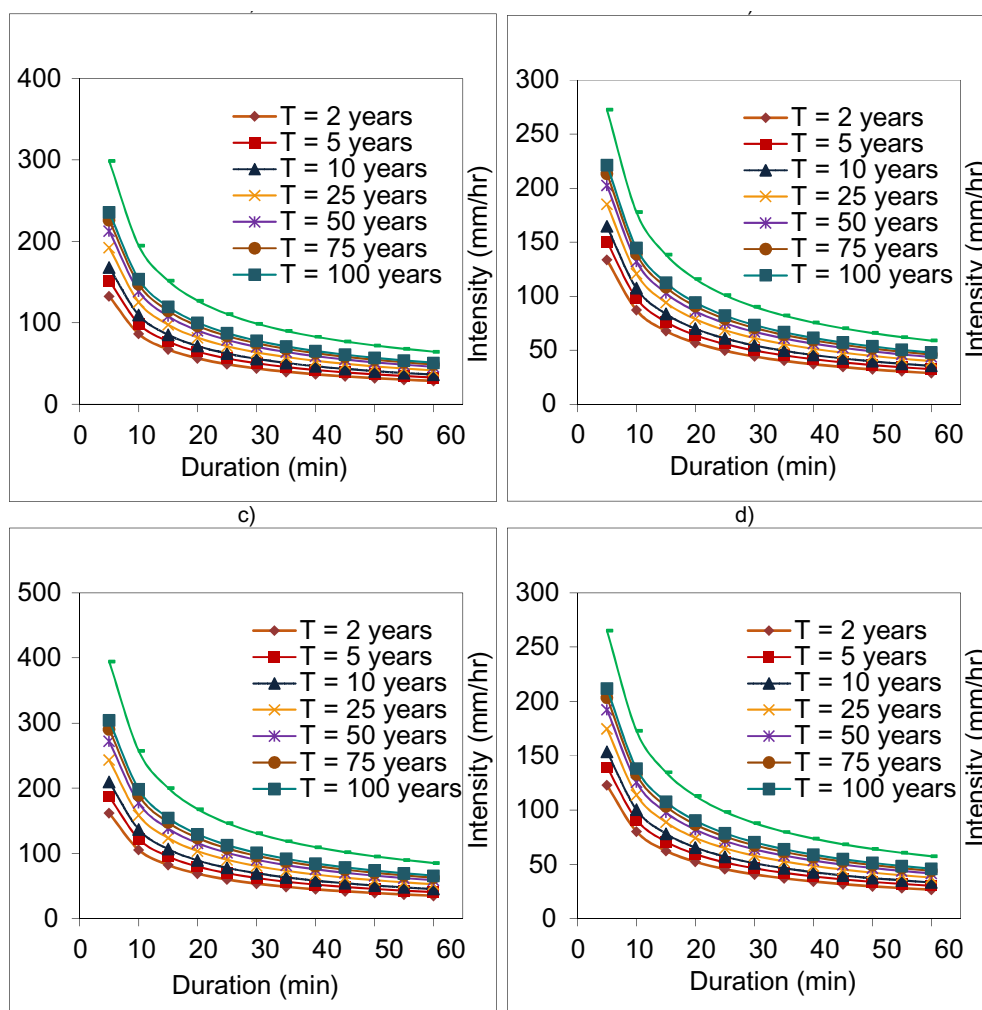


Fig. 5. Intensity-Duration-Frequency curves for stations a) Prado Velho, b) Campo Comprido, c) Ilha do Rio Claro, and d) Fazendinha

CONCLUSIONS

- The historical records showed that the Curitiba station presented the highest rainfall compared to the other stations analyzed, resulting in a more significant accumulated rainfall recorded in 24 hours, which was 146.2 mm for February 1990.
- According to the statistical analysis, it is concluded that there is a homogeneous distribution of the trend of the precipitation values in the sub-basins of the upper basin of the Iguaçu River, finding that the data of the five stations analyzed is stationary, independent, and equally distributed.
- The results reveal that the future IDF curves rose and accentuated over time. A possible climate change may occur in the future, which could be due to a meteorological phenomenon and the various factors that affect the climate.
- Finally, it was possible to obtain the parameters of the equations for calculating the intensity of precipitation events through different duration times and return periods. The verification of urban drainage structures in the cities of Curitiba and São José dos Pinhais is essential. On the other hand, the calculation of IDF curves contributes to studies on rainfall behavior in the upper basin of the Iguaçu River.

Acknowledgements: The authors are thankful to the Federal University of Technology Paraná and the financial support from the Coordination for the Improvement of Higher Education Personnel (CAPES). We also thank the anonymous reviewers for their valuable comments.

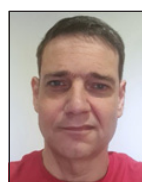
References

- Andrades, J.E., Torres Mantilla, H.A., López Hernández, J.Y. and Rojas Polanco, M. (2019) Análisis espacio temporal de la homogeneidad de estaciones de precipitación en una zona árida y semiárida del Centro Occidente de Venezuela. *Ciencia e Ingeniería*, v.40, pp.185–194.
- Beguería, S. (2005) Uncertainties in partial duration series modelling of extremes related to the choice of the threshold value. *Jour. Hydrol.*, v.303, pp.215–230. <https://doi.org/10.1016/j.jhydrol.2004.07.015>.
- Bernard, M.M. (1932) Formulas For Rainfall Intensities of Long Duration. *Trans. Amer. Soc. Civil Eng.*, v.96, pp.592–606. <https://doi.org/10.1061/taceat.0004323>.
- Boukhelifa, M., Meddi, M. and Gaume, E. (2018) Integrated Bayesian Estimation of Intensity-Duration-Frequency Curves: Consolidation and Extensive Testing of a Method. *Water Resour. Res.*, v.54, pp.7459–7477. <https://doi.org/10.1029/2018WR023366>.
- Campos Aranda, D.F. (1978) Cálculo de las curvas IDF, a partir de registros de lluvia máxima en 24 horas y Relaciones Duración LLuvia Promedio. Subdirección Regional Noreste de Obras Hidráulicas e Ingeniería Agrícola para el Desarrollo Rural., 1a Ed.
- Cardoso, C.O., Bertol, I., Soccol, O.J. and Sampaio, C.A.D.P. (2014) Generation of intensity duration frequency curves and intensity temporal variability pattern of intense rainfall for Lages/S.C. *Brazilian Archives Biol. Tech.*, v.57, pp.274–283. <https://doi.org/10.1590/S1516-89132013005000014>.
- Carvajal, L.F. and Grajales, D. (2019) Curvas Intensidad-Duración-Frecuencia no estacionarias para la cuenca del río Medellín. *Dyna*, v.86, pp.321–328.
- Da Silva, A. and Silva, R. (2017) Tendência Pluviométrica y Concentración Estacional de Precipitación en la cuenca Hidrográfica del Río Moxoto – Pernambuco – Brasil. *Revista Geográfica de América Central*, v.1, 295p. <https://doi.org/10.15359/rgac.58-1.12>.

- Da Silva, R.M., Santos, C.A.G., Macêdo, M.L.A., Silva, L.P.E. and De Macedo Machado Freire, P.K. (2013) Space-time variability of rainfall and hydrological trends in the Alto São Francisco River basin. *In: IAHS-AISH Proc. and Reports*, pp.177–182.
- Da Silva, R.M., Santos, C.A.G., Moreira, M., Corte-Real, J., Silva, V.C.L. and Medeiros, I.C. (2015) Rainfall and river flow trends using Mann–Kendall and Sen's slope estimator statistical tests in the Cobres River basin. *Natural Hazards*, v.77, pp.1205–1221. <https://doi.org/https://doi.org/10.1007/s11069-015-1644-7>.
- Duhan, D. and Pandey, A. (2013) Statistical analysis of long term spatial and temporal trends of precipitation during 1901-2002 at Madhya Pradesh, India. *Atmospheric Res.*, v.122, pp.136–149. <https://doi.org/10.1016/j.atmosres.2012.10.010>.
- Florentin, J.J., Abramowitz, M. and Stegun, I.A. (1966) Handbook of Mathematical Functions. *Amer. Mathemat. Monthly*, v.73, pp.1143, <https://doi.org/10.2307/2314682>.
- Grajales-Cardona, D. and Carvajal-Serna, L.F. (2019) Non-stationary intensity-duration-frequency curves for Medellín river basin. *DYNA (Colombia)*, v.86, pp.321–328, <https://doi.org/10.15446/dyna.v86n208.69300>.
- Gumbel, E.J. (2004) *Statistics of Extremes*, Dover Publ.
- Hosseinzadehtalaei, P., Tabari, H. and Willems, P. (2020a) Climate change impact on short-duration extreme precipitation and intensity–duration–frequency curves over Europe. *Jour. Hydrol.*, v.590, <https://doi.org/10.1016/j.jhydrol.2020.125249>.
- Hosseinzadehtalaei, P., Tabari, H. and Willems, P. (2020b) Satellite-based data driven quantification of pluvial floods over Europe under future climatic and socioeconomic changes. *Sci. Total Environ.*, v.721, <https://doi.org/10.1016/j.scitotenv.2020.137688>.
- INMET. (2009) Normais Climatológicas Do Brasil.
- Kendall, M.G. (1975) Rank Correlation Methods, *In: Griffin, C. (Ed.)*, 4th ed.
- Kite, G.W. (1988) Frequency and Risk Analysis. *In: Hydrology*. Publications, W.R. (ed.).
- Lopez de la Cruz, Jesús; Frances, F. (2014) La variabilidad climática de baja frecuencia en la modelación no estacionaria de los regímenes de las crecidas en las regiones hidrológicas Sinaloa y Presidio San Pedro. *Tecnología y Ciencias del Agua*, v.V, pp.79-101.
- Mann HB. (1945) Nonparametric tests against trend. *Econometrica*, v.13, pp.245–259.
- Molnar, P. (2011) 'Calibration'. *Watershed Modelling*. Institute of Environmental Engineering, Chair of Hydrology and Water Resources Management, ETH Zürich. Switzerland. *Watershed Modelling*.
- Mormeneo, I. and Castellví, F. (2001) Aplicación de un modelo en la generación de precipitaciones diarias. *Revista Brasileira de Agrometeorología*, Santa Maria, v.9, pp.311–315.
- Nash, J.E. and Sutcliffe, J.V. (1970) River Flow Forecasting Through Conceptual Models - Part I - A Discussion of Principles. *Jour. Hydrol.*, v.10, pp.282–290.
- Rodríguez, R., Navarro, X., Casas, M.C., Ribalaygua, J., Russo, B., Pouget, L. and Redaño, A. (2014) Influence of climate change on IDF curves for the metropolitan area of Barcelona (Spain). *Internat. Jour. Climatol.*, v.34, 643–654, <https://doi.org/10.1002/joc.3712>.
- Salas, J.D., Tabios, G.Q. and Smith, R.A. (1990) *Statistical Computer Techniques in Hydrology and Water Resources*. Colorado State University, Dept. of Civil Engineering, F.C. (ed.).
- Sansigolo, C.A. (2008) Distribuições de extremos de precipitação diária, temperatura máxima e mínima e velocidade do vento em Piracicaba, SP (1917-2006). *Revista Brasileira de Meteorologia*, v.23, pp.341–346, <https://doi.org/10.1590/s0102-77862008000300009>.
- Simard, R. and L'Ecuyer, P. (2011) Computing the two-sided Kolmogorov-Smirnov distribution. *Jour. Statist. Software*, v.39, pp.1–18. <https://doi.org/10.18637/jss.v039.i11>.
- Suárez-Aguilar, Z.E., Sepúlveda-Delgado, O., Patarroyo-Mesa, M. and Canaria-Camargo, L.C. (2020) Mathematical model to estimate curves of intensity, duration and frequency of extreme rains in Tunja, Colombia. *Informacion Tecnológica*, v.31, pp.193–206, <https://doi.org/10.4067/S0718-07642020000100193>.
- Subash, N., Singh, S.S. and Priya, N. (2011) Variability of rainfall and effective onset and length of the monsoon season over a sub-humid climatic environment. *Atmospheric Res.*, v.99, pp.479–487, <https://doi.org/10.1016/j.atmosres.2010.11.020>.
- Teixeira, C.F.A., Damé, R. de C.F. and Rosskoff, J.L.C. (2011). Intensity-duration-frequency ratios obtained from annual records and partial duration records in the locality of Pelotas -R.S., Brazil. *Engenharia Agrícola*, v.31, pp.687–694, <https://doi.org/10.1590/S0100-69162011000400007>.
- Vu, M.T., Raghavan, S. V., Liu, J. and Liong, S.Y. (2018). Constructing short-duration IDF curves using coupled dynamical–statistical approach to assess climate change impacts. *Internat. Jour. Climatol.*, v.38, pp.2662–2671, <https://doi.org/10.1002/joc.5451>.
- Weiss, L.L. (1964) Ratio of True to Fixed-Interval Maximum Rainfall. *Jour. Hydraulics Div.*, v.90, pp.77–82. <https://doi.org/10.1061/jycej.0001008>.
- Willner, S.N., Levermann, A., Zhao, F. and Frieler, K. (2018) Adaptation required to preserve future high-end river flood risk at present levels. *Sci. Adv.*, v.4, doi:10.1126/sciadv.aao1914.
- World Meteorological Organization. (2018) *Guía de Prácticas Climatológicas*, OMM-No. 100. World Meteorological Organization (ed.).
- Yan, H., Sun, N., Wigmosta, M., Skaggs, R., Hou, Z. and Leung, R. 2018. Next-Generation Intensity-Duration-Frequency Curves for Hydrologic Design in Snow-Dominated Environments. *Water Resources Research*, 54, 1093–1108, <https://doi.org/10.1002/2017WR021290>.
- Zanella, M.E. 2006. EVENTOS PLUVIMÉTRICOS INTENSOS E IMPACTOS GERADOS NA CIDADE DE CURITIBA/PR - BAIRRO CAJURU: um destaque para as inundações urbanas. *Mercator - Revista de Geografia da UFC*, 5, 61–69, <https://doi.org/10.4215/RM0000.0000.0000>.



Alvaro Javier Esteban Villota Mora, is a PhD student in Civil Engineering with a research line in the area of Sanitation and Water Resources at the Federal University of Technology Paraná (UTFPR), Brazil. His research interests cover wastewater treatments and applied hydrology.



Ronaldo Luis dos Santos Izzo received his BSc degree in Civil Engineering in 1993 from Universidade de Marília, Brazil. He is a professor of Geotechnical Engineering at the UTFPR, Brazil. His research interests cover ground improvement, pavement and geoenvironmental engineering.



Yeimy Ordoñez Muñoz, is a PhD student in Civil Engineering at the Federal University of Technology Paraná (UTFPR), Brazil. Her research interests cover geotechnical engineering, ground improvement and geo-environmental engineering.



Fernando Hermes Passig is a Sanitary Engineer from the Federal University of Santa Catarina (1993). He is a Professor at the UTFPR, Brazil. His research interests include sewage treatment, mainly in the removal of nitrogen and phosphorus, energy production with microbial fuel cells, and the use of biogas produced in anaerobic reactors.



Adalberto Matoski, Professor - UTFPR. Retired in 2023 and currently serving as a voluntary professor. He has experience in the field of Civil Construction, with emphasis on Construction Materials and Components, working mainly in the following areas: safety, engineering education, civil construction, and applied statistics.