Hybrid Method for the Estimation of Complex Natural Resonances using Cauchy and Vector Fitting

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***Abstract*—Here, we obtain the correct model order, the poles and the residues associated to a system identification for a transfer function using an hybrid algorithm that uses both the Cauchy and Vector Fitting method with a significant improvement in the validation stage.**

***Keywords*—Singularity Expansion Method, Cauchy's Method, complex natural resonance, Vector Fitting.**

I. INTRODUCTION

The Singularity Expansion Method (SEM) uses the Natural Resonance Frequencies as the key for system identification, object recognition and transfer function parameters extraction since 1971[1]. As a derivation of this method, the Cauchy Method starts treating the signal as a LTI transfer function as in Eq. 1. The parameters for the numerator and the denominator, that are, in other words, the poles and the residues for the system, can be extrapolated by solving a linear equation and estimating the model order[2].

On the other hand, Vector Fitting is based on a TLS accommodation of a previously defined set of poles, comparing the obtained signal samples to the original one[3].

The main idea in this paper is in this paper is to take advantage of both algorithms in order to improve the system identification process by combining the model order estimation and the initial poles from Cauchy, and then using the Vector Fitting TLS solution.

II. POLE EXTRACTION METHODS

Here, we present an overview of the Cauchy, Vector Fitting, and the hybrid method for Pole extraction and system identification.

A. Cauchy's Method

It makes the assumption that the response is completely LTI, thus, the transfer function can be written as in Eq. (1). Equation (2) can be used to replace coefficients a_k and b_k after having $[a]$, $[b]$ and $[C]$. The main idea is to find the solution to the linear system, which if often achieved through SVD applied to $[C]$. This matrix is reduced after a system order estimation, found with a threshold of the smallest singular value greater or equal to the number of significant decimals digits of data, as in Eq. (4), having w as the number of accurate significant decimal digits in the data from the response[2].

$$H(f_i) \approx \frac{A(f_i)}{B(f_i)} = \frac{\sum_{k=0}^{P} a_k f_i^k}{\sum_{k=0}^{P} b_k f_i^k}, \quad i = 1,2,\cdots N \tag{1}$$

$$\begin{bmatrix} A & -B \end{bmatrix}_{N\times(P+Q+2)} \begin{bmatrix} a \\ b \end{bmatrix}_{(P+Q+2)\times 1} = [C]_{N\times(P+Q+2)} \begin{bmatrix} a \\ b \end{bmatrix}_{(P+Q+2)\times 1} = 0 \tag{2}$$

$$[U][\Sigma][V]^H \begin{bmatrix} a \\ b \end{bmatrix} = 0 \tag{3}$$

Now, going back to Eq. () and considering the previous statements, one could utilize TLS approach to solve the equation and find the residues.

$$\frac{\sigma_R}{\sigma_{max}} = 10^{-w} \tag{4}$$

B. Vector Fitting

This model is based in Eq.(5), which includes the previous poles allocation of the function (whose quantity is the system order).,

$$\begin{bmatrix} \sigma(s) f(s) \\ \sigma(s) \end{bmatrix} = \begin{bmatrix} \sum_{n=1}^{N} \frac{c_n}{s-\bar{a}_n} + d + sh \\ \sum_{n=1}^{N} \frac{\tilde{c}_n}{s-\bar{a}_n} + 1 \end{bmatrix} \quad , \tag{5}$$

Where $\bar{a}_n$ represent the set of starting poles and $\sigma(s)$ is an unknown function with the same starting poles than $f(s)$.

Then, the idea is to have the linear system in Eq. (5) as in Eq.(6),

$$\sum_{n=1}^{N} \frac{c_n}{s-\bar{a}_n} + d + sh = \left(\sum_{n=1}^{N} \frac{\tilde{c}_n}{s-\bar{a}_n} + 1\right) f(s) \tag{6}$$

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Which can be solved through TLS as it is linear in all it unknowns.

C. *Hybrid Algorithm*

Vector Fitting accommodates the initial poles using a TLS and the basis shown in Eq. (6), yet it is unable to find an estimation for the system and this introduces errors for system identification and obstruct the automation process. Besides that, if the initial poles are not guessed well, the validation through reconstruction is not exact.

On the other hand, we have the Cauchy Method, whose model leads to the system's order determination and the extraction of poles (after manual elimination of incorrect ones).

The proposed method takes the order and the initial poles from Cauchy and uses that as the input for Vector fitting TLS section. The general steps for the algorithm are:

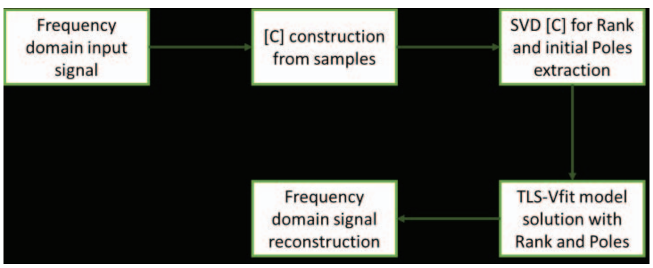

Fig. 1. General steps for the Hybrid algotihm, which begins with Cauchy matrix and then the TLS from Vector Fitting.

III. RESULTS AND COMPARISON

In order to validate the poles and residues extracted from the response signals, a 1 mm diameter and 100 mm length thin wire was simulated with CST. Figure (3) shows the frequency response, the reconstruction with the Cauchy method itself, and the reconstruction with the proposed method. Fig. (4) shows the S12 parameter obtained from two parabolic antennas of 0.9m radius in front door configuration, separated 100m.

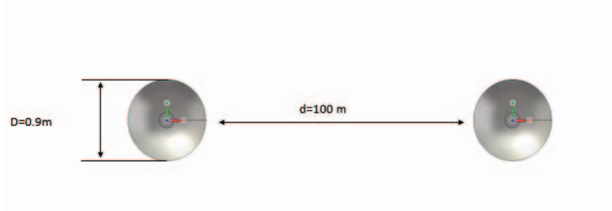

Fig. 2. Parabollic Antenna CST simulation setup.

In an effort to compare the validation of the function parameters (the quality of the reconstruction), for both methods, the R-squared statistical measure was used. Results are shown in table 1.

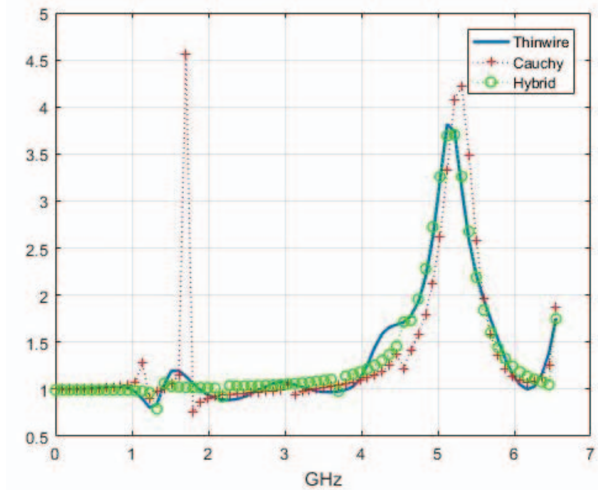

Fig. 3. Dipole frequency response and its reconstruction with Cauchy and Hybrid methods.

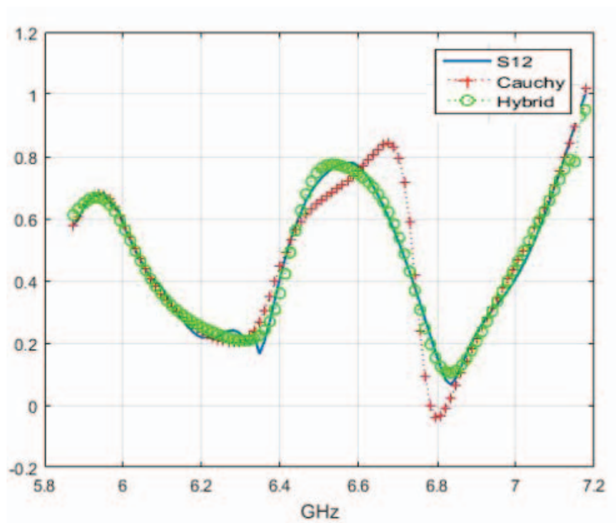

Fig. 4. Parabolic antennas S12 parameter and its reconstruction with Cauchy and Hybrid methods.

TABLE I. STATISTICAL MEASUREMENT COMPARISON

R^2		
	Cauchy Method	Hybrid Method
Antennas	0.8969	0.9844
Thin Wire	0.6343	0.9721

IV. CONCLUSIONS

The main contribution of this method is the CNR reliability increase for both Vector Fitting and Cauchy Methods. The validation shows a better reconstruction of the signals, and this hybrid method can be improved in the future by refining the general procedure, reducing computational noise in all the matrix operations, for example.

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