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Abstract: Pedestrian bridges, which are essential in urban and rural infrastructures, are vulnerable to vibrations induced by pedestrian traffic owing to their low mass, stiffness, and damping. This paper presents a novel predictive model of Human-Structure Interaction (HSI) that integrates a three-dimensional biomechanical model of the human body, and a pedestrian bridge represented as a simply supported Euler-Bernoulli beam. Using inverse dynamics, the human model accurately captures three-dimensional gait and its interaction with structural vibrations. The results show that this approach provides precise estimates of human gait kinematics and kinetics, as well as the bridge response under pedestrian loads. The incorporation of a three-dimensional human gait model reflects the changes induced by bridge vibrations, providing a robust tool for evaluating and improving the effect of structural vibrations on the properties and gait patterns.

Keywords: Human-Structure Interaction; three-dimensional biomechanical model; human gait; pedestrian bridges; inverse dynamics; structural vibrations; resonance



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1. Introduction

Pedestrian bridges are critical structures in urban and rural infrastructure, allowing the safe transit of pedestrians over obstacles, such as rivers or vehicular roads. However, these bridges exhibit high dynamic sensitivity owing to their intrinsic characteristics such as low mass, stiffness, and damping [1]. These characteristics result in low natural frequencies, generally below 10 Hz, which can coincide with the anthropic excitation frequencies generated by pedestrian traffic [2,3]. This coincidence of frequencies is known as resonance, which generates excessive vibrations that compromise the serviceability, safety, and comfort of pedestrians [4]. The most relevant cases of this problem have been observed in Millennium and Solferino pedestrian bridges, where vibrations induced by the synchronised movement of pedestrians generated significant concerns [5,6].

Following these events, research on Human-Structure Interaction (HSI) has grown significantly, and models have been proposed to evaluate the serviceability of these structures. The most widely used method is to represent a human as a periodic force that traverses the structure at a constant velocity [7]. In this case, it is convenient to model the anthropic loads by means of Fourier Series as a function of the frequency of passage, weight of the person, and Dynamic Load Factors (DLFs), which have been defined by experimental tests [8–10]. However, studies have shown that this approach generates an overestimation of up to 4 four times in the structural response compared to the experimental tests in the resonance condition [11]. This estimation error is attributed to the assumption of periodicity in anthropic loading and the omission of pedestrian interaction effects with structural vibrations [10,11].

The Mass Spring Damper (MSD) model has been proposed as a better representation of the HSI. In this model, dynamic loads are represented by modelling the pedestrian

as a single-degree-of-freedom (SDoF) system moving with constant velocity along the structure and taking a single point of contact [12]. This type of model has been widely studied and refined by adding degrees of freedom [13–16] and subsequently calibrated with experimental tests [14,17]. Nevertheless, one of its limitations is associated with the poor accuracy of the mass, stiffness, and damping parameters assumed for pedestrian systems. Several models for predicting these parameters have been proposed [13,18]; however, the high inter- and intra-subject variability makes the reliability of the results unfeasible [7]. On the other hand, the simplification of the MSD model into an sDoF means that the study of the effect of structural vibrations on the properties and gait patterns, known as structure to human interaction (S2HI), cannot be evaluated and are ignored in other studies [19]. The second method used by the authors consists of representing human gait by means of a mechanical inverted pendulum (IP) model coupled to the structural system [20], which has been sophisticated by incorporating the lateral swing condition and MSD systems for each component limb [11,21–24]. However, the limitation of this model is associated with the large number of assumptions required for its simulation, such as the control force regulating the stability for constant running added to the initial energy input which reduces the reliability of the prediction of GRFs [7].

In contrast to traditional gait models, in recent years, the authors have developed complete biomechanical models that include more Degrees of Freedom (DoF) based on specific anthropometric characteristics [25,26] shown that a realistic representation of human gait based on anthropometric characteristics improves the prediction of GRFs [27]. Anderson and Pandy [28] developed a biomechanical model of human gait based on optimization using the direct dynamics method. However, its limitation is the high CPU processing requirement with a simulation time of approximately 10,000 h. For this reason, the inverse dynamics method has become the main option for the development of human gait models based on optimization techniques added to the progress of novel numerical algorithms and robust hardware, positioning this method as a reliable, affordable, and cost-effective alternative for the simulation of human gait [26,29].

Many authors have restricted their studies on gait prediction to models in the sagittal plane and simplified the human body into fewer segments, owing to the complexity and high computational demands [25]. Some authors have utilised three-dimensional models, such as those in [25,26], which predict certain body movements based on experimentally known ones. Subsequently, [30] limited his model to the assumption of alternating leg contact and restriction of foot rotation in the coronal plane.

Although increasingly robust biomechanical models of human gait have been developed, such as [26], research has been limited to modeling human gait on static surfaces. For that reason, Aux et al. (2024) [31] developed a predictive model of vertical HSI composed of a seven-element biomechanical model and a simplified footbridge, analyzing, on the one hand, the kinematics and kinetics of gait including HSI phenomena and on the other hand the dynamic response of the footbridge. Nevertheless, this model was limited to gait in the sagittal plane and therefore the HSI analysis was only developed in the vertical direction. Consequently, it is evident that there is still a need to develop robust three-dimensional models capable of estimating the kinematics and kinetics of human gait under the effects of S2HI that allow further evaluation of anthropic loads and their implementation in HSI studies in lateral and vertical directions, which is a topic that is little addressed in the literature.

To address these gaps, this paper proposes a novel coupled HSI predictive model composed of a complete three-dimensional biomechanical model of the human body and a pedestrian bridge represented as a simply supported Euler-Bernoulli beam. The main novelties and contributions of this work are as follows: (1) The incorporation of anthropic loads defined by a realistic three-dimensional human gait model as a source of excitation for HSI, based on inverse dynamics, and able to represent the changes in gait resulting from surface vibrations (S2HI). (2) A simplified method is proposed to evaluate the three-dimensional HSI that considers the changes in the dynamic properties of the structure under anthropic

excitation under HSI effects. A description and formulation of the three-dimensional human gait model are presented in Section 2, followed by the proposed structural system in Section 3. Section 4 describes the simplified HSI formulation, and Section 5 presents the optimization scheme for the coupled HSI based on the minimization of the total mechanical energy of the pedestrian moving through the structure. Finally, Section 6 presents and discusses the results.

2. 3D Human Model

The human body was modeled as a system composed of thirteen rigid elements (body segments), connected by joints that allow free rotation in the three body planes (sagittal, frontal and coronal), Figure 1a (Table 1). A set of fifth-order Fourier series was used to represent the segmental rotations (Equation (1)), where ω_w is the walking frequency, t is the time, and a_0 , $a_n^{(i)}$, and $b_n^{(i)}$ are the coefficients associated with the series.

$$[\theta_i, \varphi_i] = a_0 + \sum_{n=1}^5 a_n^{(i)} \cos(n\omega_w t) + b_n^{(i)} \sin(n\omega_w t) \quad (1)$$

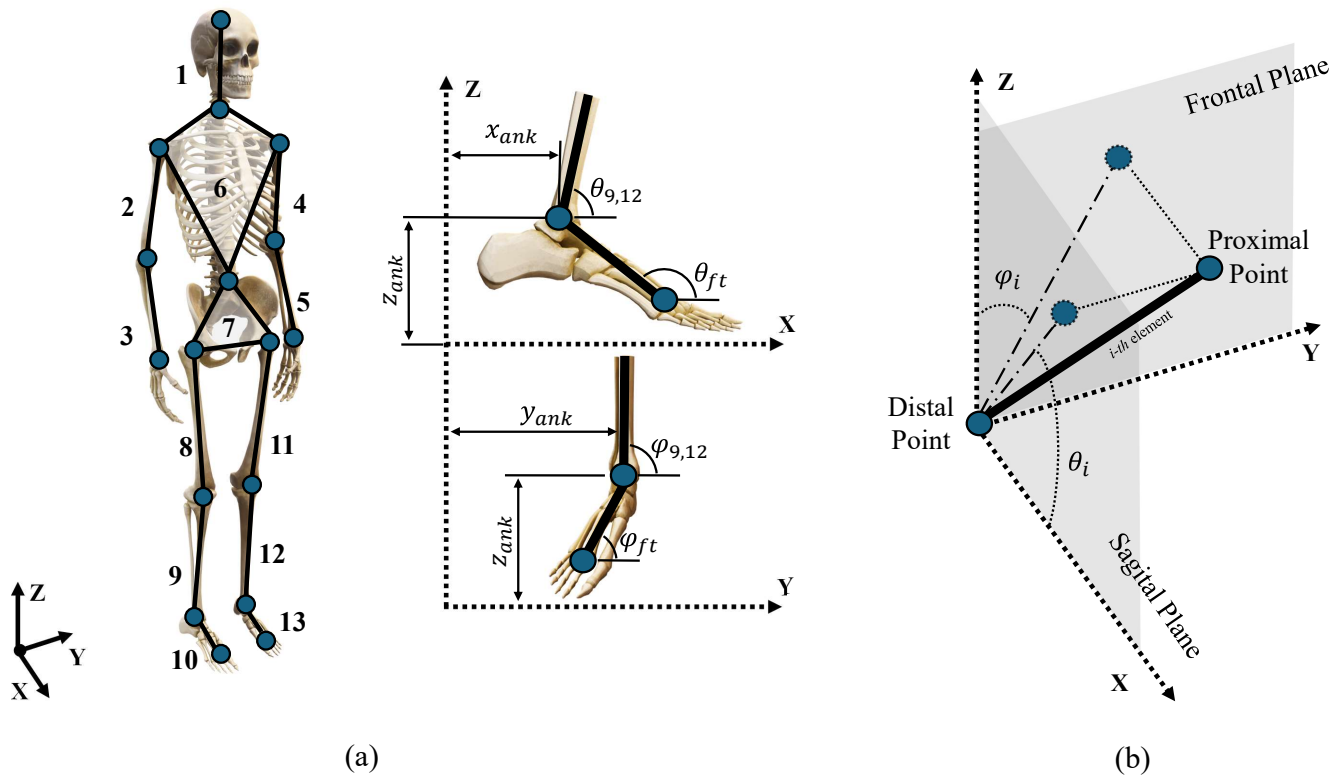


Figure 1. Human body description. (a) Segments body ID, (b) Spherical coordinate i th body segment.

Table 1. Segments body and joints included in the three human model.

ID *	Segment Body	Distal Joint	Proximal Joint
1	Head	Neck	Skull
2	Right Upper arm	Elbow	Shoulder
3	Right Forearm	Wrist	Elbow
4	Left Upper arm	Elbow	Shoulder
5	Left Forearm	Wrist	Elbow
6	Trunk	Lumbosacral	Neck
7	Pelvis	Hip	Lumbosacral
8	Right Thigh	Knee	Hip
9	Right Shank	Ankle	Knee
10	Right Foot	Rollover	Ankle

Table 1. Cont.

ID *	Segment Body	Distal Joint	Proximal Joint
11	Left Thigh	Knee	Hip
12	Left Shank	Ankle	Knee
13	Left Foot	Rollover	Ankle

* ID is the identification Segment body in Figure 1a.

2.1. Kinematics

A spherical coordinate system was used to determine the positions of the body segments throughout the gait cycle. The orientation of each element was defined by the angles θ_i and φ_i with respect to the global reference system, as shown in Figure 1b. The Inverse Dynamics methodology was used to determine the displacements of each joint center as a function of the angles θ_i and φ_i (Equation (2)), where x_{ank} , y_{ank} , and z_{ank} correspond to the positions of the ankle on the x, y , and z reference axes, respectively, Figure 2. However, L_i corresponds to the length of the i th element ($i = 1, 2, 3, \dots, 13$), and $I(j)$ corresponds to a sign function, which is equal to one when the body segment corresponds to the leg in contact and zero when it is in swing. By deriving Equation (1) twice, the accelerations at the joint centers are obtained (Equation (3)), where $\dot{\theta}_j$, $\ddot{\theta}_j$ and $\dot{\varphi}_j$, $\ddot{\varphi}_j$ are the angular velocities and accelerations of the j th element ($j = 13, 12, 11, \dots, 1$), respectively.

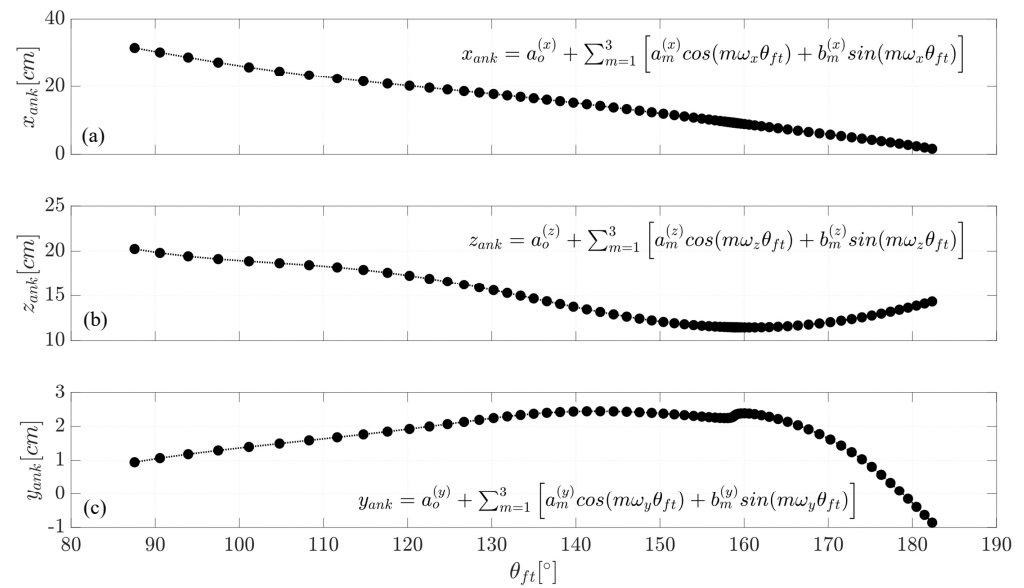


Figure 2. Ankle–foot kinematics during stance phase. (a) x coordinate of ankle joint, (b) z coordinate of ankle joint and (c) y coordinate of ankle joint.

$$\begin{aligned}
 x_i &= x_{ank} + \sum_{j=1}^i (I(j) L_j \cos \theta_j \cos \varphi_j) \\
 z_i &= y_{ank} + \sum_{j=1}^i (I(j) L_j \sin \theta_j \cos \varphi_j) \\
 y_i &= z_{ank} + \sum_{j=1}^i (I(j) L_j \sin \varphi_j)
 \end{aligned}
 \tag{2}$$

$$\begin{aligned}
\ddot{x}_i &= \ddot{x}_{ank} + \sum_{j=1}^i \left\{ -I(j)I_j \left[\cos\theta_j \left(\ddot{\phi}_j \sin\varphi_j + \dot{\phi}_j^2 \cos\varphi_j \right) - 2\dot{\phi}_j \sin\varphi_j \dot{\theta}_j \sin\theta_j + \cos\varphi_j \left(\ddot{\theta}_j \sin\theta_j + \dot{\theta}_j^2 \cos\theta_j \right) \right] \right\} \\
\ddot{y}_i &= \ddot{y}_{ank} + \sum_{j=1}^i \left\{ -I(j)I_j \left[\sin\theta_j \left(\ddot{\phi}_j \sin\varphi_j + \dot{\phi}_j^2 \cos\varphi_j \right) + 2\dot{\phi}_j \sin\varphi_j \dot{\theta}_j \cos\theta_j + \cos\varphi_j \left(\ddot{\theta}_j \cos\theta_j + \dot{\theta}_j^2 \sin\theta_j \right) \right] \right\} \\
\ddot{z}_i &= \ddot{z}_{ank} + \sum_{j=1}^i \left\{ I(j)I_j \left[\varphi_j \cos\varphi_j - \dot{\phi}_j^2 \sin\varphi_j \right] \right\}
\end{aligned} \quad (3)$$

2.1.1. Ankle-Foot Rollover Angle

The kinematics of the foot during gait has been well studied by several authors [29,32–34]. However, they are rather complex and limited to the sagittal plane. In that sense, for foot kinematics, ankle joint displacement functions (x_{ank}, z_{ank}) defined by Ren et al. (2007) [29] are proposed (Figure 2). On the other hand, in the case of ankle kinematics in the y -direction, it was determined under the assumption that the lateral acceleration of the ankle should have a behavior similar to the total body acceleration in this direction during the contact phase. The acceleration was numerically integrated to determine the position and then approximated to a Fourier series showing the trajectory of the ankle joint center as a function of the foot Rollover angle θ_{ft} (Figure 2c) (Table 2). The positions (x_i, y_i, z_i) and accelerations $(\ddot{x}_i, \ddot{y}_i, \ddot{z}_i)$ of the joint centres were determined starting at the ankle $(x_{ank}, y_{ank}, z_{ank})$ up to the i th element (Equation (2)).

Table 2. Fourier Series Coefficients of Ankle-foot Kinematics.

	a_0	a_1 [m]	b_1 [m]	a_2 [m]	b_2 [m]	a_3 [m]	b_3 [m]	ω_{ft} [1/rad]
x_{ank}	−7.896	−3.083	4.669	0.805	12.09	2.848	−0.696	0.800
z_{ank}	0.206	0.005	−0.012	−0.002	0.090	0.026	−0.006	1.814
y_{ank}	−0.007	0.034	−0.026	0.000	0.020	−0.004	−0.005	0.938

2.2. Kinetics

The inverse dynamics method was used to calculate the joint kinetics and mechanical energy expenditure during a gait cycle based solely on body kinematics and surface vibration $\vec{A}_g(t)$, and the equations of motion of the i th segment of the body are presented in Equation (4), where m_i is the mass of each body segment, $\vec{a}_i$ is the acceleration in center of mass of i th element and $\vec{g}$ is gravity. Here, the use of the upper right arrow ($\vec{}$) for the variables $\vec{A}_g(t)$, $\vec{g}$ and $\vec{a}_i$ indicates that each of these has components in the three dimensions of the x , y , and z Cartesian planes. Specifically, $\vec{F}_i$ contains the ground reaction forces GRFs in the lateral, vertical and longitudinal directions.

$$\sum \vec{F}_i = \sum_{i=1}^{13} m_i \left(\vec{a}_i - \vec{g} + \vec{A}_g(t) \right) \quad (4)$$

During the swing phase, only one foot is in contact with the ground, either right or left; therefore, the ground reactions (forces, moments, and Center of Pressures (COPs)) acting only on the supporting foot can be calculated directly from Equation (4). However, in the double-support phase, the ground reactions are indeterminate. To solve this problem, the linear transfer assumption $D(t)$ proposed by [29] was used to model the transfer of forces reactions from one foot to another during the double support phase. Finally, the mechanical energy is defined as the power integral during the time taken by the pedestrian to cross the structure, as shown in Equation (5), where T_i represents the net

torque of the i th element and $\omega_p^{(i)}, \omega_d^{(i)}$ represents the angular velocity of each joint's proximal and distal elements.

$$E_m = \int_0^L \sum_{i=1}^{13} |T_i (\omega_p^{(i)} - \omega_d^{(i)})| dt \quad (5)$$

3. Structural System

Walking forces are applied in the vertical and lateral directions to the surface. The frequencies of these loads are directly related to the step frequency f_w , which is approximately 2 Hz [35]. The vertical and lateral loading frequencies were 2 Hz and 1 Hz, respectively [4]. In this sense, the implemented pedestrian bridge was selected such that during the HSI evaluation, a critical resonance condition was generated in the first vibration modes in the lateral and vertical directions, which consisted of a supported standard IPE beam with a constant section of length (L_{br}) 30m, Figure 3. The physical and mechanical properties of these structures are listed in Table 3. Three vertical and lateral vibration modes were considered, as shown in Figure 3. The frequencies ($f_{\hat{n}}$) for each $\hat{n}th$ vibration mode was calculated using the Equation (6) and presented in Table 4.

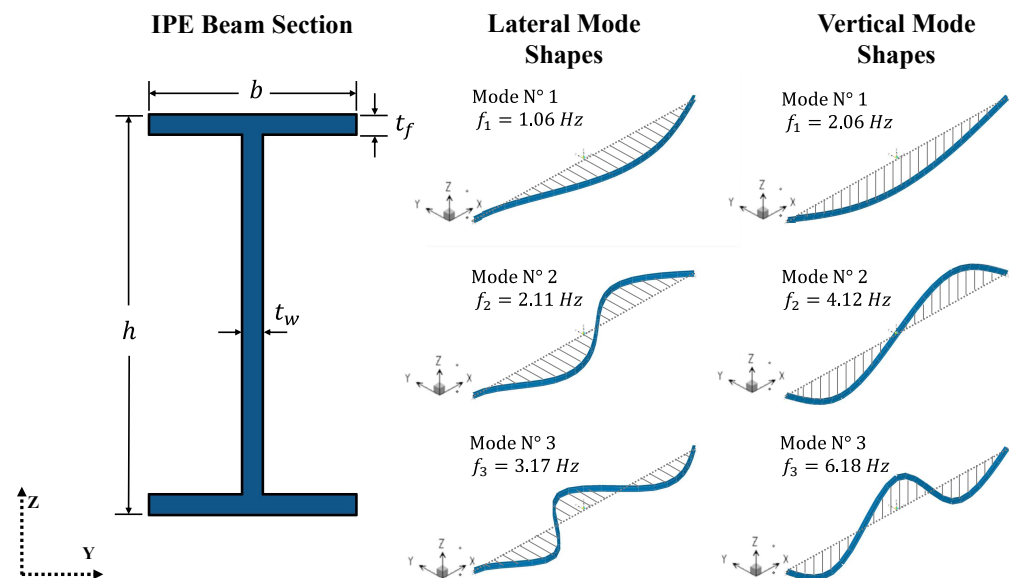


Figure 3. Pedestrian Bridge Structure. IPE Cross Section (left) and Mode shapes (right).

Table 3. Mechanical Properties of Structural System. Flange (b), Depth (h), Thickness of Web (t_w), Thickness of Flange (t_f), Length beam (L_{br}), mass per unit length ($\bar{m}$), Elasticity Modulus (E), Inertia in local axis 1–1 (I_y) and Inertia in local axis 2–2 (I_z).

Section	b [m]	h [m]	t_w [m]	t_f [m]	L_{br} [m]	$\bar{m}$ [kg/m]	E [GPa]	I_y [cm ⁴]	I_z [cm ⁴]
IPE	0.3	0.58	0.01	0.01	30	91.06	200	633.75	166.85

Table 4. Dynamic Properties of Structural System.

	f_1 [Hz]	f_2 [Hz]	f_3 [Hz]	ζ [%]
Vertical	2.06	4.12	6.18	1.00
Lateral	1.06	2.11	3.17	1.00

$$f_{\hat{n}} = \frac{(\pi \hat{n})^2}{2\pi} \sqrt{\frac{EI}{\bar{m} L_{br}^4}} \quad (6)$$

where E , I , L_{br} , and $\bar{m}$ represent the modulus of elasticity of steel, moment of inertia of the section, and length and mass per unit length of the structure, respectively. The modal shapes are described by the Equation (6) and presented in Figure 3.

$$\phi_{\hat{n}}(x) = \sin\left(\frac{\hat{n}\pi}{L_{br}}x\right) \quad (7)$$

4. HSI Formulation

A fourth-order partial differential equation (PDE) describes the deflection $u(x, t)$ at a point on the x axis of the beam at a given instant t (Equation (8)), where $p(x, t)$ represents the load generated by a pedestrian traversing the structure, M_{br} represents the mass of the structure, and c represents the coefficient of the viscous damper.

$$\frac{\partial^2}{\partial x^2} \left[EI(x) \frac{\partial^2 u(x, t)}{\partial x^2} \right] + m(x) \frac{\partial^2 u(x, t)}{\partial x^2} + c(x) \frac{\partial u(x, t)}{\partial t} = p(x, t) \quad (8)$$

Equation (8) was solved using the separation-of-variables method, which incorporates modal decomposition, as shown in Equation (9). In this context, the vibration modes are described by sinusoidal functions, as expressed in Equation (7). Equation (10) describes the motion ($Y_{\hat{n}}$) of the $\hat{n}th$ vibration mode ($\phi_{\hat{n}}$) in modal coordinates, where the natural frequency ($\omega_{\hat{n}} = 2\pi f_{\hat{n}}$) and damping ratio (ζ) are defined using the parameters listed in Table 4. The mass ($M_{\hat{n}}$) and modal loads ($P_{\hat{n}}$) were determined using Equations (11) and (12), respectively.

$$u(x, t) = \sum_{\hat{n}=1}^{\infty} \phi_{\hat{n}}(x) Y_{\hat{n}}(t) \quad (9)$$

$$\ddot{Y}_{\hat{n}}(t) + 2\zeta\omega_{\hat{n}}\dot{Y}_{\hat{n}}(t) + \omega_{\hat{n}}^2 Y_{\hat{n}}(t) = \frac{P_{\hat{n}}(t)}{M_{\hat{n}}} \quad (10)$$

$$M_{\hat{n}} = \int_0^{L_{br}} [\phi_{\hat{n}}(x)]^2 m(x) dx \quad (11)$$

$$P_{\hat{n}}(t) = \int_0^{L_{br}} \phi_{\hat{n}}(x) p(x, t) dx \quad (12)$$

Drawing from the equations that detail the kinematics and kinetics of human gait, as outlined in Section 2, the human body and the structure interact and hence are dynamically connected. This connection was established by incorporating the force generated by walking as a function of time and position in the human system, as shown in Equation (13). The transition of the left and right foot forces, $D(t)$, is defined as a function that is normalized to a range of 0 to 1. In single-stance zones, where only one foot is in contact with the ground, function $D(t)$ is equal to 1 during the Begin (BGN) to Heel Contact Right (HCR) and Toe Off Left (TOL) to Heel Contact Left (HCL) periods. However, in the double-stance zones, specifically in the shaded region between the HCR to TOL and HCL to TOR, $D(t)$ changes linearly from 0 to 1 and from 1 to 0, respectively.

$$p(x, y, t) = \left[(1 - D(t)) \sum_{i=1}^{13} m_i \left(\vec{a}_i(t) - \vec{g} + \vec{A}_g(t) \right) \right] \delta(x - x_L(t)) + \left[D(t) \sum_{i=1}^{13} m_i \left(\vec{a}_i(t) - \vec{g} + \vec{A}_g(t) \right) \right] \delta(x - x_R(t)) \quad (13)$$

$$\begin{aligned} x_L(t) &= x'_L(t) + CoP_l(t) \\ x_R(t) &= x'_R(t) + CoP_r(t) \end{aligned} \quad (14)$$

In Equation (13), the left and right feet are represented by $x'_L(t)$ and $x'_R(t)$, respectively, and the pressure centers for the left and right feet are represented by $CoP_l(t)$ and $CoP_r(t)$, respectively, in Equation (14). The accelerations induced by the structure owing to the human gait are represented by Equation (15), where $\vec{A}_g(t)$ is calculated based on the positions of the feet and the vertical modes of vibration. The applied force of human

walking in modal coordinates was obtained by combining Equations (12) and (13), resulting the Equation (16).

$$A_g(t) = \sum_{\hat{n}=1}^3 \left[(1 - D(t)) \sin\left(\frac{\hat{n}\pi x_L(t)}{L_{br}}\right) + D(t) \sin\left(\frac{\hat{n}\pi x_R(t)}{L_{br}}\right) \right] \ddot{Y}_{\hat{n}}(t) \quad (15)$$

$$P_{\hat{n}}(t) = - \left[F_L(t) \sin\left(\frac{\hat{n}\pi x_L(t)}{L_{br}}\right) + F_R(t) \sin\left(\frac{\hat{n}\pi x_R(t)}{L_{br}}\right) \right] - \left[(1 - D(t)) m_p \sin\left(\frac{\hat{n}\pi x_L(t)}{L_{br}}\right) + D(t) m_p \sin\left(\frac{\hat{n}\pi x_R(t)}{L_{br}}\right) \right] \ddot{A}_g(t) \quad (16)$$

$$\ddot{Y}_{\hat{n}}(t) + \frac{m_p}{M_{\hat{n}}} S_{\hat{n}}(t) \left[\sum_{m=1}^3 S_m(t) \ddot{Y}_{\hat{n}}(t) \right] + 2\zeta_{\hat{n}} \omega_{\hat{n}} \dot{Y}_{\hat{n}}(t) + \omega_{\hat{n}}^2 Y_{\hat{n}}(t) = - \frac{1}{M_{\hat{n}}} \left[F_L(t) \sin\left(\frac{\hat{n}\pi x_L(t)}{L_{br}}\right) + F_R(t) \sin\left(\frac{\hat{n}\pi x_R(t)}{L_{br}}\right) \right] \quad (17)$$

where,

$$S_{\hat{n}}(t) = (1 - D(t)) \sin\left(\frac{\hat{n}\pi x_L(t)}{L_{br}}\right) + D(t) \sin\left(\frac{\hat{n}\pi x_R(t)}{L_{br}}\right)$$

$$F_L(t) = (1 - D(t)) \sum_{i=1}^{13} m_i (a_i(t) - g)$$

$$F_R(t) = D(t) \sum_{i=1}^{13} m_i (a_i(t) - g)$$

Here, $F_L(t)$ and $F_R(t)$ represent the forces generated by walking on the left and right feet, respectively, without considering the influence of acceleration on the structure. The total mass of the human system is denoted as $m_p = \sum_{i=1}^{13} m_i$. By substituting Equation (16) into Equation (10), the modal coordinate equation for solving the HSI model, given by Equation (17), can be expressed in matrix form, as shown in Equation (18). In this equation, M_{br} represents the modal mass, C represents damping, and K represents the stiffness of the human structural system.

$$\left[M_{br} + m_p S^T(t) S(t) \right] \ddot{Y}(t) + C \dot{Y}(t) + K Y(t) = -\Phi(x_L(t)) F_L(t) - \Phi(x_R(t)) F_R(t) \quad (18)$$

5. Optimization Scheme

In the HSI optimization and simulation process, initial gait characteristics such as the average walking speed V_w , walking frequency f_w , and double support duration. The interior-point optimization algorithm with the Matlab R2024a *fmincon* function were implemented on a computer with the following characteristics: 64-bit Windows 11 Pro operating system version 22H2, Intel Core i9 13900 K processor @3.0 GHz and 128.0 GB RAM (Dell Technologies, Cali, Colombia). The objective function is used to find the coefficients ($a_0, a_n^{(i)}$, and $b_n^{(i)}$) that describe the trajectories of the body segments that achieve specified gait parameters, while minimizing the mechanical energy expenditure (E_m) and satisfying the constraints presented in Section 5.1.

5.1. Constrains

Linear constraints are associated with defining joint limits, hyper-extensions, and unrealistic movements for a normal gait, Table 5. The angular range of motion is defined in Equation (19), where the limits $\theta_{min}^{(i)}$, $\theta_{max}^{(i)}$, $\varphi_{min}^{(i)}$, and $\varphi_{max}^{(i)}$ are defined as 70% of the human body maxima defined in [36]. Similarly, the relative angles for the knee and elbow joints centers are presented in Equation (20).

$$\theta_{min}^{(i)} \leq \theta_i(t) \leq \theta_{max}^{(i)} \text{ and } \varphi_{min}^{(i)} \leq \varphi_i(t) \leq \varphi_{max}^{(i)}, \quad t \in [0, T_w] (i = 1, 2, \dots, 13) \quad (19)$$

$$\theta_{min}^{(1)} \leq \theta_2(t) - \theta_3(t) \leq \theta_{max}^{(1)}, \theta_{min}^{(2)} \leq \theta_9(t) - \theta_8(t) \leq \theta_{max}^{(2)}, t \in [0, T_w] \quad (20)$$

Table 5. Limit range of absolute angular displacements at joints.

Joint	Flexion		Abduction	
	$\theta_{min}^{(i)} [^\circ]$	$\theta_{max}^{(i)} [^\circ]$	$\varphi_{min}^{(i)} [^\circ]$	$\varphi_{max}^{(i)} [^\circ]$
Neck	60	120	50	130
Elbow	45	135	50	180
Wrist	45	135	50	180
Lumbosacral	60	120	75	105
Hip	60	120	75	105
Knee	30	180	75	105
Ankle	30	135	40	120
Metatarsal	45	200	65	125
Elbow *	35	180	-	-
Knee *	55	180	-	-

* Relative angular displacement at joints.

For the nonlinear constraints it is established that the foot contact ($y_{tip}(t)$) when there is contact with the ground must be equal to zero ($y_{tip}(t) = 0$). In contrast, during the swing phase, the contact must be greater than zero ($y_{tip}(t) > 0$). The second nonlinear constraint states that vertical force $F_z(t)$ is always greater than zero ($F_z(t) > 0$). The third constraint defines the relationship based on the coefficient of friction (μ), as shown in Equation (21). Finally, the last constraint defines the pitch length as $x_{ank}(T_w) - x_{ank}(0) = V_a T_w$, where $T_w = 2\pi/\omega_w$ is the walking period, x_{ank} is the position of the ankle in the x coordinate, and V_w is the walking speed.

$$\frac{\sqrt{F_x^2 + F_y^2}}{F_z} < \mu \quad (21)$$

5.2. Simulation

Human gait is considered as a process of minimum mechanical energy. In this regard, researchers such as Ren et. al. (2007) [29] determined normal gait patterns by optimization, starting from an upright position without walking. This approach was implemented in this study, for which the coefficients X_0 correspond to the result of an optimization in which the human starts from a static position.

Figure 4 presents the optimization scheme implemented in the HSI optimization, which starts by defining anthropic parameters and gait conditions. Then, the initial coefficients X_0 ($a_0^{(i)}$, $a_n^{(i)}$ and $b_n^{(i)}$) used as the starting point of the optimization are defined. These initial coefficients are adjusted in successive iterations (X) by the optimization function to reduce the mechanical energy expenditure (**Step 1**).

In **Step 2**, the X coefficients are evaluated using Equation (1), which determines the angular movements of each body segment (θ_i and φ_i). In **Step 2.1**, the Ankle-Foot Rollover was calculated using the process detailed in Section 2.1.1. The kinematics of the human body are then calculated using the methodology presented in Section 2.1, obtaining the displacements (x_i , y_i , z_i) and accelerations ($\ddot{x}_i$, $\ddot{y}_i$, $\ddot{z}_i$) for each body segment.

Step 3 involves the calculation of gait kinetics starting from the previously calculated kinematics and using the inverse dynamics methodology, as described in Section 2.2, to get the ground reaction forces GRFs (F_x , F_y and F_z). After, in **Step 4**, the bridge properties are defined and used to calculate the structural system (**Step 4.1**) as described in Section 3. Subsequently, taking as input (1) the structural system, (2) the kinematics (contact position), and (3) the kinetics (GRFs) of the pedestrian, the dynamic response of the coupled HSI structural system is calculated following the methodology presented in Section 4.

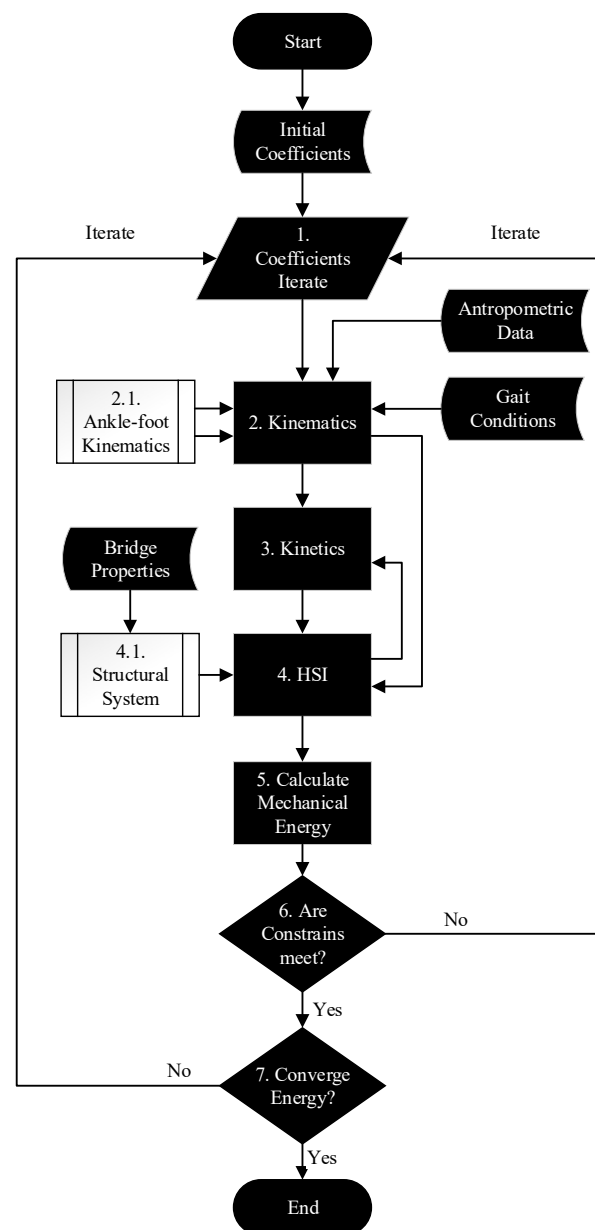


Figure 4. Optimization Scheme.

The mechanical energy during the entire time interval was determined and checked in **Step 5** (Equation (5)) to determine if the predefined constraints were met. If the constraints are not met, the process is repeated by changing coefficient X (**Step 6**). However, if the constraints are met, the process proceeds to verify if the solution converges, and if convergence has not been achieved, the process iterates again from the definition of the X_0 coefficients, adjusting them as necessary, and repeating the entire analysis. Finally, in **Step 7**, the iterative cycle continues until the constraints are met and convergence is achieved, at which point the optimization process is terminated, thus achieving an optimal solution for the system under analysis.

6. Results and Discussion

6.1. Single Pedestrian HSI Analysis

An 80 kg male subject walking along the reference structure with the anthropometric characteristics described in Table 6 was implemented for all simulation cases. The subject moves with a step length of 1.5 m at step frequency $f_w = 2$ Hz and double support duration between 33 and 50% for each gait cycle.

Table 6. Anthropometric data. Adapted from [37].

	Mass [%]	Length [mm]	* CoM [%]	Ratio Sagittal	Ratio Transverse	Ratio Frontal
Head	6.94	203.3	50.02	30.3	26.1	31.5
Upper arm	2.71	281.7	57.72	28.5	15.8	26.9
Forearm	1.62	268.9	67.51	43.88	12.31	42.93
Trunk	32.29	386.2	49.85	33.74	23.4	29.1
Hip	11.17	145.7	38.85	61.5	58.7	55.1
Thigh	14.16	422.2	40.95	32.9	14.9	32.9
Shank	4.33	434	43.95	25.1	10.2	24.6
Foot	1.37	190	44.15	25.7	24.5	12.4

* CoM is the Center of Mass of the element from distal to proximal point.

The first simulation step was to determine a normal gait pattern without considering ground vibration (rigid surface), that is, $A_g(t) = 0$. Here, the coefficients found were implemented as the starting point X_0 in the model of this study. An HSI simulation was performed considering a single pedestrian, for which, after 67 iterations and 712 s of optimization, the model found the minimum at $E_m = 196.06$ J. The coefficients a_n and b_n solutions of the optimization problem are presented in Table 7. The coefficients represent the adjusted parameters of the biomechanical model for each body segment during gait on the structure considering the HSI effects. These coefficients, derived from the Fourier series in Equation (1), describe the segmental rotations throughout the gait cycles and was used to calculate the forces on each joint using inverse dynamics.

Table 7. Optimized Coefficients for HSI.

Element	a_0	a_1	b_1	a_2	b_2	a_3	b_3	a_4	b_4	a_5	b_5
Head	1.503	−0.022	−0.069	−0.005	0.022	−0.001	0.011	−0.063	0.005	0.017	0.002
Upper arm	1.709	0.196	−0.058	0.036	0.029	−0.002	−0.265	−0.027	−0.013	0.001	0.018
Forearm	1.81	0.195	−0.058	0.036	0.029	−0.002	−0.264	−0.027	−0.013	0.001	0.018
Trunk	1.492	−0.022	−0.069	−0.005	0.022	−0.001	0.011	−0.063	0.005	0.017	0.002
Hip	1.513	0.005	−0.048	0.007	0.03	0.002	−0.014	−0.08	−0.003	0.008	−0.002
Thigh	1.727	0.223	−0.058	0.016	−0.008	0.002	−0.234	0.013	0.005	−0.004	−0.002
Shank	1.282	0.381	−0.108	−0.04	0.027	−0.004	−0.232	−0.082	0.069	−0.006	−0.005
Foot	2.478	0.59	−0.273	0.041	0.007	−0.007	−0.228	−0.128	0.117	−0.038	0.007
Head	1.275	0.01	0.007	0	0.000	0.000	−0.026	−0.012	0.006	0.002	0.000
Upper arm	−4.353	0.03	0.008	0.003	−0.001	−0.001	−0.005	−0.012	0.001	0.001	−0.001
Forearm	−4.353	0.032	0.008	0.003	−0.001	−0.001	−0.005	−0.012	0.002	0.001	0.001
Trunk	0.069	0.327	0.061	0.076	0.038	−0.004	−0.924	−0.001	0.091	0.019	0.04
Hip	0.074	0.333	0.096	0.098	0.041	−0.008	−0.994	−0.043	0.098	0.034	0.045
Thigh	1.738	−0.001	−0.016	0.005	−0.025	−0.003	0.084	0.037	−0.016	−0.011	0.001
Shank	1.675	0.016	−0.026	0.008	−0.022	−0.003	0.043	0.029	−0.012	−0.017	−0.002
Foot	−0.278	0.011	−0.026	0.01	−0.023	−0.004	0.036	0.035	−0.016	−0.016	−0.002

The ground reaction forces (GRFs) in the vertical and lateral directions as the pedestrian traverses the bridge are shown in Figures 5 and 6, respectively. The percentage differences for the GRFs, comparing the HSI with respect to those on a rigid surface, were calculated as the Root Mean Square Error (RMSE) over 10% intervals of the bridge span, as shown in Figure 7. As expected, the highest RMSE are found around the mid-span of the bridge, where the accelerations $\vec{A}_g(t)$ are greater by 63.7% in the vertical direction and 23.42% in the lateral direction, with respect to the GRFs on rigid surfaces.

The movements of the body segments are presented in Figures 8 and 9 and display typical patterns as reported in investigations [36]. The percentage differences for the angles corresponding to HSI with respect to those corresponding to the rigid surface were calculated and are very small when the pedestrian is near the bridge ends. Hence, the RMSE was calculated over the 40–60% span of the bridge, where the accelerations $\vec{A}_g(t)$

are influential, and are given in Table 8. In the case of angular movements in the frontal plane (Figure 9), the differences between the HSI and rigid surface are much greater than the differences in the sagittal plane. We believe that this is due to the lock-in phenomenon, for which the human model increases its lateral sway [24].

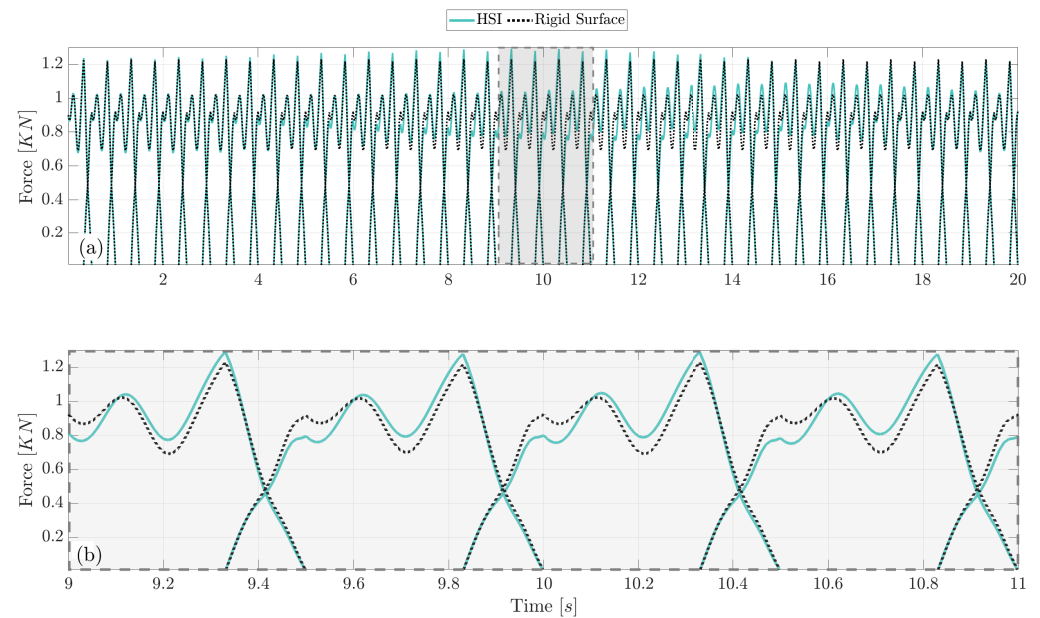


Figure 5. Vertical Ground Reaction Forces, HSI (Green solid line) and rigid surface (Black dotted line). (a) Forces over the entirety of the bridge, (b) Forces corresponding to four strides at mid-span.

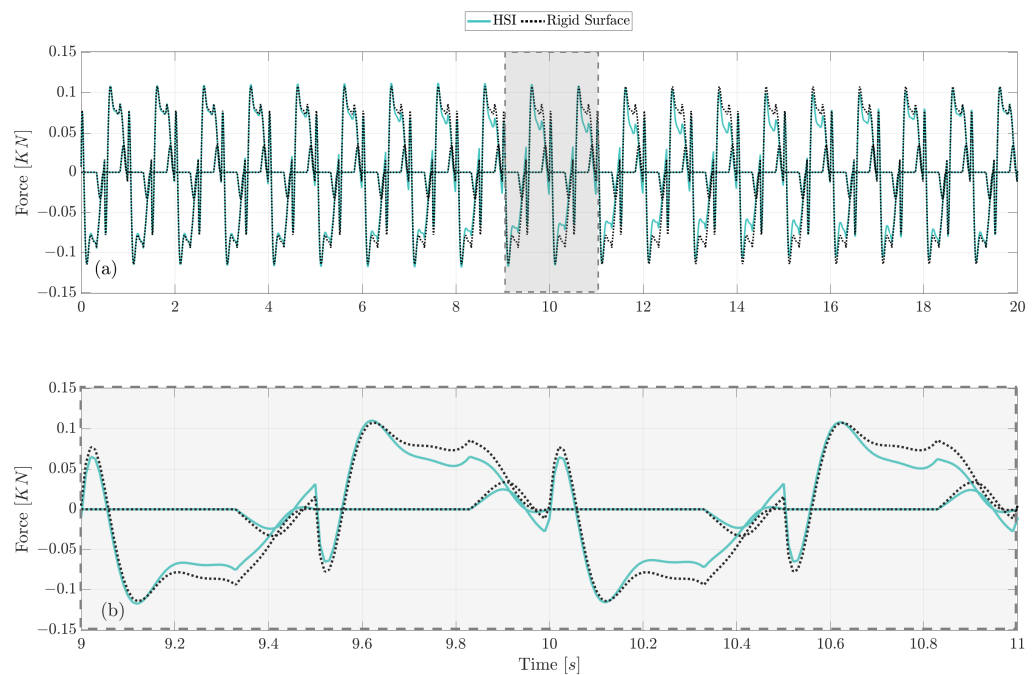


Figure 6. Lateral Ground Reaction Forces, HSI (Green solid line) and rigid surface (Black dotted line). (a) Forces over the entirety of the bridge, (b) Forces corresponding to four strides at mid-span.

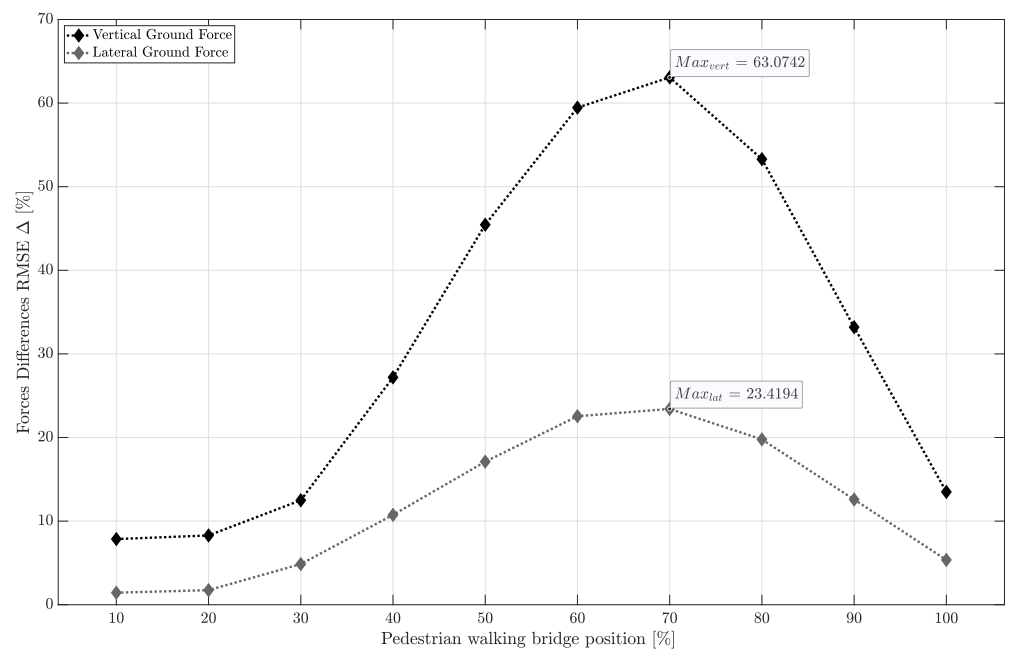


Figure 7. Forces Differences (RMSE) between HSI and Rigid Surface.

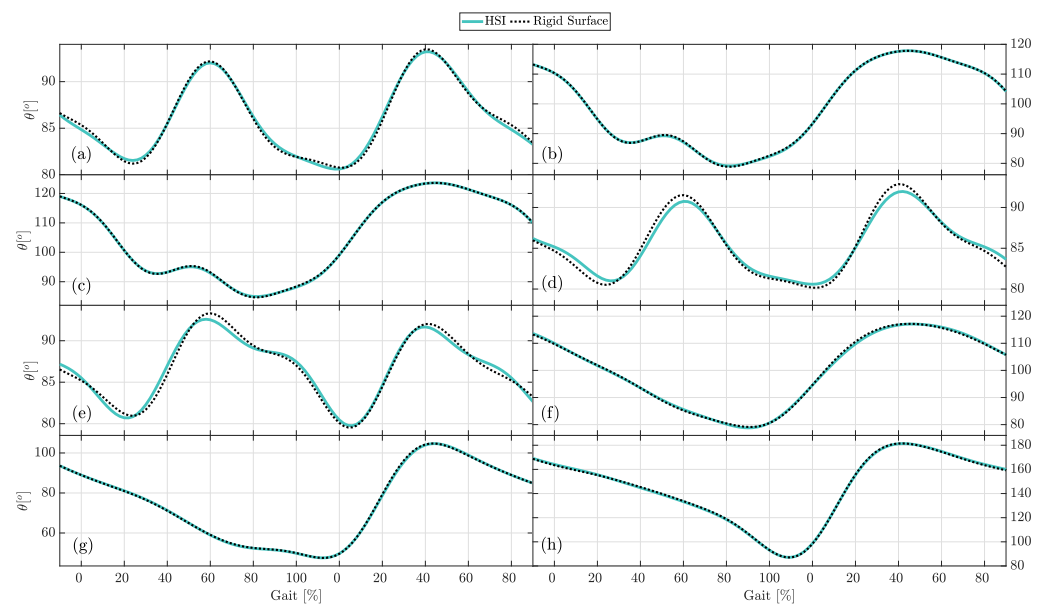


Figure 8. Sagittal Plane Angular Displacements at Mid-Span. HSI (Green solid line) and rigid surface (Black dotted line). (a) Head, (b) Upperarm, (c) Forearm, (d) Trunk, (e) Hip, (f) Thigh, (g) Shank and (h) Foot.

Table 8. Differences angular displacements during human gait between 9.5 and 10.5 s.

	Head	Upper Arm	Forearm	Trunk	Hip	Thigh	Shank	Foot
RMSE θ_i [%]	6.27	0.99	0.97	14.37	12.74	2.58	1.32	1.48
RMSE φ_i [%]	24.93	36.87	37.09	3.13	4.18	129.19	54.64	50.51

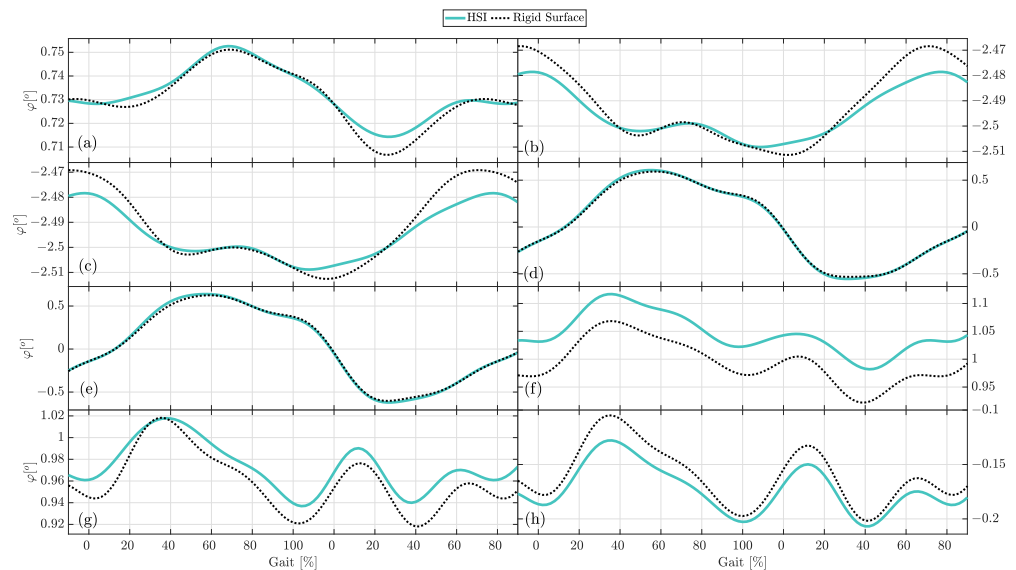


Figure 9. Frontal Plane Angular Displacements at Mid-Span. HSI (Green solid line) and rigid surface (Black dotted line). (a) Head, (b) Upperarm, (c) Forearm, (d) Trunk, (e) Hip, (f) Thigh, (g) Shank and (h) Foot.

The differences in the angular displacements presented in Table 8 indicate significant variations in the gait kinematics owing to the bridge vibrations. Specifically, the RMSE φ_i differences are greater in the lower and upper extremities (Figure 9f–h). It was observed that the abduction motion of the lower limbs (Thigh, Shank, and foot) and arms (upper and forearm) increased, that is, the lateral body sway was increased to ensure stability. This is consistent with observations made by Brady et. al. (2009) [38] in human experimental trials on an oscillating treadmill, where they recorded that the lateral step width generally increased as a consequence of vibration [39].

Figure 10 shows the instantaneous natural frequency and damping ratio of the coupled system during pedestrian transit over the structure. It can be seen that, as suggested by several studies, the effect of anthropic loads generates a change in the dynamic properties of the coupled system [40], decreasing the modal frequency and increasing the damping of the pedestrian structure system under the effects of HSI. The instantaneous frequency decreases as the pedestrian advances along the structure, reaching a minimum value when the pedestrian is at mid-span of the bridge, with a difference of 1.98% and 2.42% for the vertical and lateral directions, respectively. Similarly, the damping increases by 3.5% in the vertical direction and by 1.93% in the lateral direction.

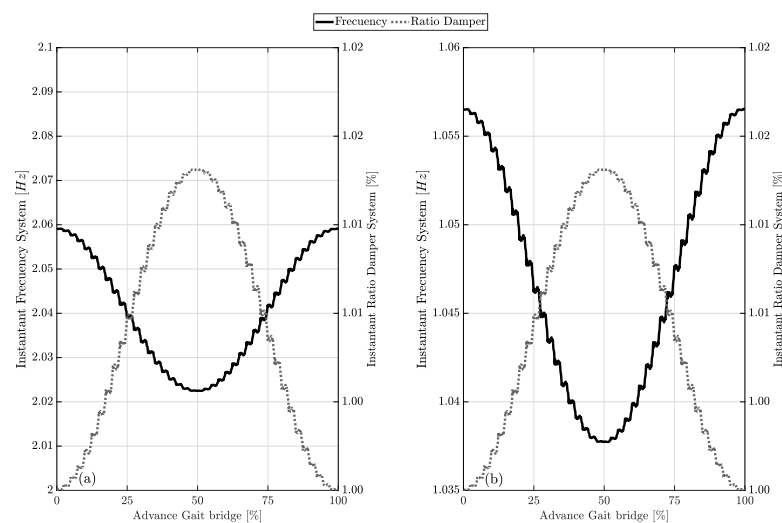


Figure 10. Instantaneous Natural Frequency and Damping Ratio of Coupled System. (a) First vertical mode, (b) First lateral mode.

The dynamic responses of the reference structure are presented in Figures 11–13. In addition to the HSI model proposed in this study, an analysis using a model omitting the effect of vibrations on the GRFs (without HSI) was considered. Figure 11 shows the bridge acceleration $A_g(t)$ at contact point in the vertical and lateral directions. The displacements $D_c(t)$ and acceleration $A_c(t)$ in the bridge mid-span are shown in Figures 12 and 13, respectively. The peak values of the structural responses are presented in Table 9, where the lower peak values were recorded for the model without HSI in the analysed cases.

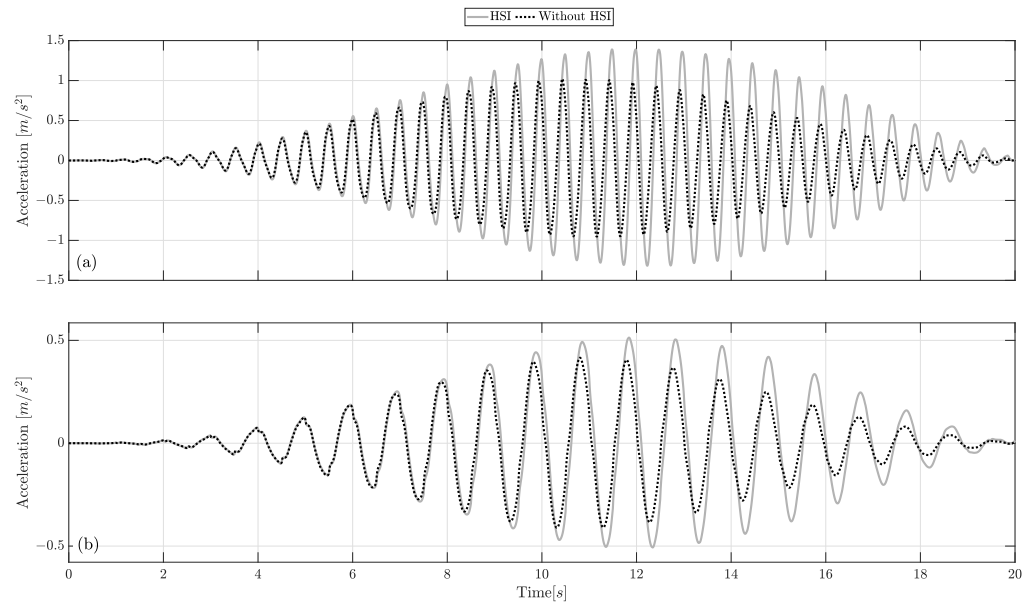


Figure 11. Acceleration $A_g(t)$ at Point of Contact. (a) Vertical direction. (b) Lateral direction.

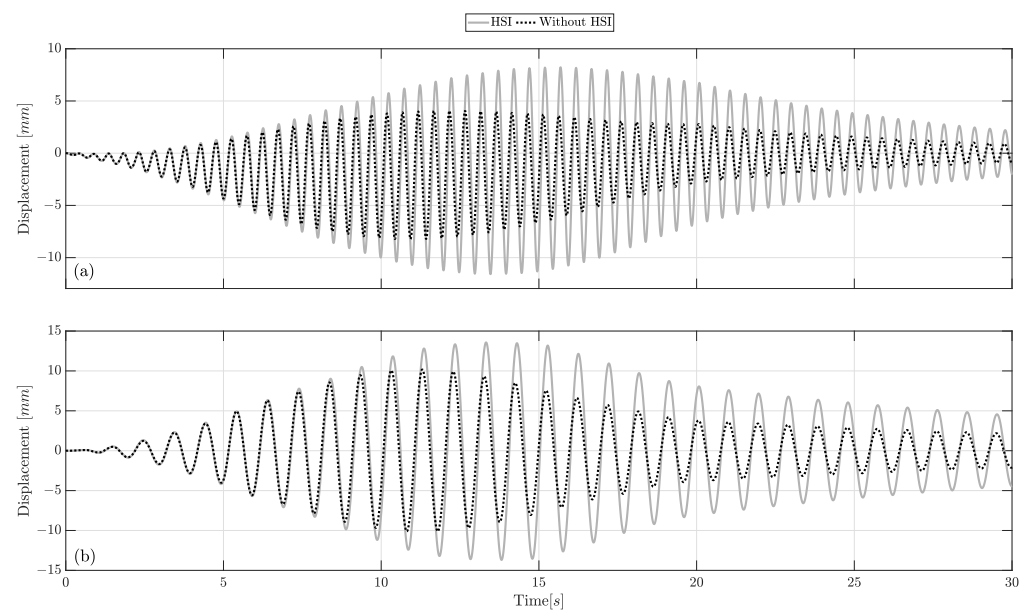


Figure 12. Displacement at Mid-Span $D_c(t)$. (a) Vertical direction. (b) Lateral direction.

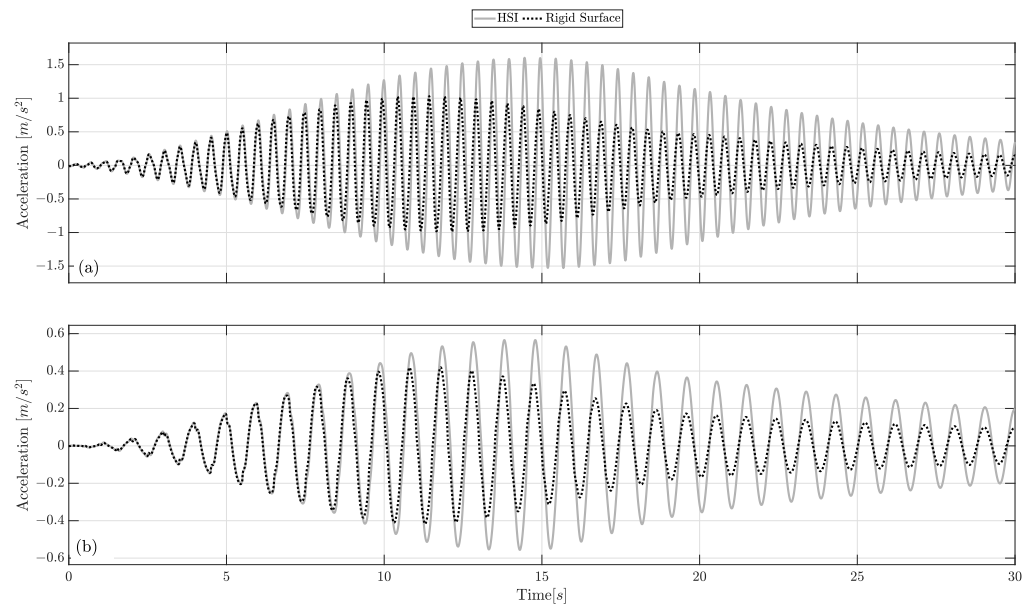


Figure 13. Acceleration at Mid-Span $A_c(t)$. (a) Vertical direction. (b) Lateral direction.

Table 9. Peak Response During Gait, With HSI and Without HSI.

	Vertical			Lateral		
	Max. HSI	Max. without HSI	$\Delta_{Vert}[\%]$	Max. HSI	Max. without HSI	$\Delta_{Lat}[\%]$
$A_g [m/s^2]$	1.39	1.02	26.41	0.51	0.42	19.08
$A_c [m/s^2]$	1.60	1.03	35.59	0.57	0.42	25.79
$D_c [mm]$	11.57	8.22	28.94	13.69	10.17	25.69

6.2. Crowd HSI Analysis

In crowd HSI analysis it is commonly accepted that the anthropic pedestrian loading F_{crowd} in the human walking frequency range (1.8–2.2 Hz), following the model of Matsumoto et al. (1978) [35] is equivalent to $\sqrt{N}F_p$ due to the small probability that if there are N pedestrians on the walkway, all of them are synchronized [41]. However, for the purpose of this study, an HSI analysis of the crowd was considered in which pedestrians are fully synchronized, defined as the worst-case loading scenario, where all pedestrians would march at the same time, applying a constant periodic load with a frequency equal to that of the bridge, for which $F_{crowd} = NF_p$.

Multiple simulations were performed using the methodology described in Section 5 considering different occupancy conditions (Mass Relation, M_{crowd}/M_{br}). The peak accelerations and displacements for each occupancy case, considering the HSI model and the model without interaction, are presented in Figure 14. The results suggest that there is an overestimation of the dynamic response of the structure in the model without interaction with respect to that of the HSI model for M_{crowd}/M_{br} greater than 15%. However, for the same conditions, considering low occupancy rates, the model without interaction underestimated the response of the structure. This phenomenon is mainly due to the crowd-structure frequency detuning ($\omega_{crowd}/\omega_{br}$), for which the frequency of the coupled HSI system decreases as the number of pedestrians increases, as shown in Figure 14 (top left).

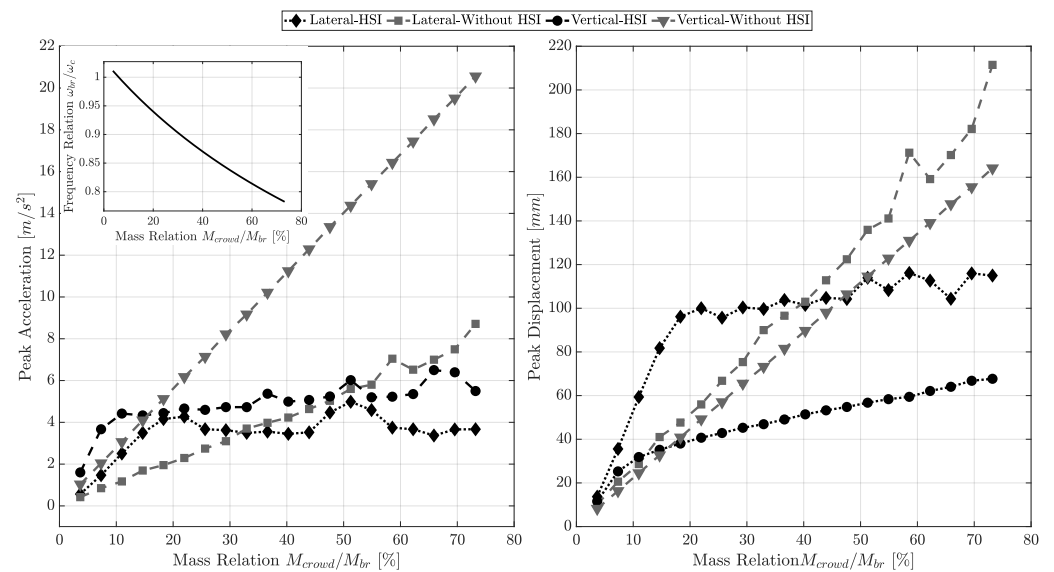


Figure 14. Peak Accelerations and Displacements at Mid-Span for Different Occupancy Levels.

The instantaneous natural frequency of the HSI system with the pedestrian at mid-span decreases less than 30% for occupancy condition levels M_{br}/M_w up to 75.0%, compared to those of the system Without dynamic HSI effects (Figure 14), for vertical and lateral directions. This is especially relevant as it suggests that pedestrian bridges with fundamental natural frequencies greater than walking frequencies, which in principle should not have excessive vibration problems, could display amplification of the dynamic response due the reduction in the natural frequency and variation in the structural damping of the coupled system, compromising the serviceability of these structures.

To reduce the effect of de-synchronization, simulations were performed by adjusting the step frequency ω_w so that the ratio of frequencies $\omega_{crowd}/\omega_{br}$ was approximately one. The simulation results are shown in Figure 15. In this case, it is observed that the peak values of acceleration and displacement are always greater than those of the model without HSI for all occupancy levels. This suggests that the traditional model underestimates the dynamic response of a structure under periodic anthropic loads.

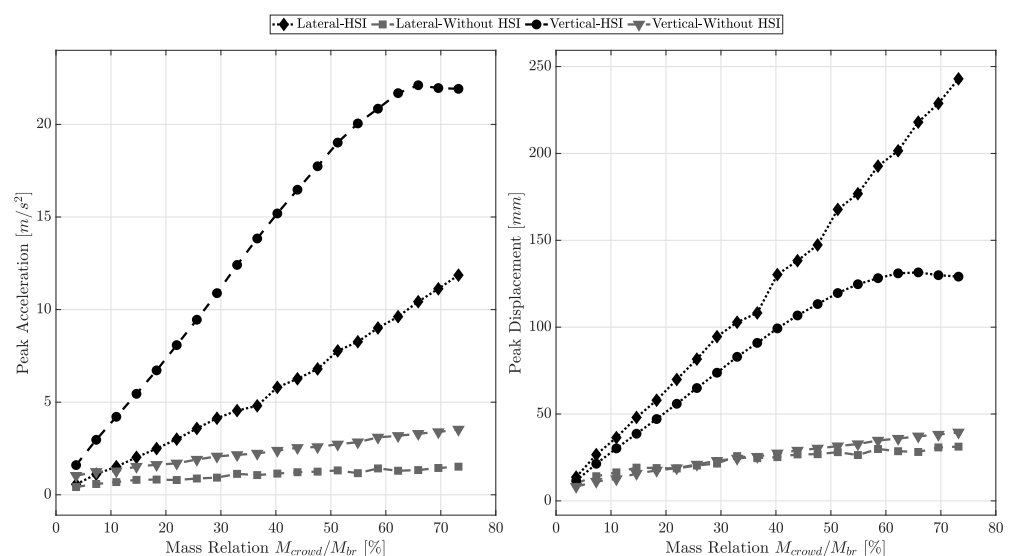


Figure 15. Peak Accelerations and Displacements at Mid-Span for Different Occupancy Levels. (Pace Frequency equal to Bridge Natural Frequency).

7. Conclusions

A three-dimensional model of the HSI was developed that consists of a whole-body biomechanical model of human gait and a structural system represented as a simple supported beam. The proposed model was used in an optimization algorithm that minimizes the mechanical energy expenditure required by a pedestrian to walk across the structure. The structural response, considering HSI, was implemented by combining inverse dynamics to predict the kinematics and kinetics of human gait and direct dynamics to solve the differential equations of the coupled system.

Multibody models such as that of Salehi et al. (2016) [26] demonstrate that predictive gait models using inverse dynamics have up to 8 times faster execution speed compared to commercial multibody models using direct dynamics. For the model presented in this research, the primary factor contributing to the computational expense is the solution of the differential equations governing the structural system and HSI. Nevertheless, the integration of direct and inverse dynamics efficiently reduces the overall time required for each optimization iteration. The computational efficiency of the model was evidenced in the simulation times obtained during the study. Each gait pattern was simulated in approximately 12 s on the computer described in Section 5, demonstrating the feasibility of the model for practical analysis compared to other approaches that require significantly longer processing times [28].

The predictions of body movements and ground reaction forces considering HSI were based solely on three simple gait characteristics: average walking speed (V_w), frequency (f_w), and step length, as well as the anthropometric data of the person and physical characteristics of the structure. It should be noted that none of the human body movements were predefined for a specific motion, and their behaviors were determined solely by the optimization process. The only constraints that were imposed corresponded to physically realizable gait patterns. The results suggest that human gait is a biological process of energy minimization, as other authors have suggested [42].

The proposed model effectively simulates the kinematics and kinetics of human gait and accurately captures the dynamic response of structures subjected to HSI effects. It is important to note that the vertical force pattern (see Figures 5 and 6) exhibits a different number of peaks compared to the existing force patterns reported in the literature. However, this discrepancy is commonly observed in models with high leg stiffness at low walking speeds, where a greater number of force peaks is generated as the walking speed decreases [43].

The results of the simulation of the HSI demonstrate the potential of this approach for modelling HSI effects pedestrian bridges. As such, it is a viable methodology for the evaluation and understanding of the impact of HSI. A comparison of the results of simulations of a person walking on a rigid surface and walking on a flexible bridge revealed significant differences in the contact forces induced by pedestrians traversing the structure. In particular, differences of 63% and 23% for vertical and lateral forces respectively, highlight the influence of HSI effects on contact forces and their implications for structural response.

Finally, as shown in Figure 15, pedestrian bridges that are not expected to have excessive vibrations may experience frequency synchronization problems if high occupancy levels are reached and the natural frequency of the coupled system decreases. This highlights the importance of taking this factor into account during the design and construction phases in order to prevent serviceability issues arising from HSI phenomena.

Limitations and Future Research

The model presented does not enable the assessment of HSI with people running or jumping. This limitation is linked to one of the main restrictions of normal walking implemented in this model, which requires that at least one of the pedestrian's feet is in contact with the ground, contrary to the activity of running, where there may be moments where both feet are in flight. Therefore, future work will extend this study by eliminating restrictions on the duration and length of double support such that activities other than

walking can be considered. Another potential area of research is to consider other types of load associated with humans crossing pedestrian bridges, such as those due to bicycles or strollers.

Biomechanical models of human gait that use reverse dynamics have been validated in previous research by proving their feasibility in predicting the kinematics and kinetics of gait (Movements and Forces) [29,31]. However, the validation of these models considering vibrating surfaces in real environments has not been addressed. The collection of experimental data where the complete body movements of the gait are recorded under HSI conditions and simultaneously structural vibrations in real bridges are configured as the next step to validate and comprehensively evaluate the approach proposed in this research. In addition, a detailed statistical study of anthropometric characteristics is just as important. This analysis will allow defining key anthropometric parameters that can be integrated into the proposed human-structure interaction models, such as the one proposed in this study, and performing comprehensive comparisons and validations, reducing the uncertainty of the physical parameters of the human system.

Finally, it is known that anthropic loads are not the only dynamic loads to which pedestrian bridges are subjected, in that sense the integration of this model with wind or earthquake effects becomes an interesting field of research that is worth exploring.

A future key aspect to be perfected within the model presented in this research is related to the kinematic model assumed for the body segment of the foot (Section 2.1.1). This is due to the biological variability of the dynamics of the foot, where the kinematics and kinetics are different according to the anthropometric conditions of the person, such as the anatomical structure of the foot, body weight, body mass index (BMI), joint range of ankle motion, or age. In this sense, a robust model that can be implemented in this model that reduces the uncertainty gap of biological variability is of great importance.

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