

PAPER • OPEN ACCESS

Analytical and experimental determination of gravity and moment of inertia using a physical pendulum

To cite this article: K L Cristiano *et al* 2019 *J. Phys.: Conf. Ser.* **1386** 012139

View the [article online](#) for updates and enhancements.

You may also like

- [Pendulum motion of a biased cylindrical tube](#)
Rod Cross
- [An experimental system for studying the plane pendulum in physics laboratory teaching](#)
Henrik B Pedersen, John E V Andersen, Torsten G Nielsen et al.
- [Dynamics of damped oscillations: physical pendulum](#)
G D Quiroga and P A Ospina-Henao



The Electrochemical Society
Advancing solid state & electrochemical science & technology

241st ECS Meeting

Vancouver, BC, Canada. May 29 – June 2, 2022



ECS Plenary Lecture featuring
Prof. Jeff Dahn,
Dalhousie University



Register now!



Analytical and experimental determination of gravity and moment of inertia using a physical pendulum

K L Cristiano¹, D A Triana¹, R Ortíz², M Pico², and A F Estupiñán³

¹ Universidad Industrial de Santander, Escuela de Física, Bucaramanga, Colombia

² Universidad Autónoma de Bucaramanga, Bucaramanga, Colombia

³ Universidad de Investigación y Desarrollo, Bucaramanga, Colombia

E-mail: dantrica@saber.uis.edu.co

Abstract. Searching to encourage and increase the desire of students to seek a vocation in the study of engineering and science, we wanted to implement and validate experimentally and numerically, the study of the movement of a mechanical oscillator using, in this case, a physical pendulum, formed by a bar and a disk. In this article has done the study the physical pendulum, combining a methodology that involves an experimental arrangement and the implementation of simulations developed in Python, with the aim objective of offering to students a visual and interactive experience, so that they can understand in a simpler way topics covered in the theoretical physics course, in such a way that is different from the typical physical-mathematical formalism. This study was carried out with low cost materials and easy access, in addition to the great social impact that I had against the acceptance and assessment by the students with whom this work was applied. This work was developed in three phases: first, to measure the period of oscillation of a physical pendulum experimentally. Second, the approach of the analytical model to compare with the experimental results. Third, the development of a dynamic simulator according to the predictions of the theoretical model. The students found a didactic and different way of studying the physical pendulum. Finally, it was possible to demonstrate a self-consistency between the experimental and numerical results of the system studied in this work.

1. Introduction

One of the most relevant applications of differential equations has been oscillatory movements, in which a physical interpretation can be given to each term present in these equations [1, 2]. The purpose for which this work was performed, was mainly the need to show, an experiment that could explain how from the study of an oscillatory movement we can indirectly find the moment of inertia of the center of mass of a composite pendulum and the value of the local gravity in which said system has been placed. One of the most interesting movements in physics is the rotational movement made by a physical pendulum [3, 4], although the effort made to calculate the severity with this type of systems is very remarkable, it is very uncommon to find the detailed explanation of obtaining the calculations analytical using the study of a simple harmonic movement for this type of systems in which the rotational dynamics of the system can be studied. In order to show the applicability and the importance of rotational movement,



using an analytical model of linear behavior to be implemented in the physics laboratory, with this can to have a better understanding in this type of movement. Is for this that in this article the authors present a novel physics laboratory, in which can be show in detail the importance and also be able to validate a physical analytical and experimental model developmented of the authors, studying the dynamic behavior of the pendulum physics, for that used laboratory materials of easy access and low cost.

This work, like those shown in references [5–8], has been of great importance in the investigation of the implementation of alternative and innovative methods in a classroom at the university level in Colombia. This paper is organized as follows: Section 2, describes the analytical study corresponding to the calculates for the physical system. In Section 3, shown the experimental procedure for the data takes. In Section 4 the results concerning to the validate of the analytic model using the experimentall method shown in the previous section.

2. Analytical study

The physical model of the system under study is shown in Figure 1. This system consists of (1) the point of support or pivot on which the system rests, (2) the disk which can be moved along the bar of iron (3).

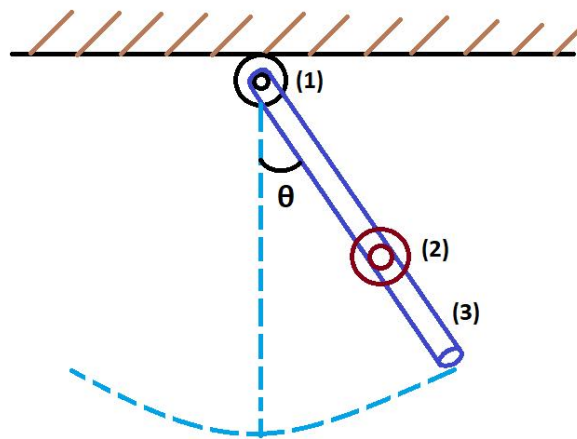


Figure 1. Physical system of pendulum to study.

To begin to analyze the dynamics of the movement to be studied, we must start from Newton's Second law for rotational dynamics, as Equation (1).

$$\sum_{i=0}^n \tau_i = I\alpha \quad (1)$$

Carrying out the analysis from the point of support (1) of the Figure 1, you can get the Equation (2) [9].

$$(-m_b g \sin(\theta)) \frac{L}{2} + (-m_d g \sin(\theta) L_y) = I\alpha, \quad (2)$$

where m_b , m_d are the mass of the bar and disk respectively. Expressing I as the moment of total inertia of the system, being the sum of the moment of inertia of the bar I_b plus that of the disk I_d (see Equation (3)).

$$I = I_b + I_d. \quad (3)$$

The algebraic expressions for the moment of inertia of bar I_b and disk I_d using the Steiner's Theorem is the Equation (4) [10]:

$$I_b = \frac{1}{3}m_b L^2, \quad I_d = m_d(L_y)^2 + \frac{1}{2}m_d R^2 \quad (4)$$

Replacing the Equation (4) in Equation (3), the moment of inertia of the system is obtained Equation (5).

$$I = \frac{1}{3}m_b L^2 + m_d(L_y)^2 + \frac{1}{2}m_d R^2, \quad (5)$$

where L are the length of bar, L_y are the length of the position of the disk respect to the pivot and R is the disk's radius.

Using the Equation (5) and replace this in Equation (2), we are obtained the Equation (6):

$$(-m_b g \sin(\theta)) \frac{L}{2} + (-m_d g \sin(\theta) L_y) = \left(\frac{1}{3}m_b L^2 + m_d(L_y)^2 + \frac{1}{2}m_d R^2 \right) \alpha, \quad (6)$$

by organizing the above equation, this can be written as Equation (7):

$$g \sin(\theta) \left[-m_b \frac{L}{2} - m_d L_y \right] = \left(\frac{1}{3}m_b L^2 + m_d(L_y)^2 + \frac{1}{2}m_d R^2 \right) \alpha, \quad (7)$$

where α is the angular acceleration, if we writing this as $\ddot{\theta}$ and take this movement for small angles $\sin(\theta) \approx \theta$, we can write the Equation (8) as follow:

$$-\theta \left[\left(m_b \frac{L}{2} + m_d L_y \right) \right] = \left(\frac{1}{3}m_b L^2 + m_d(L_y)^2 + \frac{1}{2}m_d R^2 \right) \ddot{\theta}, \quad (8)$$

Equaling to zero the before equation and comparing it with the second order differential equation corresponding to a simple harmonic oscillator [11], we have Equation (9) and Equation (10):

$$\ddot{\theta} + \left(\frac{g \left(m_b \frac{L}{2} + m_d L_y \right)}{\frac{1}{3}m_b L^2 + m_d(L_y)^2 + \frac{1}{2}m_d R^2} \right) \theta = 0, \quad (9)$$

with,

$$\ddot{\theta} + \omega^2 \theta = 0. \quad (10)$$

In this way, we can obtain the Equation (11) for the angular frequency:

$$\omega^2 = \frac{g \left(m_b \frac{L}{2} + m_d L_y \right)}{\frac{1}{3}m_b L^2 + m_d(L_y)^2 + \frac{1}{2}m_d R^2}, \quad (11)$$

now taking into account the relationship of the angular frequency with the period of oscillation ($\omega = 2\pi/T$), the following Equation (12) can be obtained:

$$\frac{4\pi^2}{T^2} = \frac{g \left(m_b \frac{L}{2} + m_d L_y \right)}{\frac{1}{3}m_b L^2 + m_d(L_y)^2 + \frac{1}{2}m_d R^2}. \quad (12)$$

Writing the Equation (12) based on the square of the period T^2 we get to Equation (13).

$$4\pi^2 \left(\frac{1}{3}m_b L^2 + m_d(L_y)^2 + \frac{1}{2}m_d R^2 \right) = \left(m_b \frac{L}{2} + m_d L_y \right) g T^2, \quad (13)$$

operating this Equation (13), we have the Equation (14):

$$\frac{4\pi^2 m_d}{g} (L_y)^2 + \frac{4\pi^2}{g} \left(\frac{1}{3} m_b L^2 + \frac{1}{2} m_d R^2 \right) = m_b \frac{L}{2} T^2 + m_d L_y T^2 \quad (14)$$

3. Experimental study

In order to bring Equation (14) to an algebraic expression of linear relationship, we can make the following substitution of physical variables (see Equation (15)):

$$b = \frac{4\pi^2}{g} \left(\frac{1}{3} m_b L^2 + \frac{1}{2} m_d R^2 \right), \quad (15)$$

where the moment of inertia of the system measured from the center of mass of each object, both the bar and the disk I_{CM} is the Equation (16) [12]:

$$I_{CM} = \frac{1}{3} m_b L^2 + \frac{1}{2} m_d R^2, \quad (16)$$

In addition we will call $\bar{m}$, x and y as the Equation (17):

$$\bar{m} = \frac{4\pi^2 m_d}{g}, \quad x = (L_y)^2 \quad \text{and} \quad y = m_b \frac{L}{2} T^2 + m_d L_y T^2. \quad (17)$$

Using the Equation (14), next we can write a linear dependence as Equation (18):

$$y = \bar{m}x + b \quad (18)$$

The dimensions and masses of the objects used in the system to be studied are recorded in the Table 1.

We perform the data collection in the following way, with the chronometer we record three times the time of 10 oscillations for the physical pendulum varying the length at which the disc is placed L_y , these data were recorded in the Table 2,

Table 1. Main measures of the physical pendulum.

Object	Length (m)	Mass (kg)
Bar	Long = 1.000	1.333
Disk	Radius = 0.055	1.611

Table 2. Data capture for the period and length of the disk location L_y in the experiment performed with the pendulum.

Lenght L_y (m)	Time (s)	Period T (s)
0.25	13.86	1.38
0.32	14.45	1.44
0.37	14.62	1.46
0.46	15.10	1.51
0.59	16.01	1.60
0.67	16.63	1.66
0.82	17.70	1.77
0.95	18.80	1.88

4. Results and discussion

To start analyzing the results that can be obtained using the analytical model of the Equation (18) using the data the Table 1 and Table 2, we are going to organize the values of x and y in the Table 3.

Table 3. Data taken from the experiment for our linear model.

$x \text{ (m}^2\text{)}$	$y \text{ (kg} \cdot \text{m} \cdot \text{s}^2\text{)}$
0.0625	2.0549
0.1069	2.4905
0.1398	2.7126
0.2152	3.2143
0.3481	4.1710
0.4583	4.8309
0.6724	6.2251
0.9063	7.7745

Realizing a lineal fitting to the experimental data [13], we can graph x in function of y , we can compare the analytical model with the data taken in the experiment, this is shown in the Figure 2.

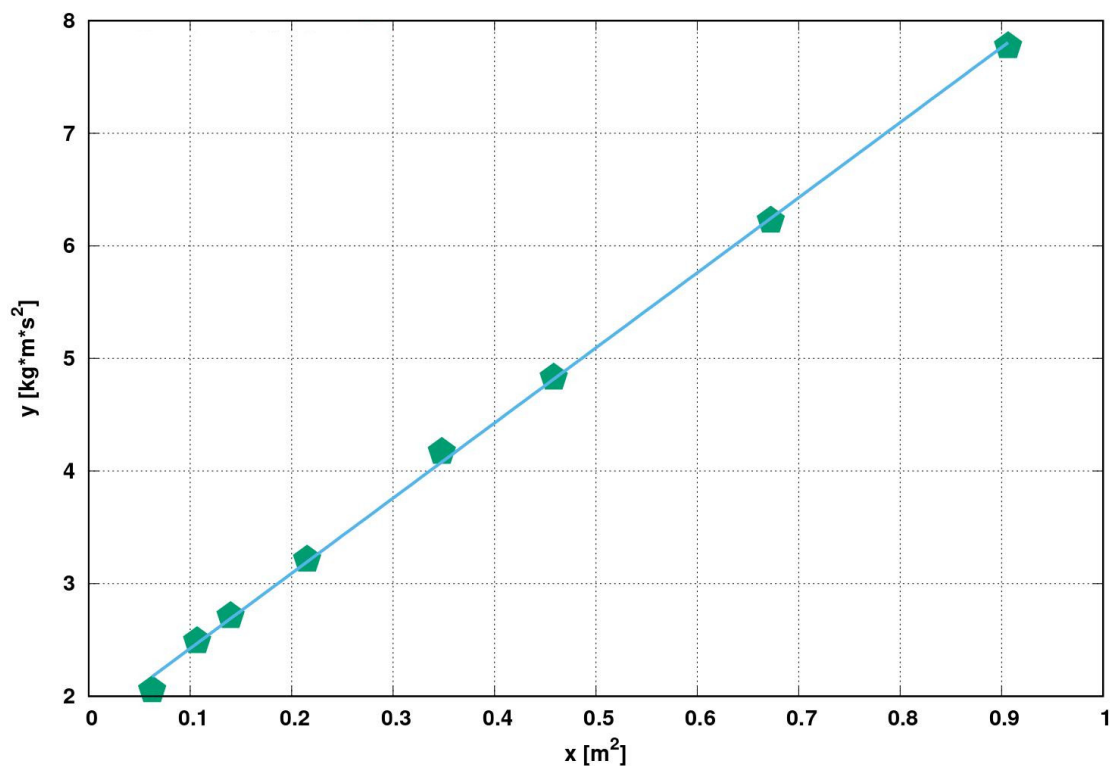


Figure 2. Experimental data are represented with green points and the analytical model are represented with blue line.

The function obtained with the analytical linear (see Equation (18)) adjustment was the Equation (19).

$$y = 6.67381 \cdot x + 1.75689. \quad (19)$$

Using the value of the constant $\bar{m}$ obtained by Equation (17) and compared with Equation (19), we can obtain the experimental value of gravity (see Equation (20)).

$$g_{(exp)} = \frac{4\pi^2 m_d}{\bar{m}} = 9.53 \quad m/s^2. \quad (20)$$

Continuing to obtain the experimental moment of inertia with respect to the center of mass I_{CM} of the system (bar and disk), it can be obtained using the Equation (15), Equation (16) and Equation (18), in the following Equation (21).

$$I_{CM(exp)} = \frac{b \cdot g_{(exp)}}{4\pi^2} = 0.42 \quad kg \cdot m^2. \quad (21)$$

Using the theoretical results of local gravity g and the moment of inertia I_{CM} shown in the Table 4, one can calculate the experimental errors of gravity and moment of inertia.

Table 4. Experimental errors obtained from the analytical model.

	Gravity (m/s^2)	Moment of inertia I_{cm} ($kg \cdot m^2$)
Theoretical value	9.81	0.44
Experimental value	9.53	0.42
Absolute Error	0.28	0.02
% Relative Error	2.85	4.54

The results of the numerical study of the system are shown in Figure 4. We have written a python script in order to reproduce the behavior of simple pendulum and physical pendulum to teaching in the physics subjects. [14, 15], a simulation in time of the evolution of the system [16, 17], besides we could corroborate the results, analytical and experimental of the physical pendulum, with the numerical study made in this simulation.

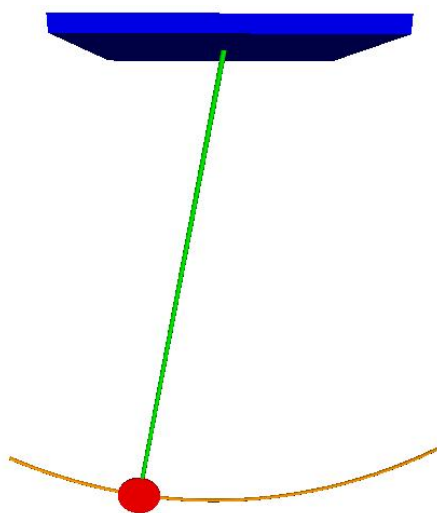


Figure 3. Schematic diagram of physical scheme simulated, when the pendulum's disk put in the extreme of this.

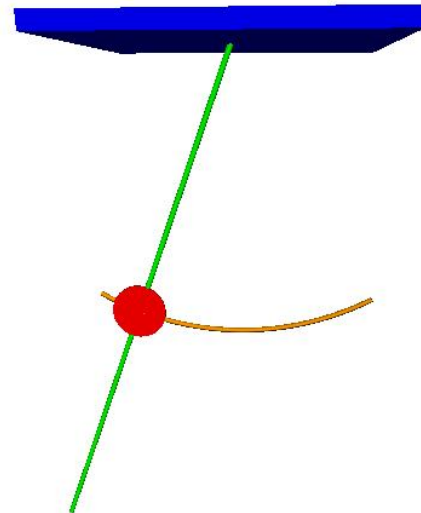


Figure 4. Simulated scheme of the system, when the pendulum's disk is ubicated in the middle of pendulum.

5. Conclusions

In this work, it was possible to perform a new analytical approach and verify it through the realization of an experimental assembly, with the purpose of measuring the gravity and moment of inertia of a physical pendulum indirectly, using mainly simple materials. The results of this article showed very small errors, below 5% which make it have a good reliability in the analytical model developed, in addition to the respective validation of this through the realization of the experiment using an innovative physical pendulum. We use this analytical approach to perform a numerical simulation of the rotational movement studied in this article, the web link of code in GitHub is: https://github.com/alexestupinan123/Teaching_physics_Python/blob/master/pendulo.py

Acknowledgments

We want give special thanks at the Universidad Industrial de Santander for the funds to show this work. Also, we give thanks the Physics Department and his Director, Jorge Martínez Téllez. We would like to thank the Universidad Autónoma de Bucaramanga, for lend us the installations and materials for carry to this experiment with which the analytical model presented in this paper could be validated.

References

- [1] Giancoli D C, Miller I A, Puri O P, Zober P J and Zober G P 2005 *Physics: Principles with applications, 6th edition* (New Jersey: Pearson/Prentice Hall)
- [2] Monteiro M, Cabeza C and Marti A C 2014 *The Physics Teacher* **52** 180
- [3] Baker G L and Blackburn J A 2005 *The pendulum: A case study in physics* (New York: Oxford University Press)
- [4] Nelson R A and Olsson M 1986 *American Journal of Physics* **54** 112
- [5] Rodríguez R, Amaya J, and Estupiñán A 2019 *Teacher Education and Curriculum Studies* **4(2)** 33
- [6] Cristiano K, Estupiñán A and Triana D 2019 *Journal of Physics: Conference Series* **1247** 012044
- [7] Angel Duarte Jesus Sanchez J A R O and Alex E 2019 *Iconic Research And Engineering Journals* **3** 6
- [8] Cristiano K and Triana D 2019 *Journal of Physics: Conference Series* **1161** 012020
- [9] Sears F W, Zemansky M W and Young H D 1987 *University physics, 7th edition* (San Francisco: Addison-Wesley)
- [10] Serway R A, Jewett J W 2005 *Física para Ciencias e Ingeniería, sexta edición volumen 1* (México: Thomson Learning)
- [11] Zill D G and Cullen M R 2013 *Matemáticas avanzadas para ingeniería: Ecuaciones Diferenciales, tercera edición volumen 1* (México: McGraw-Hill Interamericana)
- [12] Tipler P A and Mosca G 2003 *Física para la ciencia y la tecnología, 5ª Edición volumen 1A* (Barcelona: Editorial Reveté)
- [13] Janert P K 2010 *Gnuplot in action: Understanding data with graphs* (Greenwich: Manning Publication).
- [14] Backer A 2007 *Computing in Science & Engineering* **9** 30
- [15] Watters A, Ahlstrom J C and Rossum G V 1996 *Internet programming with Python* (New York: Henry Holt and Co., Inc.)
- [16] Landau R H, Páez M J and Bordeianu C C 2015 *Computational physics: Problem solving with Python* (Weinheim: John Wiley & Sons)
- [17] VanderPlas J 2016 *Python data science handbook: essential tools for working with data* (Sebastopol: O'Reilly Media, Inc.)