Electrical Power and Energy Systems 130 (2021) 106874

Contents lists available at ScienceDirect

International Journal of Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

A population games transactive control for distributed energy resources

Eder Baron-Prada [a], Eduardo Mojica-Nava [b,*]

[a] Computer, Electrical and Mathematical Sciences and Engineering Division, King Abdullah University of Science and Technology, Thuwal, Saudi Arabia
[b] Department of Electrical Engineering, Universidad Nacional de Colombia, Bogotá, Colombia

ARTICLE INFO

Keywords:
Distributed energy resources
Transactive control
Population games

2020 MSC:
00-01
99-00

ABSTRACT

Distributed energy resources are considered as a cornerstone in the path to a smarter grid. However, this evolution brings some important challenges for real-time implementation, especially those concerning control design and its integration with distribution systems. We propose a distributed transactive control algorithm based on population games to dynamically manage the distributed energy resources and smart loads in the system to reach the optimum social welfare. Agents are considered non-cooperative and they are individually incentive-driven preserving stability while guaranteeing some optimality conditions on the real-time operation. Smart loads are modeled as a flexible load with a base-load. Potential population games concepts are used to analyze the stability and Nash equilibrium of the proposed game. Simulation results of the proposed algorithm show the stability and convergence of the proposed algorithm.

1. Introduction

The electric network has been evolving towards the smart grid concept. This evolution includes developing technologies such as renewable generation and distributed energy resources (DERs) integration into the distribution systems [1,2]. DERs include distributed generators and smart loads, and its inclusion is pretended to improve power quality and reliability of the distribution system. Smart loads are able to self-regulation of the power consumption as a response to price signals. Several challenges have emerged due to the intermittency of the energy pattern of the renewable generation and the resilience requirement for robust operation [3]. Control theory and optimization algorithms have become one of the main tools to deal with these challenges [4]. Recent work has been focused on centralized algorithms for dynamic optimal dispatch in microgrids [5,6] where it is implemented an algorithm based on game theoretical methods that can achieve the system optimum state for all agents in the system. Many methods can be used to solve this problem from a centralized manner. In general, iterative methods such as projected gradient methods, Newton's method, and many others have been extensively used [7,8]. However, centralized controllers often suffer from serious computation, communication, and robustness issues in systems with many devices, since its coordination in this large-scale network is a challenging task. In addition, when agents change locally, the central system must be reconfigured to solve the new problem. Finally, a centralized optimizer could be susceptible to a single point of failure.

In response to the limitations of centralized methods, in the last decades there has been an increasing interest in the development of decentralized optimization strategies [9–11]. One of the first algorithm implemented in a distributed way was the center-free algorithm [12]. Since then, several distributed optimization algorithms have been proposed for distributed resource allocation problems[10,13]. Most of these algorithms are based on the seminal work in [14]. These algorithms are based on reaching a consensus on their estimates of an optimal solution based on local information. Despite the recent success of such algorithms, some limitations arise when they have to be implemented in real-time operation such as lack of robustness to environmental variations, dependency of the synchronization of information between agents, or the requirements of a two-time scale solution.

In contrast, game-theoretical methods have emerged as a flexible possibility to coordinate the operation of networked devices [15,16,4]. Several benefits can be exploited such as real-time adaptation and robustness to dynamic variation in environmental conditions. However, traditional game-theoretic approaches based on individuals that repetitively play a game with the other individuals seeking for an equilibrium are computationally demanding as the number of individuals grow due to the complex decision model [15,17]. In this context, population games are better suitable to deal with large number of agents since it considers the whole population interaction instead of agent-to-agent interaction. Distributed population game-theoretical control is

* Corresponding author.
E-mail address: eamojican@unal.edu.co (E. Mojica-Nava).

https://doi.org/10.1016/j.ijepes.2021.106874
Received 21 July 2020; Received in revised form 24 January 2021; Accepted 28 January 2021
Available online 13 March 2021
0142-0615/

becoming a viable strategy to handle the coordinate operation of networked devices [4]. In order to be a compelling solution, a distributed control, understanding it as a set of controllers distributed throughout the system with no central operator, has to satisfy several technical requirements. The first issue is the coordination among different devices used at low levels such as traditional generation units, renewable generation units, and controllable loads [18,19]. Then, the distributed controller has to be able to provide stability and robustness margins for the entire system in the presence of uncertainty and offering some optimality guarantee on real-time response [20]. Finally, distributed controllers rely on a communication infrastructure to work [2].

Recently, multiple demand response strategies have been proposed to deal with generation and load variation balance. A scalable coordination approach to distribution systems operation could represent several advantages over traditional control strategies. Few control strategies have been proposed to include DERs into the distribution system. On the one hand, price control methods send pricing signals to the end-use loads to activate the demand response. For instance, real-time pricing [21], or time of use [22] are based on this approach. On the other hand, a new type of control technique to integrate DERs and also microgrids by means of a market interaction has recently emerged, the so-called transactive energy (TE) control, for the coordinated operation of a large number of devices [18,23,24]. The term transactive energy, initially proposed in [25], is defined as a system integrating agents that use control techniques, which can be centralized, distributed, or hierarchical, to exchange energy based on value information under physical constraints [23,26,24,27,28]. In contrast with traditional distributed control based on consensus, where agents have a local copy of the variables and iteratively try to agree on the value of this variables. Most of the results on transactive control have been focused on the transmission system of the electrical infrastructure [23,26,29]. An important result for transmission systems using a first approach of a dynamic market mechanism is presented in [29]. In this work, the objective is to reach the market equilibrium solving a centralized optimization including the generation companies, consumer companies, and the independent system operator to achieve an optimal economic dispatch. TE control systems provide a scalable privacy-preserving framework to ensure distributed optimal coordination of DERs. A recent review of TE systems for microgrids is presented in [30], where the main concepts, architectures, distributed technologies are revised in general terms.

The main contribution of this work is a distributed dynamic transactive control algorithm to control DERs in a distribution system. We propose a two-stage optimization scheme (see Fig. 1), where in the first stage the distribution system operator (DSO) solves a static optimization problem to obtain the optimal energy price for a market period dealing with the issue of having a coordination mechanism between devices by means of a bidirectional connected graph. The DSO obtains the MCP by means of a feedback mechanism including the generated power and forecasts about the load. At the second stage, a dynamic distributed optimization is proposed based on a population games approach for solving a social welfare optimization problem between DERs using the market clearing price information. The equilibrium of the social welfare problem characterizes the outcomes of the interaction of agents with conflicting objectives. However, sometimes a hierarchy conflict between the utility of each individual and the system-level objective can arise. In particular, we implement a distributed replicator dynamics algorithm. The social welfare optimization problem considers generators as agents able to sell energy to the distribution system, while several intelligent responsive loads are connected in different points of the network. The proposed population game approach dynamically obtains the socially optimal equilibrium while resolving the hierarchical conflict between the utility of each individual and the system-level objective. The two level dynamic transactive control involves an iterative closed-loop system solving a bi-level optimization problem. This bi-level optimization problem resolve the conflict between the utility of each individual and the system level objective. A well-known result in microeconomics states that the competitive equilibrium maximizes the social welfare [31]. In the proposed approach, agents are non-cooperative, and are individually incentive-driven preserving stability while guaranteeing some optimality conditions on real-time operation.

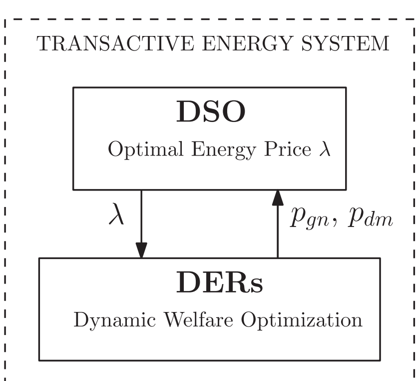

Fig. 1. Transactive Energy System in Distribution Networks.

The rest of the paper is organized as follows. In Section 2, a two-stage optimization problem is proposed to describe the distribution system including DERs and network constraints. Section 3 describes the main conceptual framework for the distributed transactive control algorithm based on the distributed replicator dynamics. In Section 4, simulations results are presented to illustrate the effectiveness of the proposed algorithm. Finally, in Section V some conclusions and future work are drawn.

1.1. Preliminaries and Notation

The graph is denoted as $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Agents are represented as a set of nodes $\mathcal{V} = \{1, 2, \ldots, V\}$. Any agent in the graph is denoted as i and j. Besides, the communication network to interchange information between agents are represented as links in the graph [32]. Furthermore, $\mathcal{E}$ is the set of communication links between agents, i.e., $(i, j) \in \mathcal{E}$ if there is a link between $i \in \mathcal{V}$ and $j \in \mathcal{V}$. The neighborhood of i-th agent is denoted as $\mathcal{N}_i$, and is defined as the set of nodes that have communication links to the i-th node. The graph $\mathcal{G}$ is connected if there is at least one path between two different nodes.

2. Transactive energy system in distribution networks

In this section, we present a transactive energy system for the distributed coordination of DERs. The TE system is defined as a system integrating agents, DERs (generators and smart loads) and the DSO, using control strategies to exchange energy while keeping the power balance (generation = demand) dynamically based on a value signal. We are assuming that the DSO has information exchange with all DERs. This can be performed through a cloud service mechanism, where the DSO upload the market clearing price (MCP) and only the DERs with granted access can download this information during a market cycle. The MCP works as a control signal from the upper level optimization to the lower level. The DSO obtains the MCP by means of a feedback mechanism including the generated power and forecasts about the load. In the second stage, DERs solve dynamically and collectively the welfare problem using the market clearing price information. The information structure between DERs is a bidirectional connected graph. A general scheme of the TE system is shown in Fig. 1. The proposed approach includes a DSO coordinator in feedback with dynamic population games solving dynamically an optimal social welfare problem. This dynamic transactive control involves an iterative closed-loop system solving a bi-level optimization problem. This bi-level optimization problem resolve the conflict between the utility of each individual and the system level objective. A well-known result in microeconomics states that the competitive equilibrium maximizes the social welfare [31].

For illustration purposes, we use an IEEE 30-bus test feeder system shown in Fig. 2 and its corresponding communication graph between loads and generators. In the next subsections, it is presented the models for the different type of agents in the TE systems.

2.1. Smart load model

In a distribution system, we consider consumers as smart loads with two profiles of energy consumption: Adjustable loads and base loads. In this work, we consider the base load as a load that each consumer can define depending on its necessities and it is the minimum load that each user need to consume, and it is predefined by the consumer. Agents of adjustable load are able to make decisions about the power consumption. If the base load is small, the adjustable load increases its request to the power network, while if the base load is high the adjustable load decreases its request to the power system. The adjustable load seeks the optimum point of consumption depending on the state of the power system.

Let m be an adjustable load such as $m \in \mathcal{M} = \{1, 2, \ldots, M\}$. The utility function for an adjustable load can be defined as a quadratic convex equation [23] as

$$U_m(p_{d_m}) = \alpha^d_m(\lambda) p^2_{d_m} + \beta^d_m(\lambda) p_{d_m} + \gamma^d_m, \tag{1}$$

where $\alpha^d \in \mathbb{R}^M, \beta^d \in \mathbb{R}^M$ and $\gamma^d \in \mathbb{R}^M$ are utility coefficients for adjustable loads. Moreover, $p_d \in \mathbb{R}^M$ is the vector of power consumed by adjustable loads, $p_d = [p_{d_1}, p_{d_2}, \ldots, p_{d_M}]$. Finally, λ is the market clearing price obtained by the DSO. The adjustable loads have a consumption limit expressed as

$$\underline{P}_{d_m} \leqslant p_{d_m} \leqslant \overline{P}_{d_m} \tag{2}$$

where $\underline{P}_{d_m}$ and $\overline{P}_{d_m}$ denotes the minimum and maximum power that an adjustable load m can consume including the base load. This means that the minimum adjustable load is the base load needed for the consumer.

2.2. Generators model

The others agents that we are considering in this system are generators. Generators are modeled considering two possibilities that are present in DERs. We are considering traditional distributed dispatchable generator and distributed generators based on storage systems. Let n be any traditional generator in the distribution system, besides let $\mathcal{N}$ be a vector in $\mathbb{R}^N$ that contains all traditional generators, where N is the number of traditional generators in the network i.e., $n \in \mathcal{N} = \{1, 2, \ldots, N\}$. The cost function for traditional distributed generators is defined as follows

$$C_n\left(p_{g_n}\right) = \alpha^g_n\left(\lambda\right) p^2_{g_n} + \beta^g_n\left(\lambda\right) p_{g_n} + \gamma^g_n, \tag{3}$$

where p_{g_n} is the amount of power generated by each generator n, and $\alpha^g \in \mathbb{R}^N, \beta^g \in \mathbb{R}^N$ and $\gamma^g \in \mathbb{R}^N$ are cost coefficients, and λ is the MCP. The distributed generators have a generation limit that is denoted as

$$\underline{P}_{g_n} \leqslant p_{g_n} \leqslant \overline{P}_{g_n} \tag{4}$$

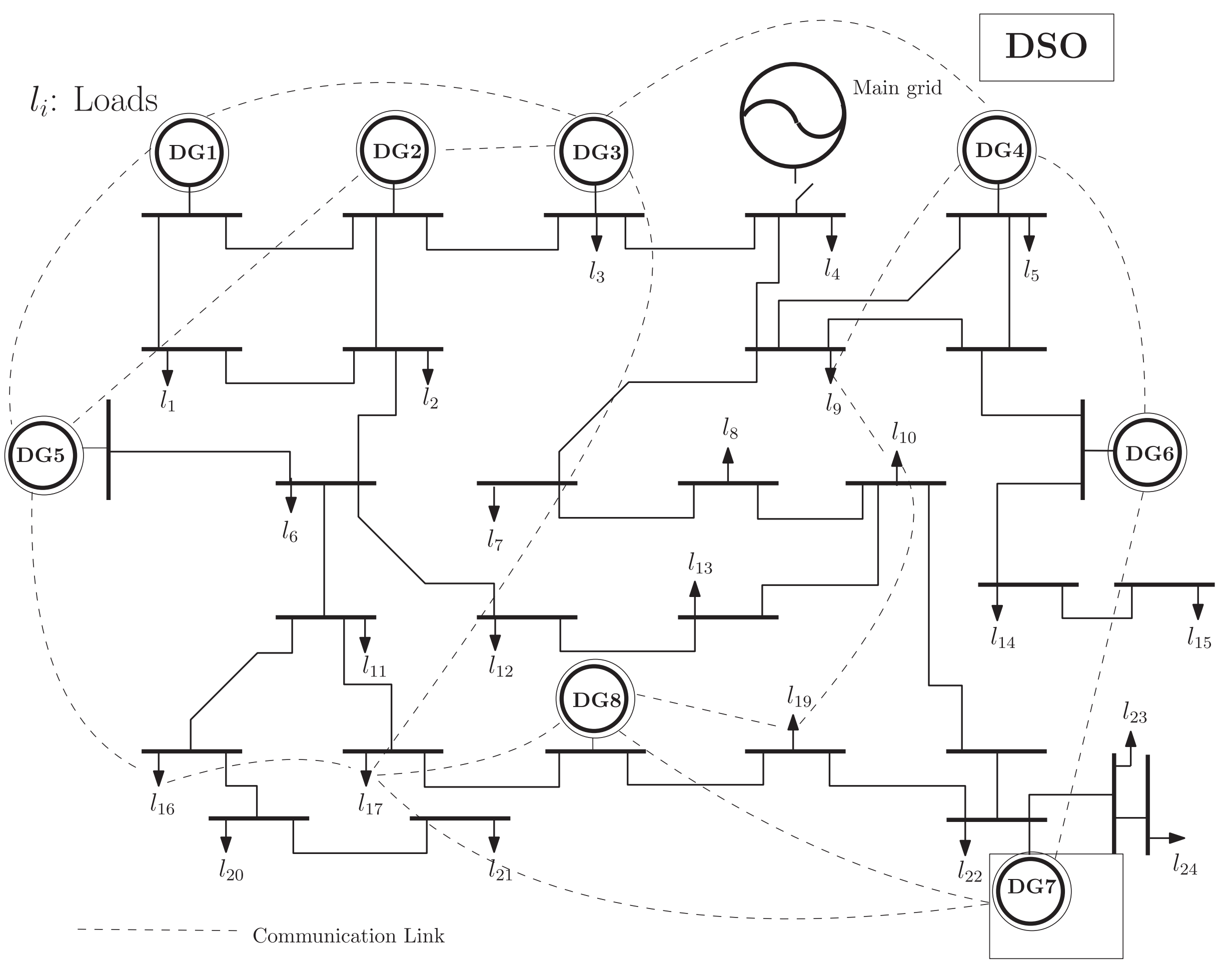

Fig. 2. General scheme of a distribution system with DERs adopted from an IEEE 30-bus.

where $\underline{P}_{g_n}$ and $\overline{P}_{g_n}$ denotes the minimum and maximum power that traditional generator n can generate, respectively.

2.3. Distributed generators based on battery energy storage systems (BESS)

On the other hand, we consider distributed generators based on storage systems. Let s be any storage system in the distribution system, besides let $\mathcal{S}$ be a vector in $\mathbb{R}^S$ that contains all storage systems, where S is the number of storage systems in the network i.e., $s \in \mathcal{S} = \{1,2,\ldots,S\}$. Cost functions for distributed generation based on storage systems have been widely studied [33]. Some cost functions are based on hourly storage cost [34] or charging and discharging cost [35]. In this work, we use the cost function proposed in [36,37]. This cost function is used to penalize the fast power charges/discharges of the storage system.

$$C_s\left(p_{b_s}\right) = \alpha_s^b\left(\lambda\right)p_{b_s}^2 + \gamma_s^b, \tag{5}$$

where p_{b_s} is the amount of power generated by each battery storage system s, and $\alpha^b \in \mathbb{R}^S$ and $\gamma^b \in \mathbb{R}^S$ are cost coefficients, and λ is the MCP. Considering the limits of the power storage in a storage system, the following constraint is imposed

$$\underline{P}_{b_s} \leqslant p_{b_s} \leqslant \overline{P}_{b_s} \tag{6}$$

where $\underline{P}_{b_s}$ and $\overline{P}_{b_s}$ denotes the minimum and maximum power that a storage system s can inject or remove from the power system, respectively.

In the modeling of the DERs, we have included the most general case of coefficients depending on the MCP. The utility and cost functions should include the price of the energy to capture the variations of the market. In this way, several possible method to obtain these coefficients can be used. Available data from the response of the DERs under several market variations might be employed in machine learning methods to obtain or adjust these coefficients. In order to illustrate a particular selection of these coefficient an example is presented.

Example 1. Consider for the cost functions in the generator model (3), a traditional economic criteria is to choose coefficients as

$$\alpha_n^g(\lambda) = \overline{\alpha}_n^g, \quad \beta_n^g(\lambda) = \overline{\beta}_n^g - \lambda,$$

then the cost function for each generator n is as follows

$$C_n\left(p_{g_n}\right) = \overline{\alpha}_n^g p_{g_n}^2 + \overline{\beta}_n^g p_{g_n} + \gamma_n^g - \lambda p_{g_n},$$

which reflects the concept of the utility of the generator ($-C_n$) as the difference between the profit of generation and the cost of production. Similarly, for the utility function of the smart loads we can choose coefficients as follows

$$\alpha_m^d(\lambda) = \overline{\alpha}_m^d, \quad \beta_m^d(\lambda) = \overline{\beta}_m^d - \lambda,$$

then the utility function for each smart load m is as follows

$$U_m\left(p_{d_m}\right) = \overline{\alpha}_m^d p_{d_m}^2 + \overline{\beta}_m^d p_{d_m} + \gamma_m^d - \lambda p_{d_m},$$

where it is clear that the utility of the consumers would be the difference between the utility of energy consumption and the cost of the energy.

2.4. Network losses model

Network losses in a distribution system are factors that can be determinant to reach the optimum social welfare. Network losses could be reduced making an optimal power flow, e.g., sometimes could be better generate with a generator more expensive if the load is near to the generator. Furthermore, losses varying through time depending on the load used by the consumers. It is not possible to solve the system in all possible cases of demand and power generation needed to solve the problem analytically [38]. However, there is a range of operation that meets all operational requirements and it is where our social welfare optimization constraints are stated. Then, we consider the network losses as constant in the range of operation and estimated by the Kron's formula as follows

$$P_{ls} = \sum_{k=1}^{N_l}\sum_{l=1}^{N_l} p_{g_k} B_{kl} p_{g_l} + \sum_{k=1}^{N_l} B_{k0} p_{g_k} + B_{00}, \tag{7}$$

where B_{kl},B_k0, and B_{00} are the loss factors, N_l is the number of generator considered in the estimation algorithm. In order to calculate an estimation of the network losses the algorithm in [39] is used. This algorithm estimates loss factors to give a robust estimation of the losses. The coefficients are considering different demand patterns using heuristic optimization algorithms. Then a robust set of coefficients for all the possible active power injections ($\underline{P}_{g_n} \leqslant p_{g_n} \leqslant \overline{P}_{g_n}$) are obtained. As a result of this estimation algorithm, it is obtained a constant power P_{ls} to be included in the algorithm as part of the load to be dispatched.

2.5. Social welfare optimization problem

The social welfare problem in a power system scenario considers to maximize the benefits of the consumers and to minimize the cost of the generators, it is based on costs (3) and utilities functions obtained from (1), the social welfare function is defined as

$$S_W(p_r, p_d, \lambda) = U(p_d, \lambda) - C(p_r, \lambda), \tag{8}$$

where the generation's cost function is the sum of (3) and (5) as follows

$$C\left(p_r, \lambda\right) = \sum_{n=1}^{N} C_n\left(p_{g_n}, \lambda\right) + \sum_{s=1}^{S} C_s\left(p_{b_s}, \lambda\right).$$

where $p_r = [p_g\, p_b]$ is a stacked vector where it is aggregated the power vector from traditional generators and distributed generators based on BESS technology. The utility function for the demand is the sum of individual utility functions as

$$U\left(p_d, \lambda\right) = \sum_{m=1}^{M} U_m\left(p_{d_m}, \lambda\right).$$

The main goal is to maximize (8) as it is presented in the following optimization problem

Problem 1.

$$\underset{p_r, p_d}{\text{maximize}} S_W(p_r, p_d, \lambda^*) \tag{9a}$$

$$\sum_{n=1}^{N} p_{g_n} + \sum_{s=1}^{S} p_{b_s} - \sum_{m=1}^{M} p_{d_m} = P_{ls} \tag{9b}$$

$$\overline{P}_{g_n} \geqslant p_{g_n} \geqslant \underline{P}_{g_n} \forall\, n \in \mathcal{N} \tag{9c}$$

$$\overline{P}_{b_s} \geqslant p_{b_s} \geqslant \underline{P}_{b_s} \forall\, s \in \mathcal{S} \tag{9d}$$

$$\overline{P}_{d_m} \geqslant p_{d_m} \geqslant \underline{P}_{d_m} \forall\, m \in \mathcal{M}, \tag{9e}$$

where $p_r = [p_g\, p_b]^\top$, and losses are represented by a constant power demand P_{ls}, which represents the topology of the network as it is explained in Section 2.4, and λ^* is the solution of Problem 2. The social welfare optimization problem (9a) can be related to a resource allocation problem, where the main objective is to allocate the requested resources by the system in an optimal way. The optimization problem (9a) is convex, the local objective function and the local constraint set are only known to the agent. Then, problem (9a) is strongly concave, and the local constraints are closed convex sets. In this work, we are not considering the transmission power flow limits or the voltage regulation in each node. We assume, as it is well-justified fro transmission networks [40], that the bus voltages magnitudes $|V_i| = |V_j| = 1$ pu for $i, j \in \mathcal{N}_i$ for all i. In addition, we assume that the bus voltage phase angles are not affected by reactive power flows. Other interesting possibility is to design a local controller on each DG to maintain the voltage within stable limits. The inclusion of these variables into the transactive energy scheme as dynamic constraints will lead to a more realistic and complete model. These further analysis are left for future work.

2.6. Distribution system operator problem

The main objective of the DSO is to determine the market clearing price (MCP) between energy producer and smart consumers. The DSO is solving a static optimization problem that maximize the welfare of the population over a market cycle (between 15 min (real-time) to hours, depending on the particular market and available technologies). We are considering a uniform optimal price λ for all DERs in the distribution system. The DSO is responsible for obtaining this optimal price anticipating the solution of the optimization problem and broadcast it to all DERs in the distribution system. We assume that the DSO is able to estimate or to forecast a base load based on previous data for a market period, which is assumed fixed during this period of time $\overline{P}_d$. It is observed that the coordination scheme casts as a Stackelberg game, as it has been noticed in [15] for demand response programs. In the objective function (10a), it is assumed that the generators will have chosen the optimal generation profile for a particular price λ considering the predicted load. This is modeled as constraint (10b), where is obtained $\overline{P}_r(\lambda)$ as the optimal generator profile for a given price λ. The optimal market clearing price is then obtained by solving the following optimization problem

Problem 2.

$$\underset{\lambda}{\text{maximize}} S_W\left(\overline{P}_r, \overline{P}_d, \lambda\right) \tag{10a}$$

$$\overline{P}_r\left(\lambda\right) = \underset{p_r}{\text{argmax}}\left(S_W\left(p_r, \ \overline{P}_d\right)\right). \tag{10b}$$

It is noticed that the DSO has as constraint another optimization problem, which based on the utility functions of agents might have a closed form. However, the proposed general method allows to consider several scenarios, where data-driven prediction techniques and advanced dynamic optimization methods could be used. In the next example, it is presented a simple case to illustrate Problem 2.

Example 2. Consider the utility and cost functions in Example 1 with the additional simplification such as that the constant terms are zero, i.e, $\gamma^g = \gamma^d = 0$, and there are no p_b. After some arithmetic, the social welfare function (8) is obtained as

$$S_W(p_r, p_d, \lambda) = U(p_d, \lambda) - C(p_r, \lambda)$$
$$= \sum_m \overline{\alpha}^d_m p^2_{d_m} - \lambda \sum_m p_{d_m} - \sum_n \overline{\alpha}^g_n p^2_{r_n} + \lambda \sum_n p_{r_n}$$

assuming $\overline{P}_d$ constant and differentiating S_W with respect to each p_r, we obtain for each r

$$\frac{dS_W}{dp_{r_n}} = -2\overline{\alpha}^g_n p_{r_n} + \lambda,$$

then we obtain the argminS_W in closed form

$$p_{r_n} = \frac{\lambda}{2\overline{\alpha}^g_n}.$$

We can now replace $p_{r_n}(\lambda)$ in the welfare function S_W and optimize with respect to λ. As a result for this particular case, we obtain the optimal price $\lambda^* = \frac{\overline{P}_d}{\sum_n 2\overline{\alpha}^g_n}$. Several computational tools are available to solve the optimization Problem 2, such as alternating method of multipliers [41], or nonlinear programming [7]. In Fig. 3, it is shown the information exchange between the DSO and DERs, where the MCP, λ^*, is obtained in Stage 1 and broadcasted to all DERs or that all DERs have access to this information. Then, in Stage 2 DERs use the MCP and solve dynamically Problem 1 considering the communication constraints between DERs. This process is repeated every market cycle when the total demand or the generation capacity could have changed. The main concern of this work is on the dynamic solution of Problem 1. In the next section, it is presented the main contribution of this work. A distributed transactive control algorithm to solve collectively the welfare optimization problem between DERs using the MCP previously solved by the DSO.

3. Distributed transactive control algorithm

In this section, it is presented the main contribution of this paper. We developed a distributed control algorithm considering distributed power integration and flexible power demand using transactive control based on population games. The transactive Control is responsible for the determination of how much energy each user will consume and how much power each generator have to generate in order to satisfy the demand while operating the system in the most economical way. This process is carried out dynamically, which means that supply and demand are continuously changing and they needs to be balanced to avoid any unstable equilibrium.

In this work, we use a distributed replicator dynamics equation to solve the social welfare problem dynamically associated to a transactive control problem between DERs using the optimal price broadcasted by

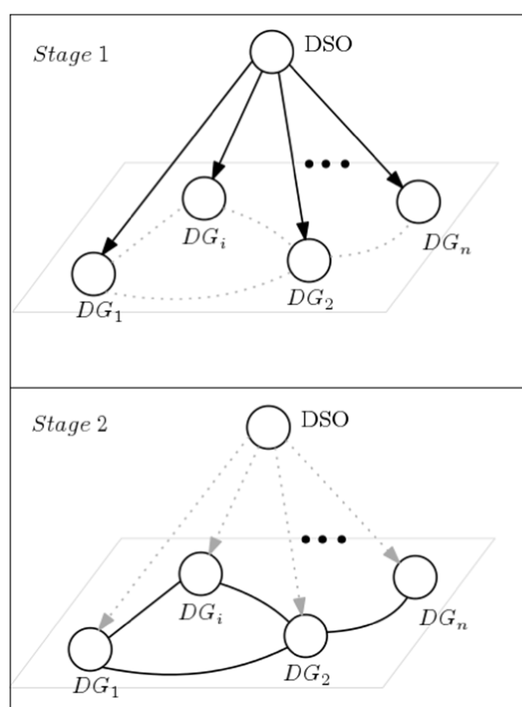

Fig. 3. Distribution system operator information exchange.

the distribution system. This model is a dynamic resource allocation problem among K DERs (generators and smart loads) given a finite amount of power, where $K = N + S + M$. The constraints in Problem 1 could be represented in the form $\mathbf{H}^\top p = P_{ls}$, where the decision variables are stacked in a vector as $p^\top = [p_r, p_d]^\top = [p_1, p_2, \ldots, p_K]^\top$, and $\mathbf{H}$ is a vector indicating the sign depending if the entry is a generator (+) or a smart load (−).The feasible set, the simplex, is defined as the set of possible trajectories of the system restricted by Problem 1 constraints as

$$\Delta = \left\{ p \in \mathbf{R}^K : \mathbf{H}^\top p = P_{ls} \right\}.$$

Some assumptions have to be considered to assure optimality conditions and the existence of solutions of Problem 1.

Assumption 1. It is assumed that the variables start inside the feasible set Δ, i.e., $p(0) \in \Delta$.

Assumption 2. The communication graph connecting each agent of the system, generators and smart loads, in the distribution system is connected.

Assumption 3. Every social welfare function $S_{Wi}(p_i)$ is assumed to be differentiable everywhere and with Lipschitz continuous derivative.

Lemma 1. There exists $p^* = (p_i^*)$ for all i with $p^* \in \Delta$, which achieves the minimum in Problem 1.

Proof. Sketch of the Proof: Given that the optimization problem is strongly concave by Kunh-Tucker first order conditions [7], it is possible to establish that the problem only has one maximum point inside the feasible set.

In the next subsection, it is introduced the basic concepts of population games, which are the cornerstone of the proposed algorithm.

3.1. Population games basics

Considering that the payoff functions of each DERs depends on the decision variables of other DERs, hence the solution of the TE system is an equilibrium. The solution concept of finding this equilibrium can be complicated because the sequential scheme of the decision making when a large set of agents is considered. Population games emerge as a distributed solution to find the Nash equilibrium when we are dealing with a large set of agents (population). One of the most well know population games is the replicator dynamics. Replicator dynamics has been used in several engineering application considering some benefits in implementation such as real-time adaptation and robustness to dynamic environmental uncertainties [4]. Replicator dynamics describe how a mass of players evolve in time while they choose a strategy. Those agents follow a strategy selected from among a finite set of pure strategies $\mathcal{V} = \{1, 2, \ldots, K\}$. Due to the size of the population, the analysis focuses through a payoff function associated with the chosen strategy. The dynamic population model describes how a pure population strategy changes through time. The replicator dynamics consider a $N+S$ finite number of generators and M limited number of consumers in the system, who adopt the i-th strategy from a finite set of pure strategies $\mathcal{V}$. Agents choosing the i-th strategy to play get a reward depending on the population state. This reward is represented by a fitness function $F_i : \Delta \mapsto \mathbb{R}$. Therefore, $F_i(p)$ defines the reward associated with the strategy $i \in \mathcal{V}$ for a particular population state p. The replicator dynamics equation has been proposed to describe evolutionary games capturing the evolutionary process where the size of the population of agents playing the most successful strategy (higher payoff than the average) tends to increase. This is represented by a simple first-order differential equation, where the population of agents playing the i-th strategy with fitness function F_i compare its success with the population of agents playing the other strategies through the average fitness function $\overline{F} = \sum_j^K p_j F_j$. This function is given by the weighted average of the fitness of the K types in the population. Considering that we normalize the equation with the term $\frac{1}{P_{ls}}$, each p_i is equivalent to the proportion of power in strategy i, and the function $\overline{F}$ can be interpreted as the expected average payoff of the population. The traditional replicator dynamics equations for each population of agents playing the i-th strategy is as follows

$$\dot{p}_i = \left(\frac{1}{P_{ls}}\right) p_i \left(F_i - \overline{F}\right), \quad \text{forall } i \in \mathcal{V}. \tag{11}$$

The replicator dynamics Eq. (11) has been originally developed for well-mixed populations, where all agents have access to the fitness functions of all the others through the average fitness function. Since we are interested in a distributed mechanism to solve the welfare optimization problem, we have to use a constrained interaction environment. Recently, the strategy-constrained interactions has been modeled using an graph $\mathcal{G}$ (see Section I.A for details of the graphs concepts) to represent which nodes are sharing information. The distributed replicator dynamics (DRD) equations based only on information from its neighbors $\mathcal{N}_i$ to evolve can be represented as

$$\dot{p}_i = \left(\frac{1}{P_{ls}}\right) p_i \left(F_i\left(p_i\right) \sum_{j \in \mathcal{N}_i} p_j - \sum_{j \in \mathcal{N}_i} p_j F_j\left(p_j\right) \right), \tag{12}$$

where F_i is the fitness function associated with each strategy, and $\mathcal{N}_i$ is the neighborhood of agent i sharing information through a communication network. In [4], we can find the simplex invariance for the distributed replicator dynamics, which means that if $p(0) \in \Delta$, the dynamic variable $p_i(t)$ evolves inside Δ. This is an important characteristic of (12), since it makes a correspondence with Lemma 1 in assuring that $p_i(t_f)$ will be in Δ guaranteeing the existence of a solution, where t_f is the final execution time of the distributed replicator dynamics.

The fundamental concept in the design of the population games is the appropriate selection of the fitness function such that the distributed replicator dynamic Eq. (12) can be used as a constrained distributed optimization algorithm. In order to obtain fitness functions to accomplish a stable and convergence solution, we use the following Lemma 2, which characterizes an optimal solution to Problem 1 satisfying Assumption 3 [42].

Lemma 2. A solution of Problem 1, p^* belonging to the feasible set Δ, is an optimal solution if and only if $\nabla S_{Wi}(p_i^*) = \nabla S_{Wj}(p_j^*)$ for all i, j.

Notice that ∇ stands for the Jacobian of a function. As it has been mentioned, the solution concept of the TE system is an equilibrium, and the Nash equilibrium is the preferred solution concept in population games. Consider the following definition of the Nash Equilibrium for population games. Let F be a population games, the Nash equilibrium set is defined as follows

$$\mathbf{NE}(F) = \left\{ p^* \in \Delta : \ p_i^* > 0 \Rightarrow F_i(p^*) \geqslant F_j(p^*), \forall\, i, j \right\}.$$

All agents (players) obtain the same profit in a Nash equilibrium. In order to associate the Problem 1 with population games, we focus on a particular type of population games: Potential games. Potential games are defined as follows.

Definition 1. Let $F : \mathbb{R}_+^K \mapsto \mathbb{R}^K$ represent a fitness vector with positive population game outcomes (payoffs) defined in the non-negative orthant. Let $V : \mathbb{R}_+^K \mapsto \mathbb{R}$ be a continuously differentiable function that meets the condition $\nabla V(p) = F(p), \forall p \in \mathbb{R}_+^K$, then F is a full potential game.

One of the main attribute of potential games is the existence of a single scalar-valued function, called potential function, which captures all relevant information about payoffs of the agents. Assuming there exists a continuously differentiable potential function $V : \mathbb{R}_+^K \mapsto \mathbb{R}$, then a potential game satisfies the following relationship for the fitness function for each agent

$$\frac{\partial V(p)}{\partial p_i} = F_i\left(p\right) \quad \text{forall } i. \tag{13}$$

Eq. (13) implies that the population game must satisfy the external symmetry defined as

$$\frac{\partial F_i}{\partial p_j} = \frac{\partial F_j}{\partial p_i} \quad \text{for all } i, j. \tag{14}$$

In potential games, there is a property that relate the Nash equilibrium to local maximizers of potential functions [43], it is stated in the following proposition.

Proposition 1. A potential game with potential function V satisfies that Nash equilibrium of the potential game is equal to the solution of the Kuhn-Tucker conditions of the optimization problem maximize $V(p)$ subject to a feasible set Δ.

On the other hand, we are interested in the stability of the game. An important result in population games about stability states that a population game satisfying (13) is a stable game if the potential function V is concave [43].

Having introduced the main concepts of potential population games, it is possible to present a definition for the fitness function relating the population game as a potential game guaranteeing optimality and stability conditions. We introduce the Lagrangian function associated with optimization Problem 1 as

$$\mathcal{L}(p,\mu) = S_W(p) - \mu^\top(\mathbf{H}^\top p - P_{ls}), \tag{15}$$

where μ are the Lagrange multipliers, S_W is the social welfare function, and $\mathbf{H}p - P_{ls}$ are the constraints in the system. Therefore, the fitness functions can be defined as

$$F_i(p_i) = \nabla_p \mathcal{L} = \nabla_p S_W - \mathbf{H}\mu, \tag{16}$$

which introduces an extended system including the Lagrange multipliers dynamics with fitness function defined as

$$F_k(\mu_k) = \nabla_\mu \mathcal{L} = -(\mathbf{H}^\top p - P_{ls}), \tag{17}$$

Accordingly, to achieve an appropriate performance in steady state, the demanded power load should be the sum of all power set points.

Finally the dynamic equation for Lagrange multipliers μ is

$$\dot{\mu}_k = \mu_k\left(F_k\left(\mu_k\right)\sum_{j\in\mathcal{N}_i}\mu_j - \sum_{j\in\mathcal{N}_i}\mu_j F_j\left(\mu_j\right)\right) \tag{18}$$

where $k = \{1, \ldots, K+1\}$ with $K+1$ as the number of constraints. In the next subsection it is presented the optimality and stability analysis of the extended dynamical system including $p_i(t)$ and $\mu_k(t)$. In order to relate the optimality condition in Lemma 2 with the distributed replicator dynamics (12), we have observed that the optimal solution (p^*, μ^*), which can be found by the Kuhn-Tucker first-order conditions for maximization establishes that p^* is a unique solution to (9a) if (p^*, μ^*) is a saddle point of $\mathcal{L}(p,\mu)$ as is shown in the next subsection.

3.2. Optimality analysis

In this subsection, we prove that our proposed game reaches an equilibrium point, and this equilibrium point is an optimum in the power system. First, consider the extended population game defined by distributed replicator dynamics (12) and Lagrange multiplier dynamics (18) with fitness functions (16) and (17), respectively. It is necessary to show that the extended population game is a potential game to guarantee optimality through Lemma 2 and Proposition 1. Then, to show the game is stable, we have to verify that the potential function V is concave and twice continuously differentiable.

Since we have defined fitness functions as (16) and (17), by definition it is clear that a potential function for the population game (12) is

$$V(p,\mu) = \mathcal{L}(p,\mu) = S_W(p) - \mu^\top(\mathbf{H}p - P_{ls}),$$

considering the form of the social welfare function S_W defined in (8), it can be shown that the game satisfies the externality symmetry (14). When the optimality condition in Lemma 2 is reached then $F_i(p_i) = F_j(p_j)$ for all i, j and it is noticed that in the distributed replicator dynamics (12) we have the term

$$\left(F_i\left(p_i\right)\sum_{j\in\mathcal{N}_i}p_j - \sum_{j\in\mathcal{N}_i}p_j F_j\left(p_j\right)\right),$$

and considering that $F_i(p_i) = F_j(p_j)$ at an equilibrium point then

$$\begin{aligned} &= \left(F_i\left(p_i\right)\sum_{j\in\mathcal{N}_i}p_j - \sum_{j\in\mathcal{N}_i}p_j F_i\left(p_i\right)\right) \\ &= \left(F_i\left(p_i\right)\left(\sum_{j\in\mathcal{N}_i}p_j - \sum_{j\in\mathcal{N}_i}p_j\right)\right) \\ &= 0, \end{aligned}$$

which implies that (12) reaches an equilibrium point. The equilibrium point is the unique equilibrium point in the system. Hence, it is the optimal point.

3.3. Stability analysis

For the stability analysis, we need to verify that the potential function $V(p,\mu)$ is concave to assure the stability of the algorithm. To prove that the function is concave, we need to check that the Hessian matrix of $V(p,\mu)$ is semidefinite negative, i.e, $\nabla^2 V(p,\mu) \leqslant 0$. The Jacobian is obtained as

$$\nabla V\left(p,\mu\right) = \begin{bmatrix} \nabla_p V \\ \nabla_\mu V \end{bmatrix} = \begin{bmatrix} \nabla S_W - \mu^\top \mathbf{H} \\ -(\mathbf{H}\mu + P_{ls}) \end{bmatrix}.$$

Hence, the Hessian is obtained as

$$\nabla^2 V\left(p,\mu\right) = \begin{bmatrix} \nabla_p^2 V \\ \nabla_\mu^2 V \end{bmatrix} = \begin{bmatrix} \nabla^2 S_W \\ 0 \end{bmatrix}.$$

The problem to check the semi-definiteness of the Hessian of V reduces to check the Hessian of $\nabla^2 S_W$. Recall that S_W is as in (8), and deriving twice it is obtained the Hessian as

$$\nabla^2 S_W = \sum_{m=1}^{M}\alpha_m^d - \sum_{n=1}^{N}\alpha_n^g - \sum_{s=1}^{S}\alpha_s^b.$$

As a result to guarantee that $\nabla^2 S_W \leqslant 0$, we obtain a relationship between the coefficients of cost functions of the generators and utility functions of the smart loads as follows

$$\sum_{m=1}^{M}\alpha_m^d \leqslant \sum_{n=1}^{N}\alpha_n^g + \sum_{s=1}^{S}\alpha_s^b \tag{19}$$

If condition (19) is satisfied then the algorithm based on the DRD is stable. In this section, it has been analyzed the main features of the proposed algorithm in terms of optimality and stability. In the next section, several case studies are presented to illustrate the effectiveness of the distributed replicator dynamics to solve the social welfare problem.

4. Simulation results

In this section, a distribution system with DERs is simulated in

Matlab to study the behavior of the proposed approach. The system is a modified IEEE 30-bus system as it can be seen in Fig. 2. There are eight distributed generators with limited power generation and a communication network between generators and consumers in this system. Generator 8 is chosen as a renewable generator with a storage capacity node, while the other generators are traditional generators. There are five consumers able to apply changes in their loads depending on the system state, all of them have a predetermined base load. Consumers aim to optimize the load to obtain the best benefit. Consumers and generators can communicate with some generators and consumers in their neighborhood through a communication network. Two simulation case studies are performed to validate the transactive model. First, it is considered the case where the BESS is giving power to the network. At this time, the BESS is considered as a generator. In the second case, the BESS is considered a load, and the dynamic transactive control is used under these conditions. The data for case studies is presented in Table 1. Finally, our method is compared with a traditional economic dispatch algorithm, where the algorithm is usually executed discretely. It means that instead of having a dynamic economic dispatch, the algorithm is executed every certain time frame.

4.1. *Case Study I: transactive control with BESS as a generator*

In this case, a BESS is acting as a generator in the system. The distribution system is composed of eight generators and four smart loads. The proposed transactive control system allows the seven generators and the BESS to generate power to meet the requirements of demand imposed by the loads. These loads are assumed as responsive loads.

In both case study, the power load in the system is varying as it is shown in Fig. 4. The power required to the network begins in 20 kW and in 500 iterations a ramp is started until it reaches 1000 iterations. At this point, the power required for the system is exactly 25 kW. The power required must be optimally supplied for the generators. Furthermore, smart loads will be sensitive to this change in the system and change consumption. Fig. 5 shows how the BESS gives power to the system 1588 W before the iteration 500. Notice that the BESS as a generator is the smallest contributor to the power system due to its highest cost reflected in its cost function. It is possible to see that generator five, which has the lowest cost, is the generator that delivers more power to the power system. In 500 iterations, it is observed a clear tendency of rising the power delivered to the power system until iteration 1500. The BESS agent rises its power delivered to 2393 W in this period of time, which is a rise considerably lower than the aforementioned generator five, which

Table 1
System Parameters Simulation

	$\overline{P_{g_i}}$	$\underline{P_{g_n}}$	β_{g_n}	ρ_{g_n}
Generators				
1	10 [kW]	0	0.04	8
2	8 [kW]	0	0.02	7
3	9 [kW]	0	0.03	6
4	10 [kW]	0	0.035	5
5	9 [kW]	0	0.03	4
6	8 [kW]	0	0.035	3
7	8 [kW]	0	0.04	2
Consumers				
	$\overline{P_{d_m}}$	$\underline{P_{d_m}}$	β_{d_m}	ρ_{d_m}
1	6 [kW]	0	0.06	8
2	5.5 [kW]	0	0.05	7
3	5.5 [kW]	0	0.04	6
4	6 [kW]	0	0.03	5
5	7 [kW]	0	0.04	4
Battery Energy System Storage				
	$\overline{P_{b_s}}$	$\underline{P_{b_s}}$	β_{b_s}	ρ_{b_s}
1	3.5 [kW]	1 [kW]	0.09	0

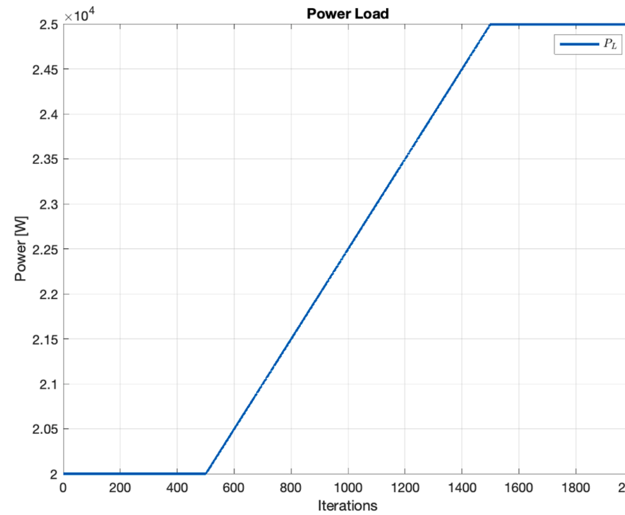

Fig. 4. Varying power load.

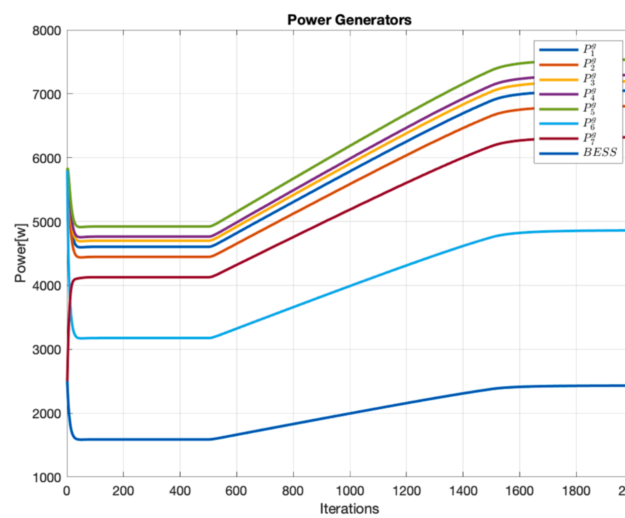

Fig. 5. Power delivered by the generators in the case study 1. BESS generator is identified as P_8^g.

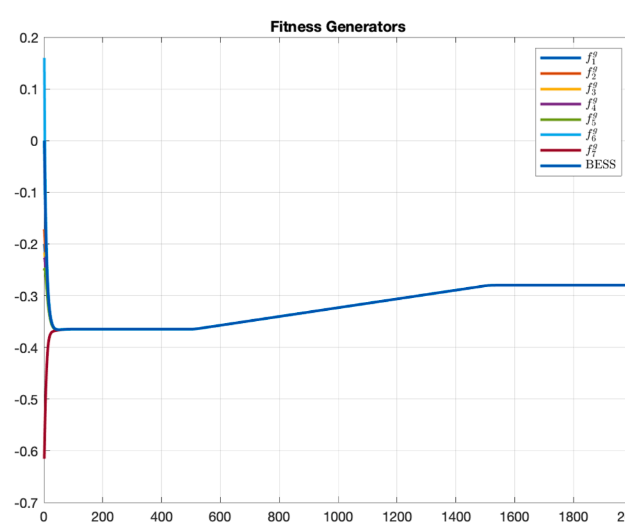

Fig. 6. Fitness function of the generators in the case study 1. BESS generator is identified as P_8^g.

rises from 4925 W to 7436 W in the same time frame. In addition, it is shown that all fitness functions converge to the same value (see Fig. 6)

Fig. 7 shows the consumer behavior in the power system. The consumers present an inverse behavior as the generators. As the fixed power load increases, the smart loads ask for lower power. It also shows how they converge to the same utility value, as it was predictable in the stability analysis section.

It is possible to observe that the more power is delivered/consumed, the more utility is perceived by the agents. Therefore, it is a natural shape of the fitness functions, which are shown in Fig. 8 and Fig. 6.

4.2. Case Study II: transactive control with BESS as a smart consumer

In this case study, a BESS is acting as a consumer in the system, i.e., it is storing energy. The distribution system is composed of seven generators and five smart loads. In this case, the proposed transactive control system allows the seven generators to generate power to meet the requirements of demand imposed by loads and the BESS. The test is the same implemented in case study I, which is shown in Fig. 4.

Fig. 9 shows a similar pattern observed in case study I. However, it is possible to distinguish that the generators are more stressed because they have to produce more power given that the BESS agent is acting as a consumer. As mentioned before, it is possible to observe that the generators fitness functions in Fig. 10 are bigger than in case I. This is also a direct consequence of generating more power. Fig. 11 shows the consumer behavior in the power system. Consumers present an inverse behavior as the generators. It also shows how they converge to the same utility value, as it was predictable in the stability section analysis (see Fig. 12).

It is shown how the BESS is the third consumer implying that for this BESS, the system maximizes its utilities consuming power at this time. Furthermore, we can see that in both cases, when the BESS system is acting as a generator or as a consumer, the fitness equilibrium point is lower than if the BESS system is disconnected.

4.3. Comparison with traditional economic dispatch

In this section a comparison between our proposed scheme and a traditional economic dispatch scheme is presented. The classical economic dispatch scheme aims to solve the generators dispatch problem each certain time frame. In our case, this time frame is defined as 400 iterations considering the load variations as external fixed input.

In Fig. 13 is shown that we overlap the traditional economic dispatch method and our proposed algorithm. Remarkably, our method tracks the

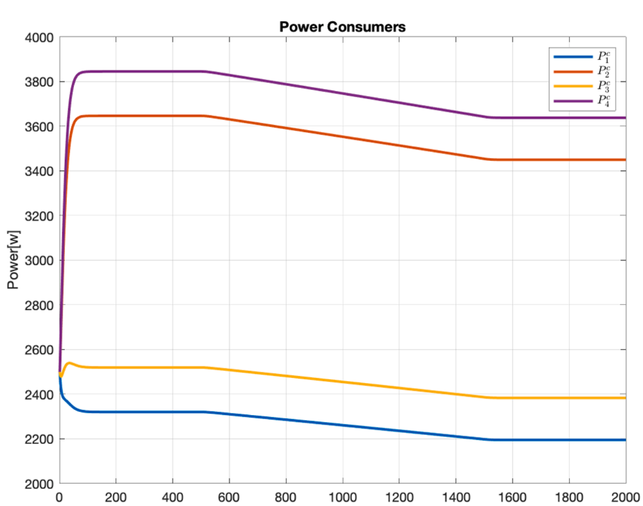

Fig. 7. Power Consumed by the smart loads in the case study 1.

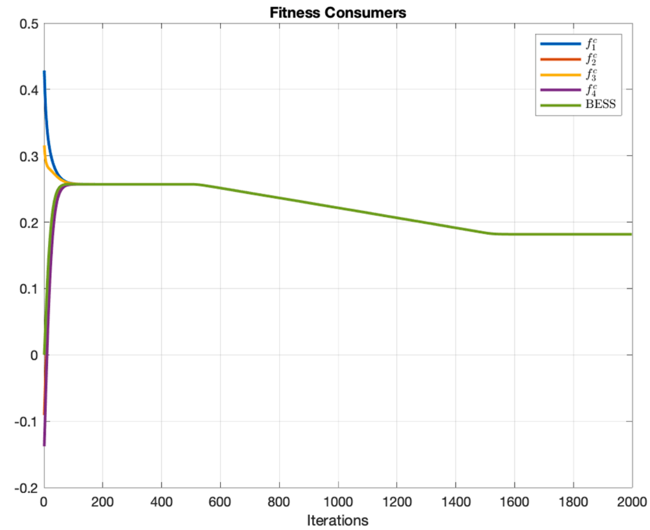

Fig. 8. Fitness function of the smart loads in the case study 1.

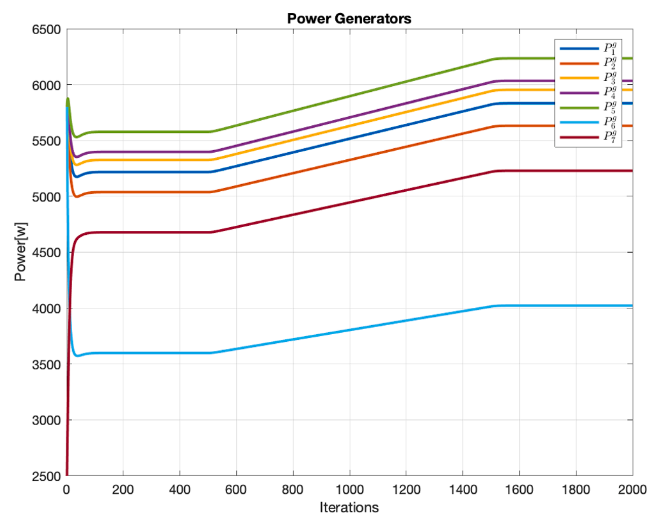

Fig. 9. Power delivered by the generators in the case study 2.

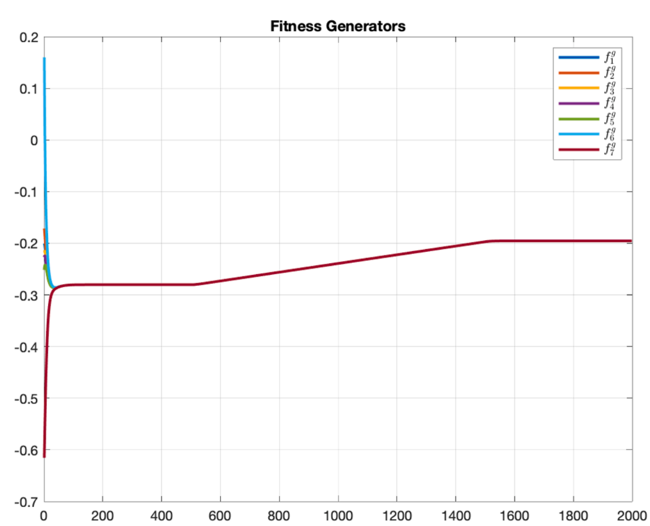

Fig. 10. Fitness function of the generators in the case study 2.

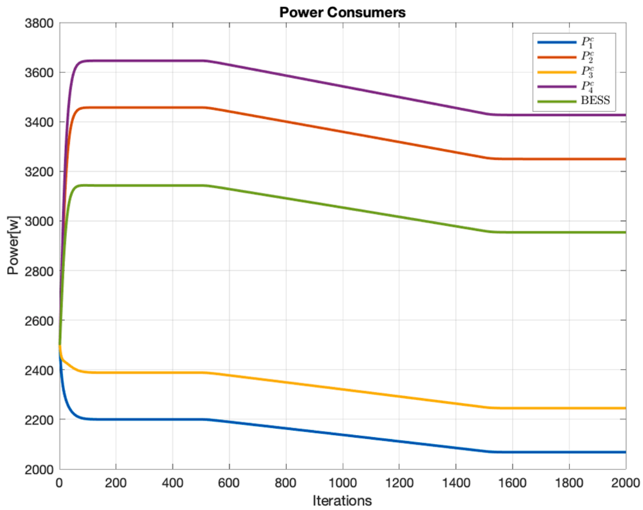

Fig. 11. Power Consumed by the smart loads in the case study 1.

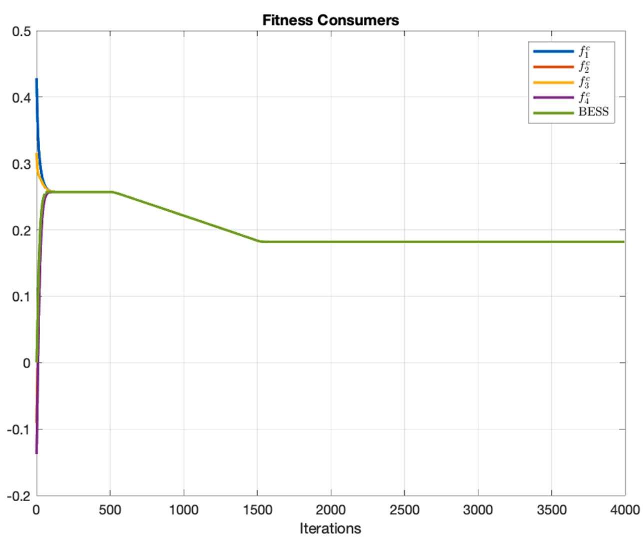

Fig. 12. Fitness function of the smart loads in the case study 1.

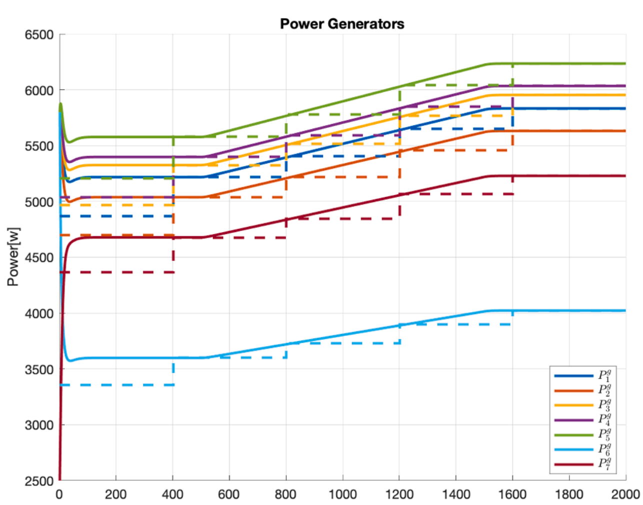

Fig. 13. Comparison between the traditional dispatch (- - -) and our proposed algorithm.

optimal points at each iteration, tracking the system optimal allocation point. However, the traditional technique is only optimal in the points where it is executed. This results in a clear advantage for the proposed strategy in terms of costs and smoothness of the responses through highly variable loads.

5. Conclusion and future work

A distributed replicator dynamics protocol has been proposed to solve a social welfare optimization problem between distributed generators and smart loads in a transactive control framework. The proposed algorithm considers utility functions of generator and consumers in order to reach the optimum social welfare dynamically while maintain some system constraints. Distributed transactive controllers are capable of handling problems with distributed information on distribution systems. As a future work, this algorithm could include a dynamical calculation of losses in the system and an algorithm running in parallel to estimate the energy valuation of each consumer agent.

Declaration of Competing Interest

None.

CRediT authorship contribution statement

Eder Baron-Prada: Conceptualization, Methodology, Software. **Eduardo Mojica-Nava:** Writing - original draft, Writing - review & editing, Investigation, Supervision.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.ijepes.2021.106874.

References

[1] Farhangi H. The path of the smart grid. IEEE Power Energy Magaz, 8, 1.
[2] Lamnabhi-Lagarrigue F, Annaswamy A, Engell S, Isaksson A, Khargonekar P, Murray RM, Nijmeijer H, Samad T, Tilbury D, den Hof PV. Control for the future of humanity, research agenda: Current and future roles, impact and grand challenges. Annu Rev Control 2017;43:1–64. https://doi.org/10.1016/j.arcontrol.2017.04.001. http://www.sciencedirect.com/science/article/pii/S1367578817300573.
[3] Anderson D, Zhao C, Hauser CH, Venkatasubramanian V, Bakken DE, Bose A. A virtual smart grid. IEEE Power Energ Mag 2012;10(1):49–57.
[4] Quijano N, Ocampo-martinez C, Barreiro-gomez J, Obando G, Pantoja A, Mojica-nava E, Ocampo-martinez POCC. The role of population games and evolutionary dynamics in distributed control systems. IEEE Control Syst 2017;37(1):70–97.
[5] Mojica-Nava E, Macana CA, Quijano N. Dynamic population games for optimal dispatch on hierarchical microgrid control. IEEE Trans Syst, Man, Cybernet: Syst 2014;44(3):306–17. https://doi.org/10.1109/TSMCC.2013.2266117. arXiv:1401.1767.
[6] Baron-Prada E, Osorio E, Mojica-Nava E. Resilient transactive control in microgrids under dynamic load altering attacks. In: 2017 IEEE 3rd Colombian conference on automatic control (CCAC); 2017. p. 1–5.
[7] Bertsekas DP. Nonlinear programming. Athena scientific Belmont; 1999.
[8] Macana CA, Pota HR. Optimal Energy Management System for Strategic Prosumer Microgrids: An average bidding algorithm for prosumers aggregators. In: Asian Control Conference - ASCC; 2017.
[9] Nedic A, Ozdaglar A. Distributed subgradient methods for multi-agent optimization. IEEE Trans Autom Control 2009;54(1):48–61.
[10] Nedic A, Ozdaglar A, Parrilo PA. Constrained consensus and optimization in multi-agent networks. IEEE Trans Autom Control 2010;55(4):922–38.
[11] Nedić A, Olshevsky A. Distributed optimization over time-varying directed graphs. IEEE Trans Autom Control 2015;60(3):601–15. https://doi.org/10.1109/TAC.2014.2364096. arXiv:1303.2289.

[12] Ho Y, Servi L, Suri R. A class of center-free resource allocation algorithms. Large Scale Syst 1980;1(1):51–62.

[13] Uribe CA, Lee S, Gasnikov A, Nedić A. Optimal algorithms for distributed optimization. Optimization 2017:1–18. arXiv:1712.00232. URL http://arxiv.org/abs/1712.00232.

[14] Tsitsiklis JN. Problems in decentralized decision making and computation., Tech. rep., PhD. dissertation, Dept. Elect. Eng. Comp. Sci., Massachusetts Institute of Technology, Cambridge; 1984.

[15] Saad W, Han Z, Poor HV, Basar T. Game-theoretic methods for the smart grid: An overview of microgrid systems, demand-side management, and smart grid communications. IEEE Signal Process Mag 2012;29(5):86–105. https://doi.org/10.1109/MSP.2012.2186410.

[16] Li N, Marden JR. Decoupling coupled constraints through utility design. IEEE Trans Autom Control 2014;59(8):2289–94.

[17] Groot N, De Schutter B, Hellendoorn H. On systematic computation of optimal nonlinear solutions for the reverse stackelberg game. IEEE Trans Syst, Man, Cybernet: Syst 2014;44(10):1315–27. https://doi.org/10.1109/TSMC.2014.2311756.

[18] Kok K, Widergren S. A society of devices: integrating intelligent distributed resources with transactive energy. IEEE Power Energ Mag 2016;14(3):34–45. https://doi.org/10.1109/MPE.2016.2524962.

[19] Asmus P. Microgrids, virtual power plants and our distributed energy future. Electr J 2010;23(10):72–82.

[20] Mojica-Nava E, Barreto C, Quijano N. Population games methods for distributed control of microgrids. IEEE Trans Smart Grid 2015;6(6):2586–95. https://doi.org/10.1109/TSG.2015.2444399.

[21] Barreto C, Mojica-Nava E, Quijano N. Incentive mechanisms to prevent efficiency loss of non-profit utilities. Int J Electr Power Energy Syst 2019;110:523–35.

[22] Samadi P, Mohsenian-Rad H, Schober R, Wong VW. Advanced demand side management for the future smart grid using mechanism design. IEEE Trans Smart Grid 2012;3(3):1170–80.

[23] Bejestani AK, Annaswamy A, Samad T. A hierarchical transactive control architecture for renewables integration in smart grids: Analytical modeling and stability. IEEE Trans Smart Grid 2014;5(4):2054–65. https://doi.org/10.1109/TSG.2014.2325575.

[24] Hu J, Yang G, Kok K, Xue Y, Binder HW. Transactive control: a framework for operating power systems characterized by high penetration of distributed energy resources. J Modern Power Syst Clean Energy 2017;5:451–64. https://doi.org/10.1007/s40565-016-0228-1.

[25] R.B. Melton, Gridwise transactive energy framework, Tech. rep., Pacific Northwest National Lab.(PNNL), Richland, WA (United States) (2013).

[26] Shiltz DJ, Cvetković M, Annaswamy AM. An integrated dynamic market mechanism for real-time markets and frequency regulation. IEEE Trans Sustain Energy 2016;7(2):875–85. https://doi.org/10.1109/TSTE.2015.2498545.

[27] Ambrosio R. Transactive energy systems. IEEE Electr Magaz 2016;4(4):4–7. https://doi.org/10.1109/MELE.2016.2614234. http://ieeexplore.ieee.org/document/7725890/.

[28] Abrishambaf O, Lezama F, Faria P, Vale Z. Towards transactive energy systems: An analysis on current trends. Energy Strat Rev 2019;26:100418.

[29] Knudsen J, Hansen J, Annaswamy AM. A dynamic market mechanism for the integration of renewables and demand response. IEEE Trans Control Syst Technol 2016;24(3):940–55. https://doi.org/10.1109/TCST.2015.2476785.

[30] Zia MF, Benbouzid M, Elbouchikhi E, Muyeen S, Techato K, Guerrero JM. Microgrid transactive energy: Review, architectures, distributed ledger technologies, and market analysis. IEEE Access 2020;8:19410–32.

[31] GJR, Mas-Colell A, Whinston M. Microeconomic Theory. New York: Oxford University Press; 1995.

[32] Bullo F. Lectures on network systems, Version 0.96, 2018, with contributions by J. Cortes, F. Dorfler, and S. Martinez.

[33] Lee J, Guo J, Choi JK, Zukerman M. Distributed energy trading in microgrids: a game theoretic model and its equilibrium analysis. IEEE Trans Industr Electron 2015;62(6):1. https://doi.org/10.1109/TIE.2014.2387340.

[34] Chen C, Duan S, Cai T, Liu B, Hu G. Smart energy management system for optimal microgrid economic operation. IET Renew Power Gener 2011;5(3):258. https://doi.org/10.1049/iet-rpg.2010.0052. http://digital-library.theiet.org/content/journals/10.1049/iet-rpg.2010.0052.

[35] García F, Bordons C. Optimal economic dispatch for renewable energy microgrids with hybrid storage using model predictive control. In: IECON 2013–39th Annual Conference of the IEEE industrial electronics society; 2013. p. 7932–7. https://doi.org/10.1109/IECON.2013.6700458.

[36] Macana CA, Pota HR, Mohiuddin SM, Mahmud MA. Online Energy Management Strategy for Islanded Microgrids with Feedback Linearizing Inner Controllers. In: 2017 IEEE innovative smart grid technologies - Asia (ISGT-Asia); 2017. p. 1–6. https://doi.org/10.1109/ISGT-Asia.2017.8378319.

[37] Shi W, Li N, Chu CC, Gadh R. Real-time energy management in microgrids. IEEE Trans Smart Grid 2017;8(1):228–38. https://doi.org/10.1109/TSG.2015.2462294.

[38] Zhu J. Optimization of power system operation, Vol. 47. John Wiley & Sons; 2015.

[39] Mojica-Nava E, Rivera S, Quijano N. Game-theoretic dispatch control in microgrids considering network losses and renewable distributed energy resources integration. IET Gener, Transmiss Distrib 2017;11(6):1583–90. https://doi.org/10.1049/iet-gtd.2016.1486.

[40] Kundur P. Power system stability and control. New York, NY, USA: CRC Press; 2007.

[41] Parra OI, Cruz J, Mojica-Nava E. Admm for transactive control of microgrids. In: 2019 IEEE Milan PowerTech, IEEE; 2019. p. 1–6.

[42] Lakshmanan H, De Farias DP. Decentralized resource allocation in dynamic networks of agents. SIAM J Optim 2008;19(2):911–40.

[43] Sandholm WH. Population games and evolutionary dynamics. MIT Press; 2010.