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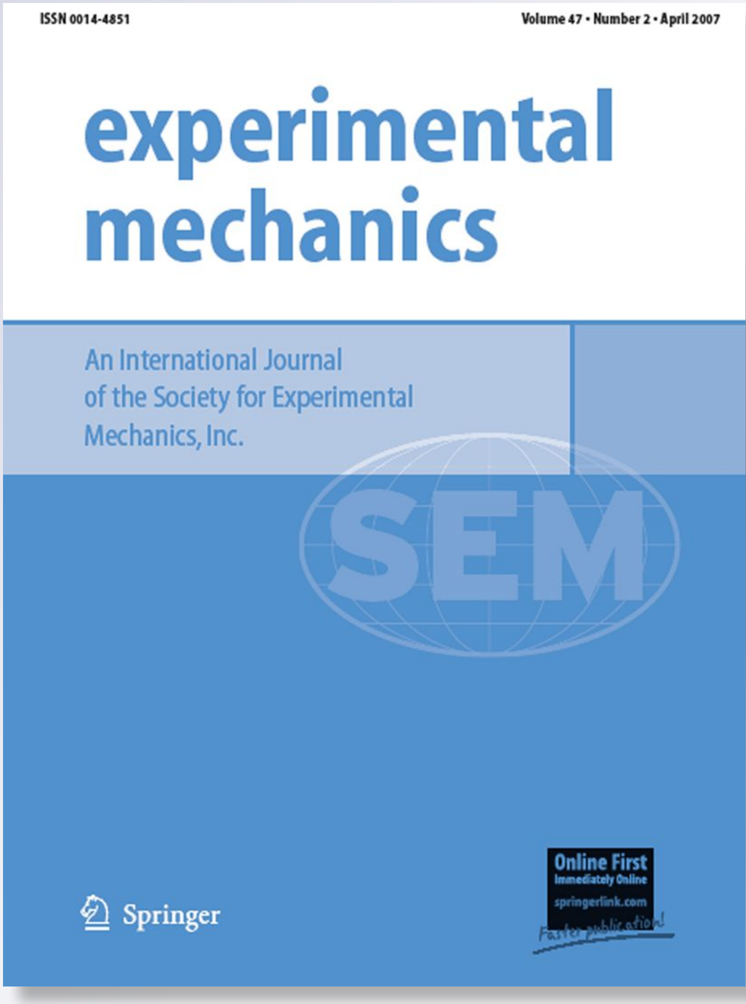

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Abstract Correlating damage level and changes in dynamic characteristics of a structure forms the basis for damage detection techniques in structural health monitoring. In reinforced concrete building structures such correlation is not well established. A damage detection technique capable of identifying the structural condition of the system based on its small amplitude vibration response is desirable because such response is easier to obtain. It is a common practice in engineering applications to estimate dynamic parameters from small-amplitude vibrations assuming a linear behavior of the structure. This simplification causes inaccurate estimation of the dynamic properties in reinforced concrete structures due to the presence of nonlinear elastic behavior. In this study no such assumption is made and a linear model is only used for sets of data corresponding to the same displacement amplitude of a nonlinear elastic system. The trends found between small-amplitude vibration dynamic properties and past levels of maximum displacement in various reinforced concrete structures are reported. In addition to analytical and numerical studies, results from a series of laboratory tests are reported to demonstrate the use of the approach. One full-scale three-story reinforced concrete flat-plate building and six small-scale reinforced concrete beams were examined. In this study, small displacements are defined as displacements below an overall drift ratio of 0.03%. The displacement dependence of the dynamic properties is considered explicitly. It was found that while fundamental frequencies of the examined reinforced concrete specimens were found to decrease uniformly as past peak displacement level increased, the equivalent viscous damping ratio was found to increase until the past peak displacement reached the neighborhood of nominal yield displacement and then observed to decrease when the specimens are pushed beyond the nominal yield displacement level, which has not been reported in literature before. Recommendations are provided as to how small amplitude vibration tests should be set up to avoid misleading observations due to nonlinear response at small amplitude response, observations that could lead to erroneous conclusions regarding the damage state of a structure.

Keywords Dynamic testing · Nonlinear elasticity · Reinforced concrete · Frequency · Equivalent viscous damping · Structural health monitoring · Damage detection

Introduction

Vibration-based structural damage detection is a growing field in different engineering disciplines. Damage detection has been typically carried out by monitoring variations in physical parameters of the structure. Extensive work has been done in the past to provide tools to estimate such physical parameters through their vibration records [10]. Most of the structural health monitoring methods are focused on finding either shifts in natural frequency or changes in structural mode-shapes [5]. The use of natural frequency as a diagnostic parameter has its basis on the assumption that natural frequencies are sensitive indicators of structural integrity [24]. Many damage detection techniques are based on the assumption that damage causes nonlinear behavior in structures [23, 27] due to development of cracks. Nonlinear behavior has been recognized to initiate

F. Consuegra
Ingetec S.A.,
Carrera 6 No. 30A-30,
Bogotá, DC, Colombia
e-mail: fabianconsuegra@ingetec.com.co

A. Irfanoglu (✉)
School of Civil Engineering and Bowen Laboratory for Large-Scale Civil Engineering Research, Purdue University,
1040 S. River Road,
West Lafayette, IN 47907, USA
e-mail: ayhan@purdue.edu

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in reinforced concrete structures in early damage stages, referred as those when the structure has experienced 5% to 9% of its failure load [20]. In order to detect damage, a comparison of the undamaged and damaged states in a structure is required [12]. In contrast to frequency and mode shapes, damping is not a parameter typically used as an indicator of damage in structures. An extensive literature review indicates that only in four out of 158 damage detection techniques damping was used as a possible damage indicator [11].

Linearized-Parameter Estimation for Nonlinear Elastic Systems and Instantaneous Frequency

The equation of motion for an elastic, viscous-damped single degree of freedom (SDOF) system subject to external forcing *F(t)* can be written as

$$\ddot{x} + 2\omega\zeta\dot{x} + \omega^2 x = \frac{F(t)}{m} \tag{1}$$

where m, ζ and ω are the mass, viscous-damping ratio and undamped natural frequency of the SDOF, and x, $\dot{x}$ and $\ddot{x}$ are the displacement, velocity and acceleration of the system, with consistent units. It should be noted that no assumption is made regarding the nature of the stiffness and viscous damping in the system. In cases where mass is constant, as in typical civil engineering structures, nonlinear behavior of the structure due variable stiffness and damping can be incorporated included into equation of motion. Such nonlinear behavior in reinforced concrete structures has been identified by several authors as a response amplitude-dependent effect [4, 6, 16, 18, 26]. While stiffness has been proposed as a displacement-dependent parameter in most of the cases, damping has been proposed as either a displacement-dependent or a velocity-dependent parameter [13, 28].

Estimates of natural frequency and damping in civil engineering structures are made analyzing structural response data, obtained under harmonically forced or free vibration conditions, using the Fourier spectrum and the logarithmic decrement techniques. These estimates correspond to linear-elastic formulations due to the stationarity assumption inherent in the Fourier Transform [21] and the constant linear-system assumption on which the logarithmic decrement technique is based [7]. In this presented work, behavior of nonlinear elastic systems with displacement-dependent stiffness and viscous damping is investigated. Following is an example of linearized (constant) frequency and damping estimates obtained through the aforementioned methods, which assume linear-elastic behavior, applied on a nonlinear elastic system. First, the nonlinear elastic system is defined with its displacement-dependent stiffness and damping parameters. Next, linearized frequency and damping parameter estimates are given. Hilbert Transform is introduced as a way to track vibration frequency changes in the response of the nonlinear elastic system.

Linearized Frequency Estimation for a Nonlinear Elastic System

Figure 1(a) illustrates the load–displacement curve of an elastic nonlinear softening system. Without loss of generality, mass is taken as unity, i.e., $m=1$ lbf-sec^2/in. The displacement-

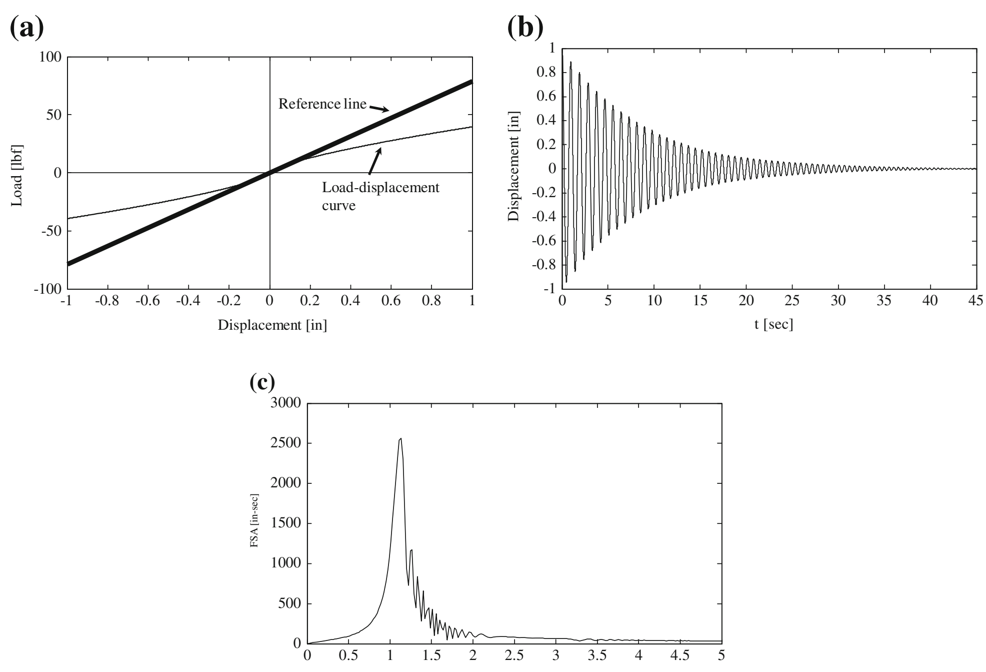


Fig. 1 Frequency estimate from free vibration of a nonlinear elastic system. (**a**) Load–displacement curve. (**b**) free-displacement response. (**c**) Fourier spectrum

dependent parameters of stiffness (secant stiffness) and damping are given as $k(x)=4\pi^2|x|^{-0.3}$ and $\zeta(x) = 0.01 + 0.01|x|$. A damping ratio linearly dependent on displacement was selected for illustration purposes. Later it will be shown that such an assumption was also based on actual tests performed in reinforced concrete structures at small amplitudes [9]. Runge–Kutta algorithm is used to solve the dynamic equation for the current system as well as in other numerical examples given in this study [19].The free vibration response of the system starting from an initial unit displacement and zero velocity is shown in Fig. 1(b). The corresponding Fourier Spectrum of the response is given in Fig. 1(c). The vibration frequency associated with the largest peak in the Fourier Spectrum is 1.1 Hz, however smaller peaks are observed in the spectrum due to the nonlinear nature of the system. Given the displacement-dependent nature of the stiffness, the largest peak observed in the Fourier Spectrum will be different if the system is released from a different initial displacement.

Linearized Viscous Damping Estimation for a Nonlinear Elastic System

Figure 2 shows the estimate of damping obtained by applying the logarithmic decrement method on the free vibration response of the nonlinear system of Fig. 1. In Fig. 2(a) the peaks used to compute the equivalent linear viscous damping ratio are shown and in Fig. 2(b) the equivalent linear viscous damping is compared to the actual displacement-dependent viscous damping defined for the system. A maximum difference of more than forty percent is found between the estimated values and the actual values.

As illustrated in this section vibration frequency of a nonlinear-elastic system will change as a function of the

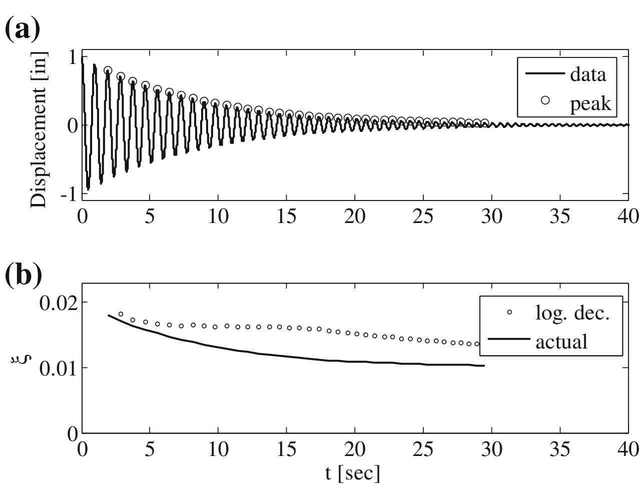


Fig. 2 Viscous damping ratio estimate from free vibration of a nonlinear elastic system. (**a**) Free-displacement response. (**b**) Comparison of logarithmic decrement viscous damping ratio estimate and actual viscous damping ratio of the system

displacement amplitude. A common practice in civil engineering applications is to define the natural frequency of a system as the frequency corresponding to the largest peak in the Fourier Spectrum without further investigation of nonlinearity. Herein, the term natural (fundamental) frequency will be used to refer to the frequency with largest Fourier amplitude while being aware of the displacement-dependent nature of the stiffness in a nonlinear elastic system. In other words, the natural frequency (and the corresponding damping ratio) will be associated with a particular displacement response level in the structure unlike in a linear system where natural frequency and damping ratio are indifferent to displacement response level.

Instantaneous Frequency Through Hilbert Transform

The Hilbert Transform of a signal $x(t)$ is defined as

$$x'(t) = \int_{-\infty}^{\infty} \frac{x(u)}{\pi(t-u)} du \tag{2}$$

Using equation (2), it is possible to obtain an analytical signal $X(t)$ of the original signal $x(t)$ as

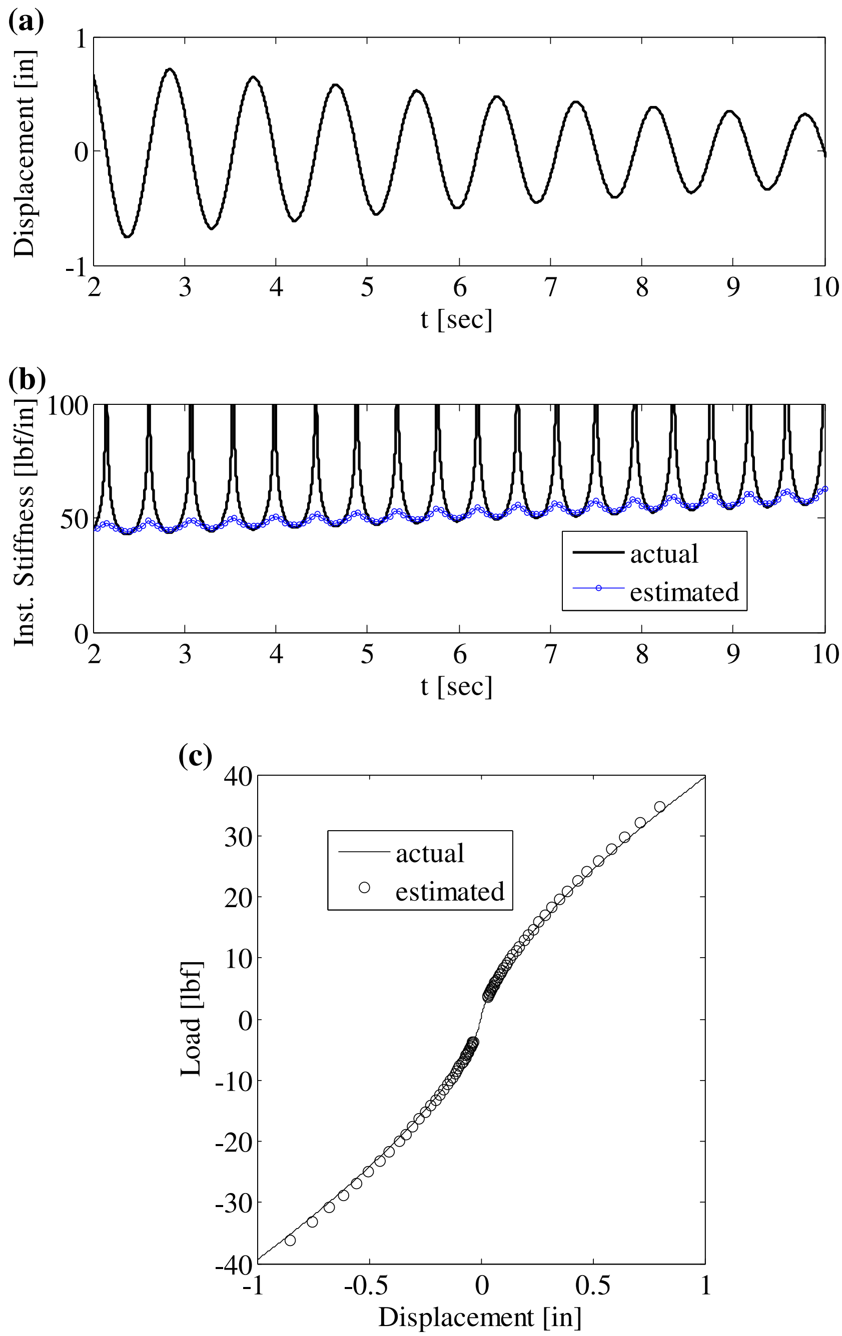
$$X(t) = x(t) + ix'(\mathrm{t}) = \mathrm{A}(\mathrm{t})\mathrm{e}^{\mathrm{i}\theta(\mathrm{t})\cdot\mathrm{t}} \tag{3}$$

where A and θ are the instantaneous amplitude and phase angle, respectively. Using equation (3), the instantaneous frequency of the signal can be estimated as time derivatives of the phase angle [2].

$$f(t) = \left(\frac{1}{2\pi}\right)\frac{d\theta(t)}{dt} \tag{4}$$

Equation (4) is used to estimate the instantaneous frequency of the freely vibrating SDOF given described in Fig. 1. Other techniques such as the time-dependent Fourier analyses [21], the moving-block analysis [3] and the Cohen-class type of time-frequency distributions [8] are available in the literature to track frequency variation of signals. The vibration record used to investigate response frequency through Hilbert Transform must satisfy the condition of being a mono-component signal [17], i.e. generated by a single dynamical process. That is satisfied by a nonlinear elastic SDOF system, in which the system exhibits a unique vibration frequency at each instant. The corresponding instantaneous secant stiffness is computed as $k=4\pi^2 f^2$ for a unit vibrating mass. Figure 3 (a) shows a portion of the free vibration response in Fig. 1 and the corresponding estimated instantaneous stiffness is shown in Fig. 3(b). In Fig. 3(b) the actual secant stiffness defined in the system $(k(x) = 4\pi^2|x|^{-0.3})$ is plotted for comparison. It is seen that instantaneous stiffness estimated through Hilbert Transform produces accurate results at instants of peak-amplitude

Fig. 3 Instantaneous stiffness. (**a**) Free vibration response. (**b**) Estimated and actual secant stiffness. (**c**) Estimated and actual load–displacement curve



displacements but not elsewhere. The natural frequency estimated through Hilbert Transform on a nonlinear elastic system is a combination of slow varying and fast-oscillating functions [14]. Because natural frequency is based on instantaneous amplitude calculated from equation (3), estimates of frequency are related to the characteristic stiffness exhibited at the amplitude of the current vibrating cycle. Refined methods to obtain estimates of instantaneous frequency where slow and fast-oscillating functions are treated separately can be found in the literature [13]. Figure 3(c) shows the comparison between the actual load–displacement curve and the estimated curve according to the procedure presented in this work. It is seen that the discrete version of the load–displacement curve calculated at the peak displacement points is in good agreement with the actual curve. In the simplified method presented herein only estimates of frequency at instants of peak displacements are used. This method is proposed as an alternate approach that does not require extensive computational processing. The effectiveness of the proposed method will be shown in "Full-scale three-story flat-plate building" when it is applied to a real civil engineering structure.

Experimental Measurements in Reinforced Concrete Structures

A full-scale three-story flat-plate building and six small-scale cantilever beams were tested in the laboratory. The dynamic properties were measured at small displacements after different large displacements were induced in the structures. The small displacements at which the dynamic tests were performed are limited to a drift at which typical reinforced concrete buildings are expected to remain uncracked. Because of absence of inelastic behavior in reinforced concrete structures below their cracking point [25], dynamic tests performed at such displacements are used to estimate the condition of the structures at a given stage. Under the double-curvature boundary conditions, lateral drift lateral ratio in a prismatic column may be calculated as $\Delta/\mathrm{h} = \mathrm{h}^2 * \mathrm{M}/6 * \mathrm{E} * \mathrm{I} = \mathrm{f}_{\mathrm{cr}} * \mathrm{h}/6 * \mathrm{E} * \mathrm{y}$, where Δ, h, M, E, I, f_{cr}, and y are lateral column distortion, floor height, moment at the ends of the column, modulus of elasticity, moment of inertia of the column cross-section, tensile stress capacity of concrete, and distance from the neutral axis to extreme fiber in cross-section, respectively. Given the compressive strength of the concrete f'c, E and f_{cr} may be calculated through known expressions [1]. For a typical reinforced concrete building with h=10ft, f'c=4ksi and 20 in×20 in square columns, the cracking drift level estimated as described above is 0.03%. This drift level was established as the drift limit for small amplitude vibration tests in the reinforced concrete structures. The maximum large displacement demand level used was 3.0% overall drift ratio for the full-scale structure [15] and 13% overall drift ratio level for the small-scale beams [9].

Full-Scale Three-Story Flat-Plate Building

The test specimen was a full-scale two-span by one-span three-story reinforced concrete flat-plate structure consisting of six columns spaced at 20 ft in each direction supporting a 7 in. thick slab at each story (see Fig. 4(a)). The story height of the building was 10 ft. An average compressive cylinder test of f'c=4ksi was measured at the time tests were started in the building. All results presented herein correspond to the fundamental mode of the longest plan dimension, i.e. left-to-right direction, of Fig. 4. A punching-shear failure (see Fig. 4(b)) was observed when the structure was pushed to 3.0% overall drift ratio.

Table 1 Linearized dynamic-parameter estimates from free vibration response in the full-scale structure. Fundamental frequency and viscous damping ratio: f_1 and ζ_1 respectively

Condition	Δ roof [in]	f_1 [Hz]	ζ_1 [%]
As-built	0.020	1.9	0.8
After 1.5% drift	0.020	1.3	2.4
	0.040	1.2	2.4
	0.100	1.2	2.9
After 3.0% drift	0.020	1.0	2.4
	0.040	1.0	2.6
	0.100	0.9	3.0

Table 1 lists the estimates of fundamental mode frequency and viscous damping ratio of the full-scale structure obtained from free vibration response tests carried out after different levels of large displacements. Natural frequency has been estimated by the largest peak in the Fourier Spectrum and equivalent viscous damping ratio is estimated using the logarithmic decrement approach. Although small participation of higher modes was seen, the records were filtered to isolate the fundamental mode response. It is seen that dynamic properties measured at a given condition in the structure are sensitive to the initial displacement given at the roof level (Δ_{roof}). However, because of the nature of free vibration tests, i.e., the response amplitude decreases, dynamic properties estimated via linearity assumption-based methods provide only a rough estimate of the displacement-dependent properties of the structure.

The nonlinear-elastic behavior of the structure illustrated in Table 1 is studied through the Hilbert transformed as explained in "Instantaneous frequency through Hilbert Transform". Figure 5(a) shows the measured roof-displacement during free vibration response after the structure experienced 3.0% overall drift and the corresponding instantaneous stiffness obtained through Hilbert Transform. Values of stiffness computed at instants of peak displacements (envelopes) are plotted in

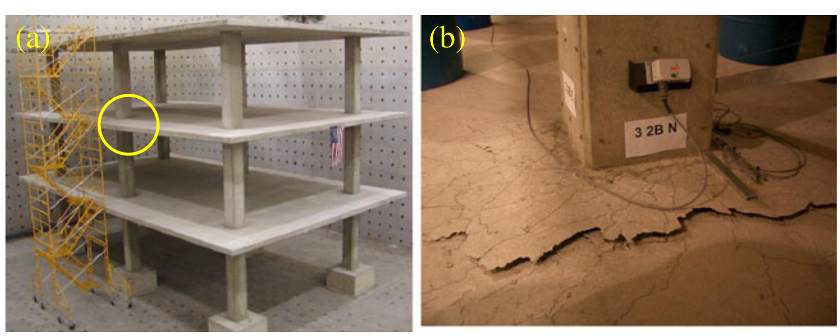


Fig. 4 Full-scale three-story reinforced concrete flat-plate building at the Bowen Laboratory of Purdue University. (**a**) General view and location of punching-shear failure. (**b**) punching-shear close-up

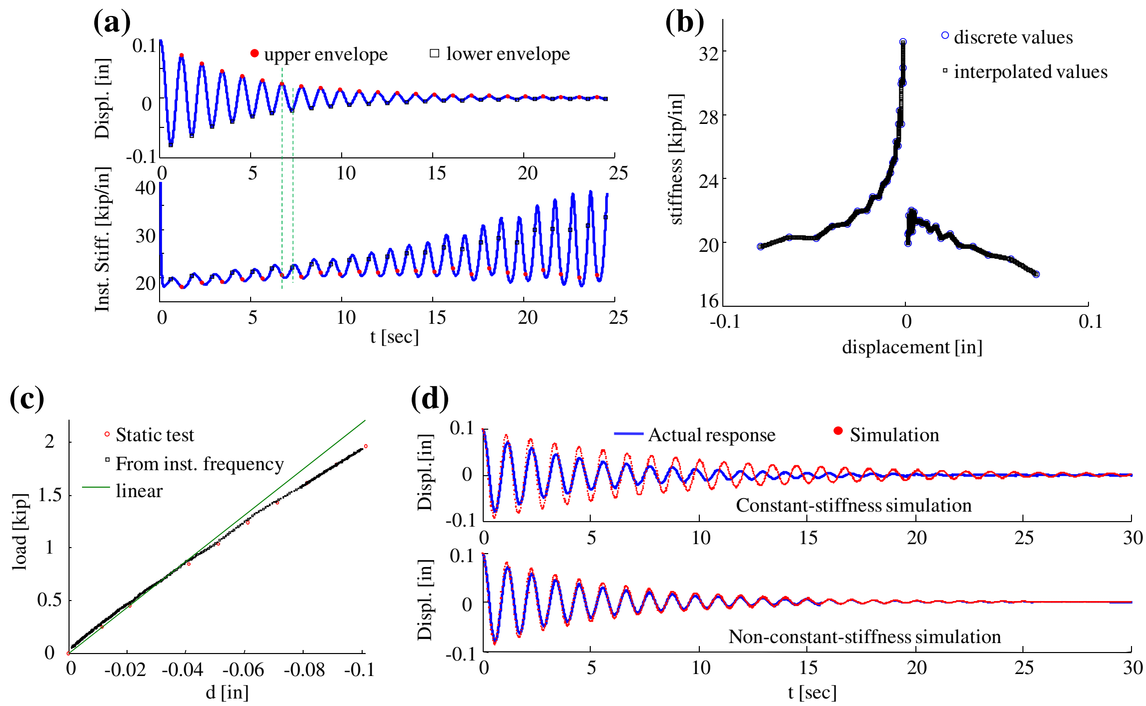


Fig. 5 Nonlinear-elastic response of the full-scale structure. (**a**) Free displacement response and instantaneous stiffness through Hilbert Transform. (**b**) Stiffness-displacement relationship. (**c**) Load–displacement relationship. (**d**) Comparison of actual and simulated response

Fig. 5(b). The stiffness estimate was done on the isolated first mode response of the structure, guaranteeing the mono-component nature of the signal as required by Hilbert Transform. Linear interpolation between those discrete values was used to define the stiffness of the system for displacements between the discrete points. The comparison of the corresponding load versus displacement curve with that measured during pseudo-static part of the test can be seen in Fig. 5 (c). A linear load–displacement relationship is also plotted in Fig. 5(c) for reference. It is seen in Fig. 5(c) that the load–displacement curve inferred from free vibration response compares well with that measured directly through static testing in the laboratory. In order to verify the nonlinear elastic response of the structure and check the above described procedure to recover load–displacement curves, the actual response is compared to two types of simulated responses as shown in Fig. 5(d).

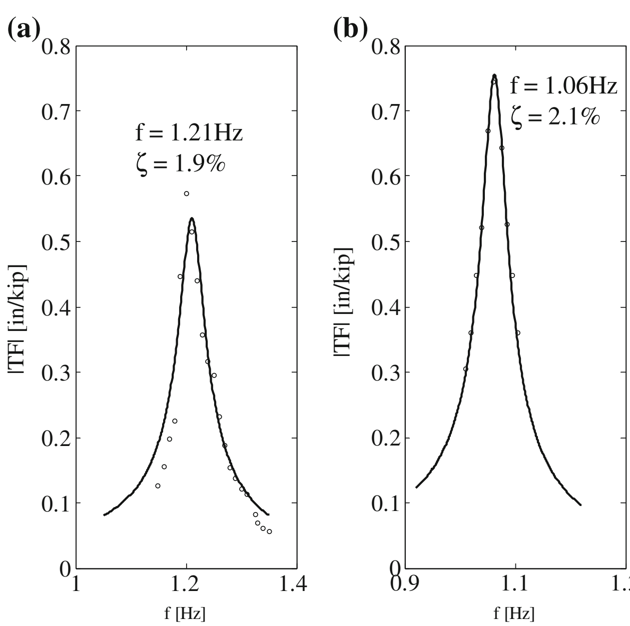


Fig. 6 Transfer functions for the full-scale structure. (**a**) non-constant-displacement-response TF. (**b**) constant-displacement-response TF

Table 2 Dynamic-parameter estimates from constant-displacement-response forced vibration tests in the full-scale structure. Fundamental frequency and viscous damping ratio: f_1 and ζ_1 respectively

Condition	Δ roof [in]	f_1 [Hz]	ζ_1 [%]
As-built	< 0.01(ambient)	2.0	-
	0.01	1.9	0.8
After 1.5% overall drift	< 0.01(ambient)	1.5	-
	0.01	1.2	2.1
	0.02	1.2	2.0
	0.04	1.1	2.1
	0.10	1.0	2.0
After 3.0% overall drift	< 0.01(ambient)	1.1	-
	0.01	1.0	2.2
	0.02	0.9	2.6
	0.04	0.9	2.5
	0.10	0.8	2.5

The simulation named as 'constant-stiffness simulation' used the stiffness measured at the end of the pseudo-static part of the test, i.e., just before the release of the structure for its free vibration response. On the other hand, 'variable stiffness simulation' was done by using the stiffness-displacement relationship of Fig. 5(b). A constant equivalent viscous damping ratio amount was used in both simulations. Use of nonlinear elastic behavior investigated through equations (2), (3) and (4) (Hilbert transform) matches the actual response of the system much better. Asymmetry of the stiffness-displacement relationship was also detected by the technique as shown in Fig. 9(b).

One can obtain estimates of the dynamic properties in a fashion that allows checking for their possible response amplitude-dependence using the Hilbert Transform method as described. Alternately, steady-state forced vibration tests could be performed at identical peak displacement levels, regardless of the forcing frequency, to eliminate variation of properties due to peak displacement level during tests. In forced-vibration tests on buildings, it is common practice to set constant the stroke or the generated force of the dynamic linear actuator, or the masses of the eccentric-mass shaker, exciting the structure. Then, different excitation frequencies in the vicinity of the natural frequency of interest are swept and a Transfer function (TF) is obtained as the ratio output/input for a given frequency. Equation (5) shows the analytical TF for a linear SDOF system with constant parameters when the output is displacement and the input is the force (k: stiffness, ω: natural frequency, ζ: viscous damping ratio, Ω: excitation frequency). The type of Transfer Function (TF) obtained from the described type of forced-vibration test corresponds to results obtained at different amplitude displacement responses with increasing peak displacement

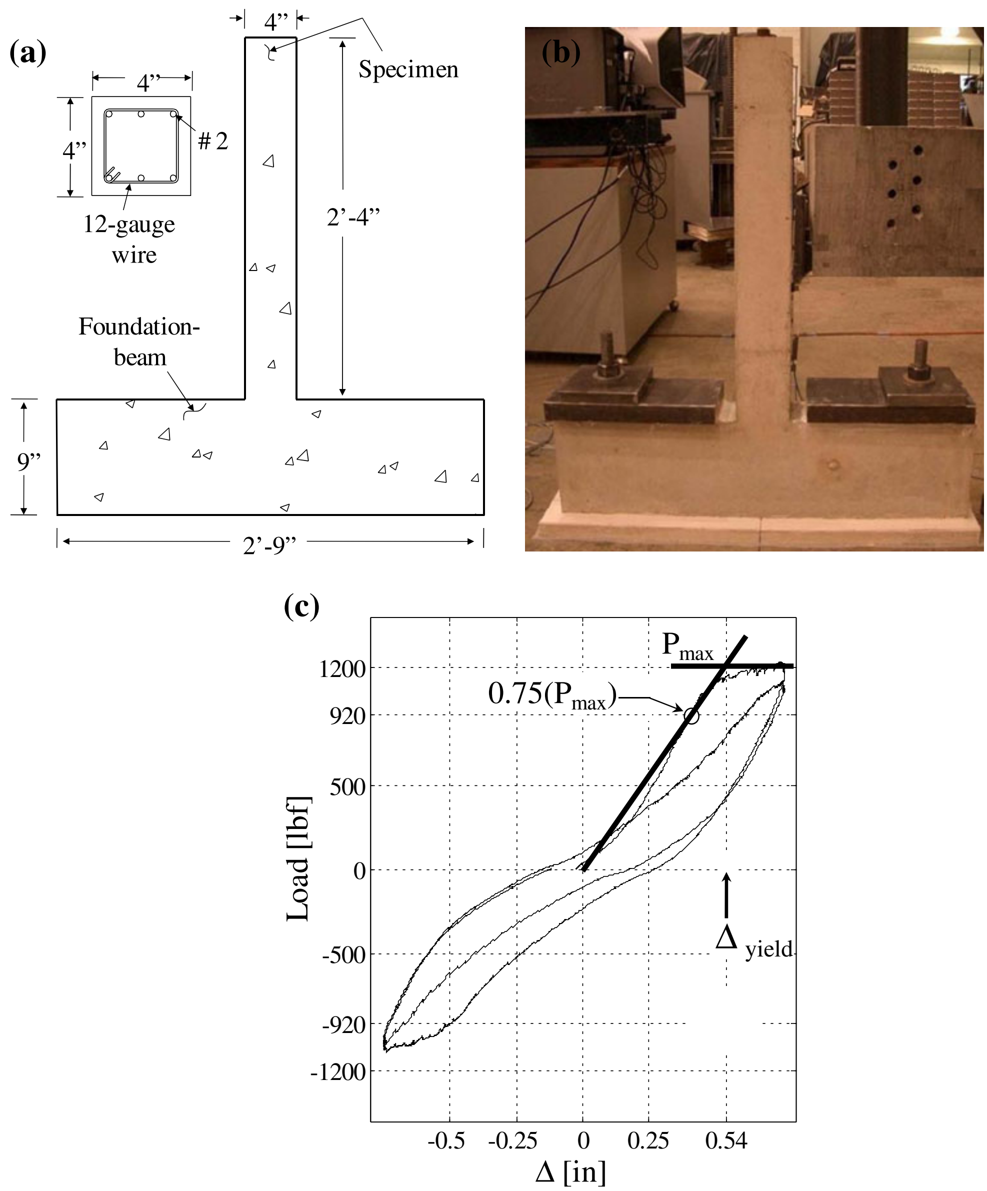


Fig. 7 Small-scale cantilever beam. (**a**) Reinforcing details. (**b**) General view. (**c**) Yield displacement in the Load–displacement curve of Specimen 1

response as the excitation frequency gets closer to the natural frequency of the structure. This type of TF is named herein as the "non-constant-displacement-response TF". Another type of TF, named as the "constant-displacement-response TF", can be obtained if the force amplitude is adjusted at each frequency step such that the peak displacement response of the structure is kept constant regardless of the forcing frequency. These two types of TFs would be identical if the tests were performed on a linear, constant parameter system. Figure 6 shows a comparison of the two types of TFs obtained for the aforementioned three-story flat-plate reinforced concrete building (Fig. 4) after the building has experienced an overall drift ratio of 1.5% with both input-force and output-displacement measured at roof level. A least-square best fit on each of the experimental TFs shown in Fig. 6 is made using equation (5) using parameters k, ω and as ζ the parameters. Natural frequency and damping ratio estimated from the non-constant and constant displacement-response TFs, thus Fig. 6(a) and (b) respectively, differ by more than 10%. The fact that a better fit is obtained on the constant-displacement-response TF demonstrates that the structure exhibits nonlinear behavior even at such small displacements (below 0.03% roof mean drift ratio). This nonlinear behavior cannot be captured by using non-constant displacement response TF, i.e. the typical approach in building tests. Constant-displacement-response TFs are viable in outfield structures with the use of an external-force mechanism of variable force and frequency (hydraulic actuator) to generate the input and an accelerometer to measure the output. Displacements inferred from accelerations measured during tests have been seen to be reliable to obtain a constant-displacement-response TF [9].

$$|TF(\Omega)| = \frac{1/k}{\sqrt{\left[1-(\Omega/\omega)^2\right]^2 + [2\zeta\,\Omega/\omega]^2}} \tag{5}$$

Table 2 summarizes the frequency and damping measured at different small displacements (below 0.03% overall drift ratio) after different large drift levels in the full-scale structure. Properties in Table 2 were obtained through constant displacement response TF type of tests. Natural frequency is observed to change as much as 45% in the range of low amplitudes for a given drift level, indicating the extreme dependency of natural frequency to the dynamic test displacement amplitude. From the constant-amplitude dynamic tests done before and after an event (condition), using properties estimated at identical low-amplitude displacement levels it is seen that the natural frequency is sensitive to the past large overall drift level. If the identical low-amplitude displacement level approach is not followed, however, erroneous conclusions could be made. For example, in Table 2, comparison of fundamental frequency measured at 0.10-in drift after experiencing 1.5% overall drift ratio and measured at 0.01-in after experiencing 3.0% overall drift ratio (i.e. damaged condition) would at best lead to indifference even though the structure sustained punching shear failure once it experienced 3.0% overall drift ratio.

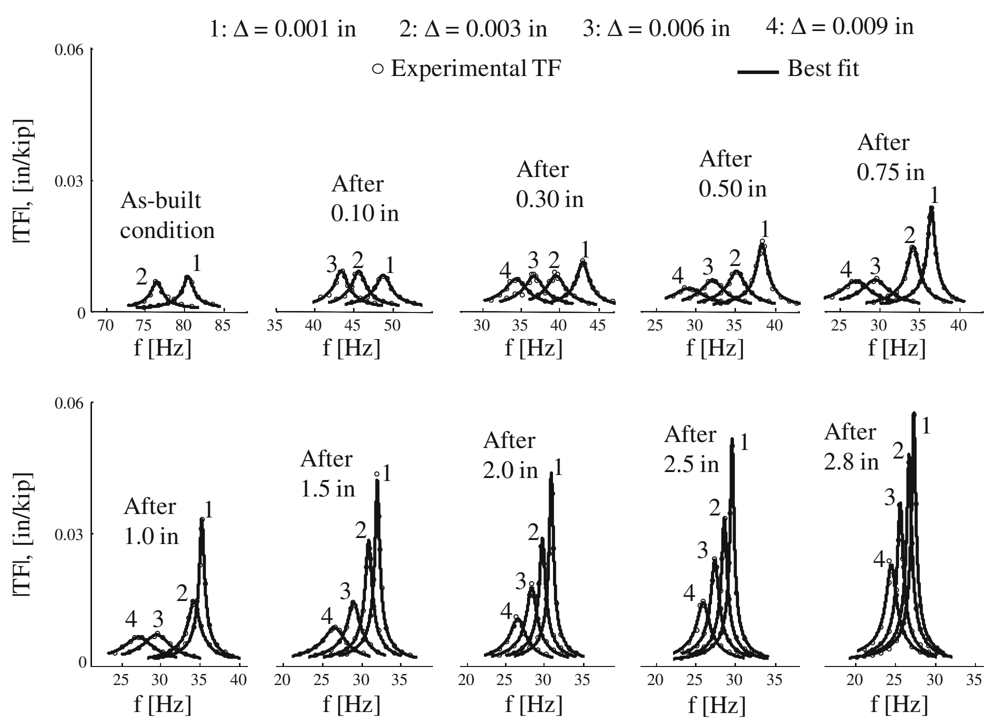


Fig. 8 Constant-displacement response TFs for Specimen 6 of the small-scale cantilever beams

Small-Scale Beams

An extensive testing program was developed on six small-scale reinforced concrete cantilever beams (Fig. 7(a) and (b)). Two concrete mixes with average compressive strength of 3,740 psi at 28-days were used to cast the six specimens. The beams were subjected to pseudo-static cycles of increasing large amplitudes (as referenced in Fig. 8) and the small-amplitude dynamic properties were obtained by means of free, forced, and ambient vibration tests [9]. Measurements of input-force and output-displacement were taken at the tip of the beams. Yield displacement of beams was estimated to be about 0.5in based on the secant method [22] as shown in Fig. 7(c).

Figure 8 shows the constant-displacement response TFs measured in specimen 6 before and after each of the pseudo-static loading cycles imposed in the beams. Tests carried out at beam tip displacement amplitudes of 0.001 in, 0.003 in, 0.006 in and 0.009 in, are labeled as 1, 2, 3 and 4, respectively. Best-fit analytical models (equation (5)) were used to estimate the properties of each specimen at each stage. It can be seen by simple inspection that carrying out constant-displacement response TFs allows a good fit at different small-amplitude levels. Figure 9 shows the parameter estimated from Fig. 8 after the beam has experienced 0.75in overall displacement. The fundamental frequency varies between 27 and 37 Hz depending on the small displacement level used in testing. The range in the fundamental frequency corresponds to a stiffness variation factor of 1.9. As for the viscous damping ratio, Fig. 9 shows that a linear displacement-dependent assumption may offer reasonable representation of viscous damping models as implemented in "Linearized frequency estimation for a nonlinear elastic system". Tables 3 and 4 summarize the dynamic parameter estimates obtained similar to those given for the full-scale 3-story flat plate reinforced concrete structure in Tables 1 and 2. Constant-displacement response TF (Table 4) allows detection of sensitivity of dynamic parameters to small-amplitude displacement levels better than free-vibration response estimates do (Table 3). The differences between estimates in Tables 3 and 4 come from the fact that estimates from free vibration response are linear (constant parameter) estimates of an elastic system responding nonlinearly, whereas values in Table 4 represent an equivalent linear system of a nonlinear elastic system responding at a unique displacement level. In other words, both estimates are equivalent linear estimates, but estimates obtained via constant-displacement response TF approach are representative of a nonlinear elastic system responding with a displacement-dependent stiffness. Clearly the constant-displacement response TF approach yields estimates that offer better representation than the displacement-independent parameter estimates (see Fig. 6 for comparison).

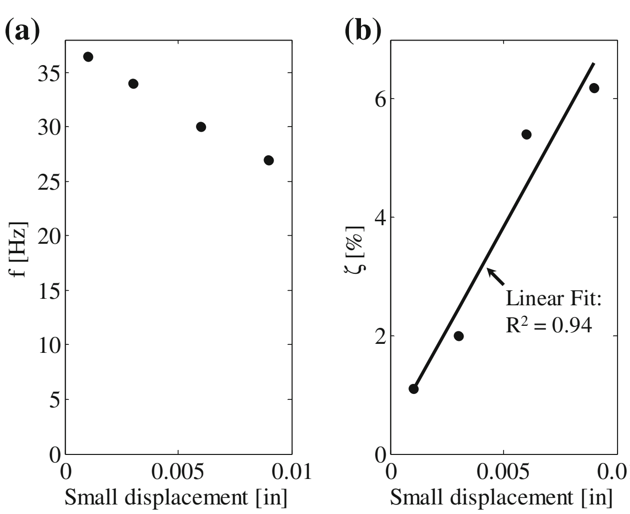


Fig. 9 Varation of dynamic properties with small-amplitude displacement level in Specimen 6. (**a**) Frequency. (**b**) Viscous damping ratio

Table 3 Linearized dynamic-parameter estimates from free vibration response in specimen 6 of small-scale beams. Fundamental frequency and viscous damping ratio: f_1 and ζ_1 respectively

Condition	Δ top [in]	f_1 [Hz]	ζ_1 [%]
As-built	0.003	81.5	0.9
After 0.10 in	0.003	50.2	1.7
	0.006	47.8	1.9
After 0.30 in	0.003	45.9	1.6
	0.006	42.5	1.8
	0.009	41.2	1.9
After 0.50 in	0.003	41.1	1.4
	0.006	39.1	1.6
	0.009	36.7	1.8
After 0.75 in	0.003	37.6	1.1
	0.006	36.5	1.3
	0.009	36.0	1.4
After 1.0 in	0.003	35.6	1.0
	0.006	34.8	1.2
	0.009	34.5	1.3
After 1.5 in	0.003	32.4	1.0
	0.006	31.8	1.1
	0.009	31.6	1.2
After 2.0 in	0.003	31.1	1.0
	0.006	30.6	1.1
	0.009	30.3	1.3
After 2.5 in	0.003	29.8	1.0
	0.006	29.4	1.1
	0.009	29.0	1.2
After 3.7 in	0.003	27.3	0.9
	0.006	27.0	1.1
	0.009	26.6	1.2

Table 4 Dynamic-parameter estimates from constant-displacement-response forced vibration tests in specimen 6 of small-scale beams. Fundamental frequency and viscous damping ratio: f_1 and ζ_1 respectively

Condition	Δ top [in]	f_1 [Hz]	ζ_1 [%]
As-built	< 0.001 (ambient)	84.3	0.4
	0.001	80.5	0.7
	0.003	76.5	0.8
After 0.10 in	< 0.001 (ambient)	52.7	1.1
	0.001	48.8	1.8
	0.003	45.6	1.7
	0.006	43.4	1.8
After 0.30 in	< 0.001 (ambient)	46.7	1.0
	0.001	43.0	1.7
	0.003	39.5	2.8
	0.006	36.7	3.1
	0.009	34.4	3.8
After 0.50 in	< 0.001 (ambient)	41.7	0.9
	0.001	38.3	1.5
	0.003	35.2	2.9
	0.006	32.2	4.2
	0.009	29.3	6.5
After 0.75 in	< 0.001 (ambient)	37.9	0.6
	0.001	36.5	1.1
	0.003	34.2	2.0
	0.006	29.6	5.4
	0.009	27.3	6.2
After 1.0 in	< 0.001 (ambient)	35.9	0.6
	0.001	35.2	0.8
	0.003	33.6	1.4
	0.006	30.6	3.7
	0.009	27.3	6.1
After 1.5 in	< 0.001 (ambient)	32.5	0.5
	0.001	32.0	0.8
	0.003	30.9	1.2
	0.006	29.0	2.6
	0.009	26.6	4.9
After 2.0 in	< 0.001 (ambient)	31.7	0.5
	0.001	30.9	0.7
	0.003	29.7	1.2
	0.006	28.4	2.1
	0.009	26.6	3.4
After 2.5 in	< 0.001 (ambient)	30.4	0.5
	0.001	29.6	0.7
	0.003	28.6	1.0
	0.006	27.4	1.6
	0.009	26.0	2.8
After 3.7 in	< 0.001 (ambient)	27.9	0.6
	0.001	27.3	0.8
	0.003	26.7	1.0
	0.006	25.6	1.2
	0.009	24.5	2.4

Figures 10 and 11 show the variation of absolute values of natural frequency and equivalent viscous damping ratio with large displacement levels (past maximum displacement) in the small-scale beams. Despite different absolute values of frequency and damping are obtained for different specimens, trends of variation of those properties with past maximum past displacement are consistently observed for all the specimens. Results from small-amplitude displacements of 0.001in, 0.003in, 0.006in and 0.009in (~0.03% overall drift) are presented. Figures 10 and 11 have identical horizontal axis on the corresponding specimens that illustrates tests performed at as-built condition (zero) through at the condition of maximum overall drift ratio of 13% (3.7in tip displacement in specimen 6). The natural frequency is observed to decrease continuously with past maximum displacement, with steeper decrease in early cycles. On the other hand, starting from the as-built state, the viscous damping ratio exhibits an early increase and then decrease with past maximum displacement. The past maximum displacement at which the viscous damping ratio goes from increasing trend to decreasing trend corresponds to a

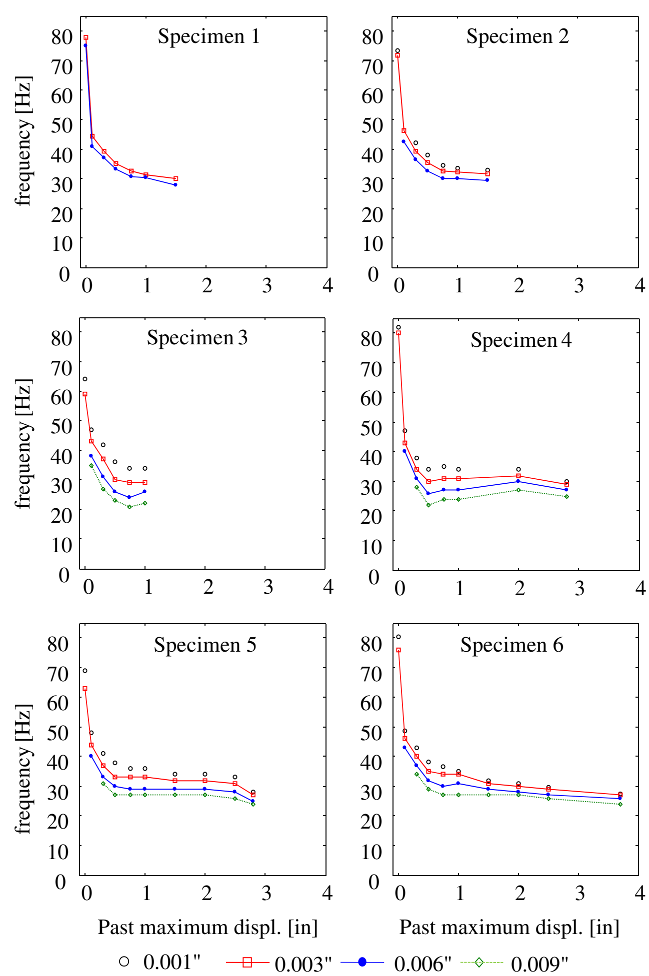


Fig. 10 Variation of frequency with past maximum displacement in the small-scale cantilever beams

displacement near the yield displacement. This displacement (shown in Fig. 7(c)) was identified during the pseudo-static cycle testing as the displacement at which yield capacity of the member was developed (approximating the nominal moment capacity *Mn* as *As* x f_y x *d*, i.e. area of the steel reinforcing times yield stress times the distance between the center of the longitudinal rebars from one side to the face of the beam on the other side). The fact that trends observed in frequency and damping with past maximum displacement depends on relative variation of those parameters rather than absolute values is corroborated by Figs. 12 and 13 which are normalized versions of Figs. 10 and 11 respectively. In Figs. 12 and 13 dynamic properties values have been normalized with respect to those values measured at initial condition therefore only data series of displacement tested in the initial condition are shown. Regardless the scatter observed in the experimental data, as seen in Figs. 10, 11, 12 and 13, the general trends in fundamental mode frequency and associated equivalent viscous damping are well identified.

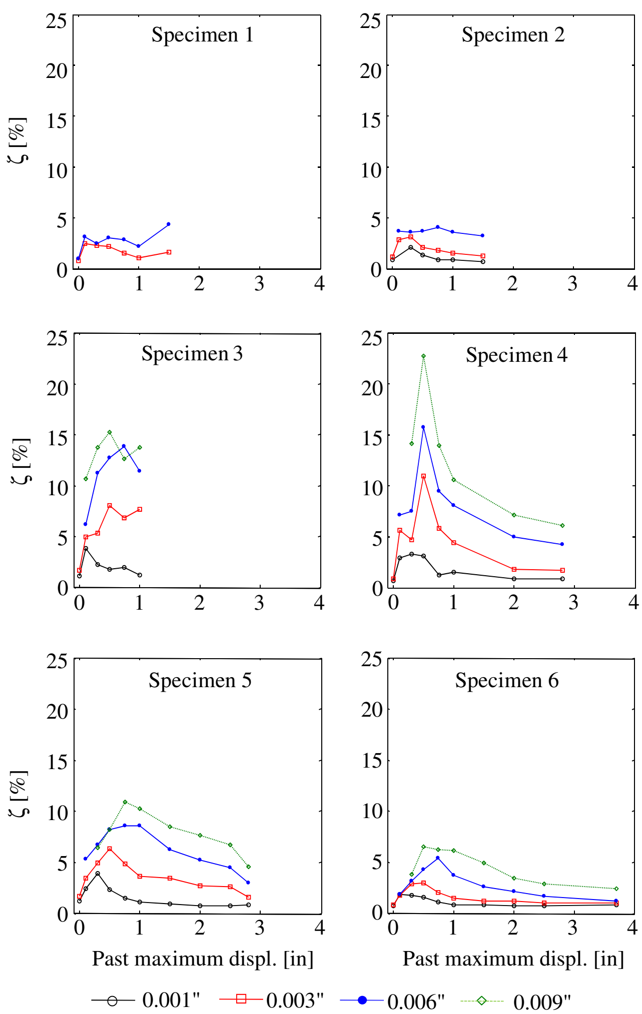


Fig. 11 Variation of equivalent viscous damping with past maximum displacement in the small-scale cantilever beams

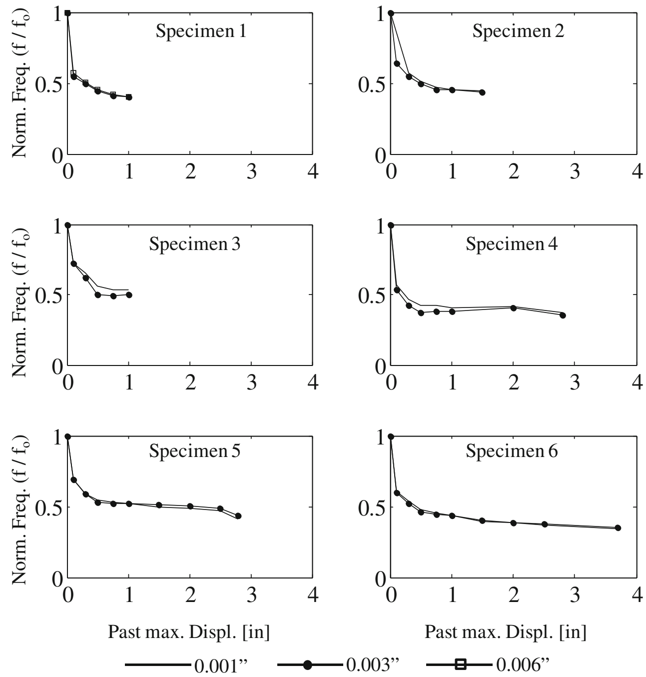


Fig. 12 Relative variation of frequency with past maximum displacement in the small-scale cantilever beams

Conclusions

In reinforced concrete structures, dynamic properties measured at small displacements (below 0.03% overall drift

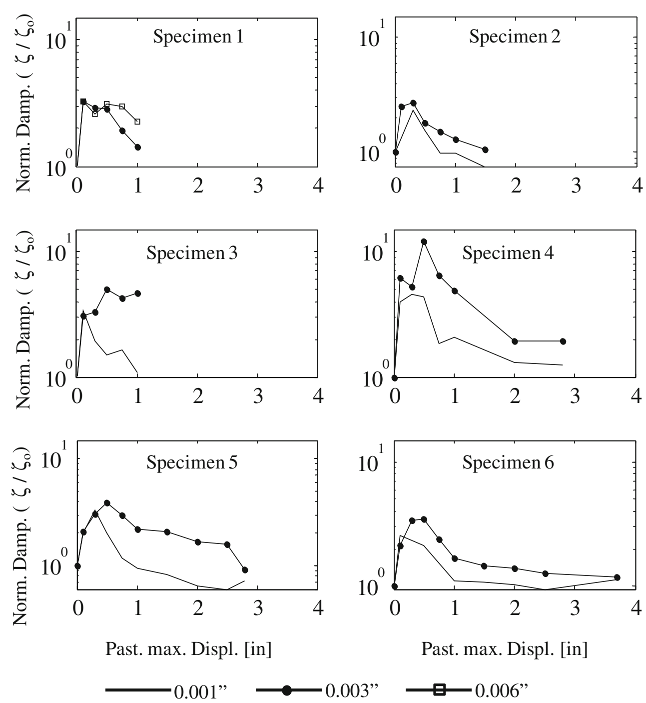


Fig. 13 Relative variation of equivalent viscous damping with past maximum displacement in the small-scale cantilever beams

ratio) are susceptible to nonlinearity in the response of the structures. Consequently, it is important to verify the validity of linear elastic model assumptions to determine if properties estimated through such models are exclusively associated with a given displacement amplitude. Common forced vibration tests performed on buildings result in data obtained at a wide range of displacement responses. Although these "non-constant displacement response" type forced vibration tests can illustrate the nonlinear behavior of the structure at small displacements, they cannot provide a complete description of the relationships between dynamic properties and displacement amplitude unless a nonlinear elastic model is considered while seeking the best-fit dynamic parameters. Because of the complexity of such models use of an alternative test approach in which the structure is forced to respond at a certain peak displacement level regardless of the forcing frequency is recommended. This "constant displacement response" approach leads to a more consistent implementation of typically employed linear-response based models. Variation in small-amplitude dynamic properties with past large peak displacement levels in reinforced concrete structures could be tracked using data obtained from identical-level small displacement forced vibration tests. A practical finding of this study is that the trend in equivalent viscous damping ratio changes from increase to decrease when the past maximum displacement reaches the neighborhood of nominal yield displacement of the structure (plastic behavior). This information could be used in health monitoring of structures if information recorded at small-amplitude vibrations at different epochs of the life of the structure are available. The applicability and validity of the introduced method to recover load–displacement curves from free vibration responses was verified on a full-scale 3-story flat-plate reinforced concrete structure.

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