

ON THE RELATIONSHIP BETWEEN SMALL AMPLITUDE VIBRATION
DYNAMIC PROPERTIES AND THE PAST MAXIMUM DISPLACEMENT IN
REINFORCED CONCRETE STRUCTURES

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To my Mother who keeps smiling at me from heaven.
To my Father who gave me the music and the engineering.

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ABSTRACT

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Damage detection in structural health monitoring is usually done by correlating the level of damage in a structure to its dynamic parameters estimated from vibration records. In buildings such correlations are not well established. A vibration-based damage detection technique that is able to identify the structural condition of the system based on small amplitude vibrations is desirable because such kind of data are typically available. This study focuses on the relationships between small-amplitude vibration dynamic properties and past levels of displacement in reinforced concrete structures. Small displacements are defined as displacements below an overall drift ratio of 0.03%. Laboratory tests on a full-scale flat-plate structure and a series of small-scale beams were done. It is observed that natural frequency and equivalent viscous damping ratio are displacement-dependent parameters even at small displacement levels. Such displacement-dependency of dynamic parameters was captured by using an elastic single-degree-of-freedom (SDOF) system with variable stiffness and variable equivalent viscous damping, i.e., nonlinear-elastic SDOF, to represent the small-amplitude vibration response of the reinforced concrete structures. It is found that change in the dynamic parameters with past maximum displacement is tracked best using tests done at identical displacement. Observations from tests are compared to those extracted from experiments done by other researchers. In the small-scale beams, the equivalent viscous damping ratio measured at small displacements found to increase from the as-built level with

displacement initially but then decrease once the past maximum drift reaches approximately the yield level. A similar behavior is observed in a full-scale 7-story structure. A condition based on relative change in fundamental frequency and equivalent viscous damping ratio is suggested to define, and identify, a characteristic displacement associated with yield in reinforced concrete buildings. However, more data from full-scale building tests are necessary to justify the extension of the observations made from small-scale test for use in structural health monitoring of actual buildings. Nonlinear-elastic SDOF model representation has encountered difficulties when applied on currently existing data sets from full-scale buildings. A practical finding of this study is that the fundamental frequency in a typical reinforced concrete building reduces to approximately $2/3$ of that measured at as-built condition after the structure is subjected to approximately 1.5% maximum drift ratio level.

CHAPTER 1. INTRODUCTION

1.1. Background

Vibration-based structural damage detection is a growing field in different disciplines. Extensive work has been done in the past to provide tools to estimate physical parameters of structures based on vibration records. The usual approach in structural damage detection also known as structural health monitoring, is to monitor changes in those parameters due to a particular event. However, the relationships between change in dynamic response characteristics and structural condition of reinforced concrete structures are still not established, and the sensitivity of these relationships are not well-understood.

Yielding identifies a particular stage in the behavior of reinforced concrete structures. The structural condition of buildings that have experienced displacement demands beyond yield level is likely to be a deteriorated one. Inspection of such buildings during post-earthquake reconnaissance is of priority in order to preserve the public safety. Small amplitude vibration, including the ambient vibration that occur naturally during regular use of a building, is the easiest approach to collect response data that would permit estimation of dynamic properties of the building. It is also desirable that sufficient data could be obtained from buildings by use of a few sensors. For example, ambient vibrations recorded by an accelerometer installed in the building could be used to estimate the natural frequencies of the structure through a Fourier analysis. It is suggested that damping parameters in a building could be estimated from the same vibration record through the so-called random decrement technique (Cole 1973). Ambient vibration signatures are the easiest type of data to collect from a given

structure. Determining the inelastic deformation experienced by a structure using observations based on ambient vibration response of the structure would have important practical applications but remains a grand challenge. Therefore, it is important to investigate the relationships between vibration parameters at small displacements, including those at ambient levels, and inelastic deformation levels.

1.2. Objective and Scope

The objective of this study is to establish relationships between frequency and equivalent viscous damping ratio at small displacements and maximum displacement previously experienced by a reinforced concrete structure.

The scope of the study included (a) instrumentation and testing of a full-scale three-story flat-plate reinforced concrete building to measure small-amplitude dynamic properties after successive pseudo-static loading cycles, (b) construction and instrumentation of six small-scale reinforced concrete beams to be tested pseudo-statically and dynamically to obtain small-amplitude dynamic properties, (c) development of methods to extract dynamic properties from vibration records of nonlinear-elastic systems.

It has been commonly assumed that, at small displacements, reinforced concrete structures behave as linear-elastic systems. A critical aspect of the presented study is the detailed investigation of the existence of nonlinear-elastic behavior in structures vibrating at small amplitudes. The premise is that consideration of nonlinear-elastic behavior would yield important information about the structural system in a building.

1.3. Organization

Chapter 2 reviews the existing research on damage detection using vibration records classified by the type of dynamic parameter used. Current methods available for estimation of instantaneous frequency and damping are illustrated.

Chapter 3 describes the methods used in this study to extract natural frequency and equivalent viscous damping ratio from free vibration records of nonlinear-elastic systems.

Chapter 4 summarizes the experimental results from the specimens tested as part of this research.

Chapter 5 provides an analysis of the experimental results presented in chapter 4. Behavior of frequency and equivalent viscous damping with past maximum displacement is described based on observations presented in chapter 4.

Chapter 6 is a comparison of behavior of frequency and equivalent viscous damping with past maximum displacement described in chapter 5 and those observed in specimens tested by other researchers. Different types of reinforced concrete structures, small and full-scale, are used for comparison.

Summary, conclusions and recommendations for future work are presented in Chapter 7.

CHAPTER 2. EXISTING RESEARCH

2.1. Introduction

Dynamic characteristics extracted from vibration data recorded in a building are indicators of the physical properties of the building. For example, natural frequencies are directly related to the stiffness of the structural system. A large body of research on correlation between the structural condition of a structure and its dynamic properties exists in literature.

Several damage-detection methods based on vibration characteristics have been developed in the field of structural health monitoring (Doebling 1996; Doebling et al. 1998; Sohn et al. 2003). The methods are based on correlation between damage and dynamic properties such as natural frequency, mode shapes and damping ratios. These damage detection techniques require structural model parameters such as flexibility and stiffness coefficients, which need to be estimated based on the vibration data. Many methods correlate damage to those model parameters rather than to the dynamic parameters, such as natural frequencies, equivalent viscous damping ratios, or mode shapes that could be measured directly.

Estimating characteristic dynamic properties of a structure from vibrations recorded therein has been a common practice in the field of structural engineering (Gulkan and Sozen 1977; Kaminoso et al. 1985; Snieder and Safak 2006; Trifunac 1972). To develop a simple and effective way to detect the presence and extent of damage, dynamic parameters that are sensitive to damage and could be obtained from vibration records need to be chosen. This

selection will depend on the definition of damage or structural condition, which could require careful inspection and/or repairs. Finding robust relationships between dynamic characteristics and presence of the undesired condition continues to be a research motivation.

Following is a summary of existing research on dynamic properties of reinforced concrete structures classified by the property investigated.

2.2. Natural frequency

The use of natural frequency as a diagnostic parameter has its basis on the assumption that natural frequencies are sensitive indicators of structural integrity (Salawu 1997). Most of the structural health monitoring methods are focused on finding either shifts in natural frequency or changes in structural mode-shapes (Chang et al. 2003). There are two sources of natural frequency shifts in reinforced concrete structures. One is what produces permanent loss of stiffness of the system, i.e. cracking of sections, and the other is the nonlinear-elastic behavior. In the latter case, mechanical properties of the system change during the deformation but the structure recovers back to its initial state when the deformation is removed.

Several studies that show variation of natural frequencies as a result of stiffness degradation of the structural system can be found in the literature (Abrams and Sozen 1979; Bonacci 1989; Healey and Sozen 1978; Moehle and Sozen 1980; Wood 1985). In these studies, reinforced concrete structures underwent inelastic deformations caused by earthquake simulations and fundamental frequency of the structures were measured between earthquakes with different displacement demands. Frequency was measured from free vibration tests conducted at displacement levels that did not introduce inelastic deformations in the structures. Ratio between fundamental mode frequency at the initial condition and at the end of the earthquake simulations ranged between 1.2 and 4.1.

Clinton (2006) observed that the fundamental frequency in an undamaged structure varied 20% during tests. The data he used corresponded to an overall drift ratio of 0.3%, derived from acceleration record. Yu (2005) observed similar variation of the fundamental frequency in a series of tests performed on a damaged 4-story building. In his experiments, the maximum overall drift ratio, derived from acceleration records, was 0.01%. The previous two examples illustrate that the observed natural frequency of a reinforced concrete structure can change by 20% during typical vibration tests when it has no visible damage or when no additional damage is introduced in the structure. They indicate the existence of nonlinear-elastic behavior in reinforced concrete structures while vibrating at low amplitudes and regardless of the structural condition of the structures.

There are several signal processing tools to analyze the evolution of frequency in dynamic response of a structure. These tools are known as time-frequency distribution methods and are used to estimate the dominant vibration frequencies at each instant of the record. The Spectrogram is the most basic time-frequency distribution method. It consists of a moving-time-window Fourier spectral analysis. However, due to the “Heisenberg Indeterminacy principle” (Flandrin and Stöckler 1999) there is a trade-off between time and frequency resolution. As an extension of the classical Fourier analysis, the Spectrogram has limitations due to the assumption of stationarity of the data.

The Wigner-Ville distribution (Ville 1948; Wigner 1932) is an extension of the Spectrogram and offers better resolution in the time-frequency plane. However, fictitious frequencies are introduced into the results during the process. Some of the concerns about Wigner-Ville distribution approach have been addressed by Bradford when the Wigner-Ville distribution is used on data from some civil engineering applications (Bradford 2006).

The wavelet approach is an adjustable window Fourier spectral analysis (Mallat 1999). In wavelet approach, harmonic functions of the Spectrogram are replaced by functions that satisfy certain general conditions and give a better fit on the data.

Hilbert transform permits estimating an analytical signal of the original response data (Peters 1995). This analytical signal is made up of instantaneous amplitude and phase angle. The instantaneous frequency is then computed as the time derivative of the instantaneous phase angle. However, the Hilbert transform approach assumes that a single process is involved in the generation of the data. The ambiguity in the instantaneous frequency in a signal with multiple frequency components restricts the use of the Hilbert transform. An alternate method to decompose the original signal with contributions from multiple dynamic processes into a collection of well-behaved Hilbert-transform functions (mono-frequency components), has been proposed by Huang et al. (1998).

2.3. Mode shapes

Mode shapes have been used primarily to locate damage in the structure. Methods to locate damage based on information from a few mode shapes have been applied to truss structures (Gao et al. 2007; Messina et al. 1998; Shi et al. 2006) . Applicability of such methods demands a large number of sensors in order to define a given modeshape accurately. However, it is known that modeshapes that dominate structural response may not be sensitive to damage. For example, experimental results showed that significant structural damage did not affect the modeshapes appreciably in an 11-story reinforced concrete building (Galambos and Mayes 1978) where the fundamental period of the structure increased by 110% but no more than 10% change was observed in the corresponding modeshape.

2.4. Shear wave velocity

Dynamic response analysis of multistory buildings using shear wave propagation has been formulated by several authors (Hall et al. 1995; Kohler et al. 2007; Safak 1999; Snieder and Safak 2006; Westergaard 1933). In this formulation the total response at a given floor level is computed as superposition of the reflected and transmitted waves from the floors below and above. This formulation implies that the building is assumed to deform only in shear, although participation of shear and flexure deformation exists in reinforced concrete buildings. A few examples of damage detection in a real structure by using the shear wave velocity are found in the literature (Ivanovic et al. 1999; Todorovska and Trifunac 2008a; Todorovska and Trifunac 2008b; Trifunac et al. 2003). In those studies damage was detected by changes in the travel-time of waves. A dense network of sensors was used in those studies. In particular, in the study by Ivanovic, infeasible information to obtain such as direction and angle of the incoming earthquake-waves arriving to the structure were needed.

2.5. Damping

Research on damping as an instrument to detect damage is scarce. From a literature review done by Doebling (Doebling 1996), only in four out of 158 damage identification techniques damping was used. A few other studies that use damping as a parameter to detect damage in reinforced concrete were found in the literature. Modena and his colleagues (Modena et al. 1999) investigated change in dynamic properties in precast hollow floor panels made out of reinforced concrete. Results showed that visually undetectable cracks caused negligible variations in natural frequencies but increase of as much as 50% in damping. Curadelli et al. (Curadelli et al. 2007) showed how damping was more sensitive to damage than natural frequency in reinforced concrete beams. In their study, natural frequency reduced 12% while damping coefficient increased 40%. Damping coefficient in their study was obtained through wavelet transform of free

vibration responses measured in the specimen. Drift ratio levels, derived from accelerations, were approximately 0.002% in their tests.

Damping in structural systems, unlike frequency, does not relate to well-identified physical properties. For example, for a given mass and stiffness distribution in a structure, which can be estimated from dimensions and physical properties of the structure, the natural frequencies of the structure could be calculated. However, no such direct method exists to calculate damping in a structure. This is perhaps the largest difficulty in obtaining a unique damping model for structures. Damping has been modeled by using different approaches in the literature (Bert 1973; Wyatt 1977) . One of the several types of damping is the Coulomb damping, defined as the constant magnitude friction force opposing the movement. The main drawback of this model is that the dynamic load factor, which is an essential parameter obtained from forced vibration tests of a structure, is undefined when the system has Coulomb damping and the frequency of the harmonic excitation is identical to the natural frequency of the system (Nashif et al. 1985).

Another type of damping is the so-called structural damping where the damping force is proportional to the displacement and is in phase with velocity (Myklestad 1952). Structural damping, also called rate-independent damping, has its roots in analytical models that attempt to simulate the constant energy dissipation of materials observed in small-amplitude vibration tests (Kimball and Lovell 1927; Wegel and Walther 1935). The disadvantage of using this model of damping is the complexity of the equation of motion for an arbitrary external force (Chopra 1995). Another difficulty with this model is that it does not provide a mathematical representation of free vibration response (Scanlan and Meldenson 1962), which eliminates use of data obtained during free vibration dynamic tests of structures.

Viscous damping is the most popular model for damping in civil engineering applications. Viscous damping model, assumes that a damping force proportional

to the velocity exists in the structure. Its formulation allows derivation of analytical solutions for the dynamic equation of motion for steady-state and free vibration responses. According to Lazan (Lazan 1968), as early as 1784 Coulomb recognized that mechanism of damping operative at low stress levels may be different from those at high stress levels. Recent research shows that, in reinforced concrete structures, significantly smaller values of viscous damping are obtained in the small amplitude vibrations range than in the large amplitude vibrations range (Celebi 1996). For small amounts of damping, there is a good equivalence between viscous and structural damping (Soroka 1949).

Damping phenomena in the range of inelastic deformations in reinforced concrete structures have been studied by different researchers (Gulkan and Sozen 1977; Iwan and Gates 1979). In those studies, maximum inelastic response of a real system was determined based on the properties of an equivalent best-fit single-degree-of-freedom. The equivalent system was defined by its period and its equivalent viscous damping ratio.

Logarithmic decrement method is commonly used in civil engineering applications to compute damping of structures. This method estimates damping as a fraction of the critical viscous damping. Logarithmic decrement assumes constant stiffness and amplitude-independent viscous damping. Despite the fact that stiffness and energy dissipation characteristics change with displacement response amplitude, the logarithmic decrement method is used indistinctly in civil engineering applications (Kwok et al. 1990; Li et al. 2002). Alternate methods to estimate the viscous damping ratio from vibration data such moving-block analysis (Bousman and Winkler 1981) and through Hilbert-transform (Yang et al. 2004), are also based on linear elastic, constant damping ratio dynamic model considerations. In the first method Fourier transformation of data within a given window of the signal allows estimation of frequency and damping of transient type of signal. In the second method, frequency and damping are estimated

based on the known relationships of instantaneous amplitude and phase with time.

Studies that show variation of equivalent viscous damping ratio with inelastic displacement levels in reinforced concrete structures can be found in the literature (Eberhard and Sozen 1989; Moehle and Sozen 1978; Morrison and Sozen 1981). In those studies, viscous damping ratio was obtained through the logarithmic decrement technique applied on free vibration records. Each of the tests was performed with similar initial pull force. Consequently, different displacement response was observed in the structures depending on the level of stiffness deterioration at the time of tests. There was a general trend of damping to increase after inelastic level demands. Viscous damping ratio in reinforced concrete beams was measured after increasing inelastic displacement levels by Lakshmanan (Lakshmanan et al. 1991). Viscous ratio was observed to increase after initial, lower displacement levels and then decrease after larger displacement levels.

Energy dissipation mechanism in a structure might be caused by presence of different types of damping. Work to estimate the viscous damping ratio (due exclusively to viscous damping portion) of systems with viscous and friction damping has been done (Wu et al. 2007). Methods to estimate the damping coefficient of systems with displacement-dependent stiffness and velocity-dependent damping have been proposed (Feldman 2006; and 2007).

CHAPTER 3. TOOLS TO ESTIMATE NONLINEAR STIFFNESS AND NONLINEAR DAMPING OF ELASTIC SINGLE DEGREE OF FREEDOM STRUCTURES

3.1. Introduction

In the most general form an elastic SDOF structure can exhibit both variable stiffness and variable damping. Estimation of nonlinear stiffness of the system can be done through time-frequency representations mentioned in section 2.2. In this chapter, simulated responses of elastic systems with constant or variable stiffness and constant or variable viscous damping ratio are used for observation of frequency and viscous damping ratio. Estimation of nonlinear secant stiffness and damping of the systems was done through the instantaneous frequency obtained via Hilbert transform. The chapter starts with a review of the Fourier transform as an introduction to frequency-content investigation of signals. Then, the Hilbert transform formulation is presented and examples of comparison with the Fourier transform are given. Approaches to extraction of elastic restoring force versus displacement and damping-versus displacement curves from simulated free vibration responses are presented.

3.2. Fourier transform

The most common tool used in civil engineering applications to analyze the frequency-content of vibration records is the Fourier transform. The Fourier transform converts an original time domain signal $x(t)$ to the frequency domain $X(\omega)$ as

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-i\omega t} dt \quad \text{Eq. 3.1}$$

where, $i = \sqrt{-1}$. The discrete version of equation 3.1, named as the Discrete Fourier Transform (DFT) (Oppenheim et al. 1999), can be written as

$$X_k = \sum_{j=0}^{N-1} x_j e^{\left(-2\pi i * \frac{j}{N} k\right)} \quad \text{Eq. 3.2}$$

where x_j is the time domain data and X_k is its representation in the frequency domain; N is the number of data samples. Typically, the DFT is implemented in the form of a Fast Fourier Transform (FFT). In the Fourier transform, the original signal is expressed as summation of sine and cosine functions properly weighted. The signal is assumed to be stationary. Frequency resolution of the Fourier transform in digital formulation is $\Delta f = 1 / N \cdot \Delta t$, where Δt is the time-step of the original signal, and Δf is the frequency of the harmonic function with the longest period that could be used in the representation.

Figure 3-1 (a) and (b) show a harmonic signal and its corresponding frequency representation through the Fourier transform, obtained using FFT. Different Fourier Spectra amplitudes were obtained using different time portions of the record. The precision of the FFT is improved as larger time portions of the record are considered ($\Delta f = 1 / N \cdot \Delta t$). Additional information about location of frequency in time (time-frequency distribution) through Fourier transform can be done by use of the Spectrogram. The Spectrogram computes the FFT of consecutive sliding time-window portions of the original signal to give estimates of frequency content over time.

Consider the free vibration response of a 2-DOF linear system with contributions from each mode in the form of $x_i(t) = e^{-\zeta_i \omega_i t} \left(\cos \omega_{di} t + \zeta_i / \sqrt{1 - \zeta_i^2} \sin \omega_{di} t \right)$, where ω_i , ω_{di} and ζ_i are the undamped and damped circular natural frequencies, and

equivalent viscous damping ratio of mode i . Total response $x(t)$ is computed as contribution of first and second mode as $x(t)=x_1(t)+x_2(t)$. Figure 3-2 (a) shows the total displacement response $x(t)$ obtained as summation of $x_1(t)$ and $x_2(t)$. Figure 3-2 (b) shows the corresponding frequency representation through the FFT with modal frequencies correctly identified labeled as f_1 and f_2 ($\omega_1=2\pi f_1$ and $\omega_2=2\pi f_2$).

Figure 3-2 (c) to (f) show the spectrograms of $x(t)$ for different time-window selections. On the vertical axes, a pair of values indicating the time-window in seconds and corresponding frequency-resolution in Hz are given. It can be seen that as the length of the time-window is increased the frequency resolution is improved which leads to better estimates of the two frequency components. However the finer frequency resolution is obtained only at the expense of lower time resolution. The best frequency resolution obtained from Fourier transform is that obtained when the transformation is applied to the full record at once. This condition, in turn, limits the frequency resolution obtained through the Spectrogram.

3.3. Analytical signal and Hilbert Transform

The Fourier Transform (FFT) of a real signal with N terms is a complex vector with N terms. Physical frequency content of the signal is determined by the first group of terms, $N/2+1$ terms (if N is even number) of the FFT series. The last group of terms in the FFT series (from $N/2+2$ to N term) is a “mirror” of those that determine the frequency content of the signal. The “mirror” frequency with respect the frequency information is replicated is known as the Nyquist frequency (half the sampling rate). Positive and negative frequencies in signal processing are designated as the frequencies separated by the Nyquist frequency.

The analytical signal is a representation that allows removing the negative frequency from the given original signal. If the original signal is a real data series its analytical signal will be a complex data series. The Hilbert transform of an original signal is the imaginary part of its analytical signal and therefore it allows estimating its analytical signal. Any signal $x(t)$ with a defined Hilbert transform, can be represented in its analytical signal form $X(t)$ as follows:

$$X(t) = x(t) + i * x'(t) = A(t)e^{i\theta t} \quad \text{Eq. 3.3}$$

where, $i = \sqrt{-1}$, A is the instantaneous amplitude, θ is the instantaneous phase angle, and $x'(t)$ is the Hilbert Transform defined as (Bendat and Piersol 1986)

$$x'(t) = \int_{-\infty}^{\infty} \frac{x(u)}{\pi(t-u)} du \quad \text{Eq. 3.4}$$

The Instantaneous frequency is defined as the time derivative of the phase angle

$$f(t) = \left(\frac{1}{2\pi} \right) \frac{d\theta(t)}{dt} \quad \text{Eq. 3.5}$$

3.4. Comparison of the Fourier and the Hilbert transforms

Three examples of harmonic signals are given below to compare the Fourier and the Hilbert transforms. Frequency content estimated through the Fourier transform (using FFT) is compared to the frequency content estimated from Hilbert transform formulations using equations 3.3, 3.4 and 3.5. It is observed that in the case of mono-component signals, i.e., where at any given time only one frequency is associated with the process that generated the signal, analysis using the Hilbert transform has the advantage over the Fourier transform due to

fact that the Hilbert transform can locate the frequency in time and can even provide finer frequency resolution. On the other hand, when the signal analyzed is not a mono-component signal only Fourier can give accurate estimations of frequency content, although it cannot capture the possible presence of a non-stationary of the signal.

3.4.1. Example 1

Consider the pure sinusoidal signal $x(t) = \sin(2 \cdot \pi \cdot f \cdot t)$ with a total duration of 10 seconds and vibration frequency 'f' defined as: $f = 1.0$ Hz for $t \leq 5$ seconds and $f = 1.05$ Hz for $t > 5$ seconds. Figure 3-3 (a), (b) and (c) show the original signal, the instantaneous frequency estimated through equations 3.3, 3.4 and 3.5 (Hilbert transform) and the Fourier spectrum of the whole record, respectively. The shift in frequency at time $t=5$ seconds is detected by the instantaneous frequency computed through the Hilbert transform. On the other hand, Fourier spectrum is unable to detect the two different frequencies in the signal because the frequency resolution of the FFT is $\Delta f = 1 / T_{record} = 1 / 10 \text{ sec} = 0.1 \text{ Hz}$.

The instantaneous frequency of Figure 3-3(b) shows a discontinuity toward the ends of the signal, i.e. near $t=0$ and $t=10$ seconds. This discontinuity is due to the fact that the Hilbert transform was computed through the FFT but with negative frequencies removed from the Fourier transform of the original signal. The Hilbert transform was obtained as the inverse Fourier transform of the modified, no-negative frequency content representation (The MathWorks 2004). The Fourier Transforms are exposed to the Gibbs phenomenon (Oppenheim et al. 1999), which introduces large oscillations in the Fourier series near discontinuities of the signal. In the case of the beginning of the signal, the oscillation in the Fourier series is caused by the change from zero (for $t < 0$) to finite values of the signal ($t > 0$) and it has effect up to approximately 1.5 seconds.

3.4.2. Example 2

Consider a signal with a frequency changing linearly with time (chirp function). The instantaneous frequency of the signal is set at $f = 1$ Hz at time $t = 0$, and $f = 10$ Hz at time $t = 10$ seconds. Figure 3-4 (a), (b) and (c) show the original signal, the instantaneous frequency computed through Hilbert transform and the Fourier spectrum of the signal, respectively. . As seen in Figure 3-4(b), the Hilbert transform captures the varying frequency accurately. The Fourier spectrum also shows the components of the range of frequencies in the whole signal. The Fourier transform is capable of identifying the frequency variation in the signal successfully because the wide range of frequency shift (from 1 to 10 Hz) in the signal is larger than the frequency precision (0.1 Hz) of the Fourier transform. However, by definition, the Fourier spectrum cannot indicate the instant at which a given frequency exists in the signal. Rather, it assumes that all of frequencies shown in the spectrum exist at all times, i.e. the signal frequency content is stationary.

3.4.3. Example 3

Consider a signal made up of two stationary harmonic functions along the whole record, $x(t) = \sin(2 \cdot \pi \cdot f_1 \cdot t) + \sin(2 \cdot \pi \cdot f_2 \cdot t)$ with frequencies $f_1 = 1$ Hz and $f_2 = 2$ Hz. Unlike the previous two examples, this signal is not a mono-component signal because at each instant it has two components with distinct frequencies, f_1 and f_2 . Figure 3-5 (a), (b) and (c) show the signal, its time-frequency distribution obtained through Hilbert transform, and its Fourier spectrum, respectively. The instantaneous frequency computed through the Hilbert transform identifies two different frequencies, one at $(f_1 + f_2)/2 = 1.5$ Hz (Deering and Kaiser 2005) and another at around $f = 400$ Hz. Both frequencies are fictitious. However, the Fourier spectrum correctly identifies the two component frequencies of the signal. This simple example illustrates the advantage of the Fourier transform over the Hilbert transform in analyzing the frequency content of a signal. This example

also illustrates the required condition of having a mono-component signal to determine the frequency content of the signal accurately through the Hilbert transform. If the signal being analyzed is not a mono-component signal, filters need to be used to retain only the frequency of interest in the signal before applying the Hilbert transform.

3.5. Physical interpretation of the instantaneous frequency

The dynamic equation for a SDOF system can be written as

$$m \ddot{x} + c \dot{x} + kx = F(t) \quad \text{Eq. 3.6}$$

where m is the mass; c is the damping coefficient which for the viscous damping formulation can be stated as $c = 2\zeta\omega m$, with ζ and ω being the viscous damping ratio and the circular natural frequency respectively; k is the stiffness of the system; and $F(t)$ is the external forced applied. The stiffness can be expressed in terms of its natural frequency f [Hz] as

$$k = 4\pi^2 f^2 m \quad \text{Eq. 3.7}$$

3.5.1. Free vibration response of a linear system

Figure 3-6 (a) shows the free vibration response of a linear system with mass $m = 1$, stiffness $k = 4\pi^2$ (consequently natural frequency $f = 1$ Hz) and equivalent viscous damping ratio of 1% of the critical ($\zeta = 0.01$). The instantaneous amplitude $A(t)$ of the analytical signal of equation 3.3 is plotted in Figure 3-6 (a). Figure 3-6 (b) and (c) show the corresponding instantaneous stiffness computed by equations 3.5 and 3.7 and the Fourier spectrum of the vibration response respectively. Figure 3-6 (b) shows that the constant stiffness of the system k is

accurately estimated through the Hilbert transform. The constant stiffness is also represented by a unique peak in the Fourier spectrum of Figure 3-6 (c).

3.5.2. Free vibration response of a nonlinear elastic system

Figure 3-7 illustrates two different types of nonlinear elastic systems described by their stiffness-displacement and load-displacement relationships. The secant stiffness defined as the ratio between the net force applied and the displacement measured in the system has been used to define the systems. Figure 3-7 (a) and (b) correspond to a nonlinear softening system while Figure 3-7 (c) and (d) are defined for a nonlinear hardening system. Both systems have a limit displacement $|d_{\text{linear}}| = 0.1$, below which the systems behave linearly (constant stiffness). This constant stiffness zone has been defined in order to address representation of realistic mechanical structures that below certain displacement levels might exhibit a constant stiffness, i.e. linear elastic behavior. The secant stiffness below the elastic limit has been defined as $k_o = 9\pi^2$ in both systems of Figure 3-7.

Figure 3-8 (a), (b) and (c) show the displacement response, the instantaneous frequency computed by equation 3.5, and the Fourier spectrum of the free vibration response of the nonlinear softening system of Figure 3-7 (a) and (b). The mass of the system and the equivalent viscous damping ratio were defined as $m = 1$ and $\zeta = 1\%$. Consequently, natural frequency below the elastic limit ($|d_{\text{linear}}|$) is computed as $f_o = \sqrt{k_o / m} / 2\pi = 1.5 \text{ Hz}$. Amplitude of the analytical signal $A(t)$ defined in equation 3.3 is plotted in Figure 3-8 (a). It is observed that the instantaneous frequency increases as the vibration amplitude decays as illustration of the nonlinear softening behavior. After the displacement amplitudes have fallen below the elastic limit $|d_{\text{linear}}|$, the instantaneous frequency reaches a constant value associated with the constant stiffness k_o ($f = f_o = 1.5 \text{ Hz}$). Figure 3-8 (c) illustrates the presence of different frequencies in the total response of the

system as well. Different peaks in the Fourier spectrum can be interpreted as different vibration frequencies in the SDOF system. The magnitude of each of those peaks is associated to the amplitude of vibration at that given frequency. Larger peak-magnitudes are associated to lower frequencies because the system stiffens as the vibration amplitude decays. However, the Fourier spectrum cannot tell which unique frequency was governing the response at what time.

In order to develop a physical interpretation of the instantaneous frequency, values of the instantaneous stiffness computed through the Hilbert transform and the analytical expressions that defined the systems shown in Figure 3-7 (a) and (b) are compared. Figure 3-9 (a) shows a short duration of the free-vibration response of the nonlinear softening system. In Figure 3-9 (b) the actual stiffness values of the system ('defined') and those obtained by using the Hilbert transform, i.e. equations 3.5 and 3.7, on the free vibration response ('estimated') are plotted. It is observed that the instantaneous stiffness obtained via the Hilbert transform matches the actual stiffness only at peak-amplitude vibration points. This comparison reveals the limitation of the Hilbert transform in giving accurate results at instants different than the peak-displacement vibration points.

Similar analyses have been developed for the nonlinear hardening system of Figure 3-7 (c) and (d). The displacement response, the instantaneous frequency via equation 3.5 and the Fourier spectrum of the free vibration response of the system are plotted in Figure 3-10 (a), (b) and (c), respectively. The mass of the system and the equivalent viscous damping ratio were set as $m = 1$ and $\zeta = 1\%$. Evolution of instantaneous frequency with time reveals the nonlinear hardening behavior. The instantaneous frequency of Figure 3-10 (b) decreases until the vibration amplitudes in the system fall below the linear limit $|d_{\text{linear}}|$, at which point the instantaneous frequency reaches a constant value corresponding to the constant stiffness and frequency ($k_o = 9\pi^2$, $f_o = 1.5 \text{ Hz}$). Similar observations

can be made from the Fourier spectrum in Figure 3-10 (c), where multiple peaks correspond to different vibration frequencies. The magnitude of each of those peaks is associated to the amplitude of vibration at that given frequency. Larger peak-magnitudes are associated to higher frequencies because the system softens as the vibration amplitude decays. However, the Fourier spectrum itself cannot identify this behavior due to the fact that Fourier series representation of a signal assumes stationary frequency content over the duration that the Fourier transform is applied. Figure 3-11 (a) shows a shorter portion of the response in Figure 3-10 (a). Figure 3-11 (b) shows a comparison of the instantaneous stiffness estimated by equation 3.5 ('estimated') and actual stiffness of the system ('defined'). The instantaneous frequency estimated via the Hilbert transform is accurate only at peak-amplitude vibration points. The comparison between actual and estimated stiffness through the Hilbert transformation evidences, one again, the limitation of the Hilbert transform to estimate the instantaneous frequency at instants different than those displacement peaks.

3.6. Extracting load-displacement curve from free-vibration response

The free-vibration response of a SDOF is a mono-component signal and exhibits a single vibration frequency at each instant of the response. This holds true even if the system has a displacement-dependent stiffness. The well-behaved Hilbert transform applied on the response of a SDOF system permits estimating the corresponding stiffness-displacement and load-displacement relationships. As noted in section 3.5.2, instantaneous frequency estimate from the Hilbert transform matches the actual frequency only at the peak displacement points. Accordingly, estimates of stiffness, obtained through equation 3.7, are valid only for those instants. This condition implies that the recovered load vs. displacement relationship will be a discrete version of the full curve with information at the peak-displacement points only.

Figure 3-12 shows the estimated stiffness-displacement and load-displacement relationships of the system described in Figure 3-7 (a) and (b). Peak-displacement points used in the estimation are identified with circles in Figure 3-12 (a). Values of stiffness estimated using equation 3.5 and 3.7 are shown in Figure 3-12 (b) and the corresponding load versus displacement relationship in Figure 3-12 (c). Actual curves plotted for comparison have been labeled as 'defined'. Good agreement between actual and estimated values of stiffness of the system is observed.

3.7. Logarithmic decrement method to estimate equivalent viscous damping ratio

Estimation of damping from free vibration response of civil engineering structures is commonly done by logarithmic decrement

$$\zeta = \frac{1}{2\pi n} \ln \left(\frac{x_i}{x_{i+n}} \right) \quad \text{Eq. 3.8}$$

where ζ is the equivalent viscous damping ratio obtained using peak amplitudes x_i and x_{i+n} of the response after n number of cycles. Logarithmic decrement is a method formulated for linear systems with constant viscous damping (Clough and Penzien). It is suggested that damping could be estimated by using the Random Decrement Technique (Cole 1973). This technique has been successfully applied to civil engineering structures by several researches (Jeary 1997; Li et al. 2002)

Damping estimation through logarithmic decrement was analyzed on one linear and two nonlinear elastic systems. Figure 3-13 (a) shows the free vibration response of a linear system with mass, stiffness, and equivalent viscous damping ratio defined as $k = 4\pi^2$, $m = 1$, and $\zeta = 1\%$, respectively. Figure 3-13 (b) shows the equivalent viscous damping ratio estimated by equation 3.8 plotted versus

time based on the peaks illustrated in Figure 3-13 (a). The damping through equation 3.8 was estimated in two different ways. In the first approach, the initial peak response and the current peak response is used ('init-to-current'); in the second approach adjacent pair of peaks is used ('adjacent pair'). Actual damping ratio defined in the system is plotted as a line for reference in Figure 3-13 (b). Good agreement between damping estimates and the actual value is observed.

The free vibration responses of an elastic variable-stiffness system with constant damping (Figure 3-14) and variable damping (Figure 3-15) are considered. Secant stiffness k of the system has been defined through the displacement x as $k(x) = 4\pi^2 \cdot |x|^{-0.2}$. The damping was defined as $\zeta = 0.01$ in the case of constant damping and as $\zeta(x) = 0.01 + 0.01 \cdot |x|$ (linear-displacement-dependent). Part (a) of Figure 3-14 and Figure 3-15 shows the free vibration displacement response and peaks used to estimate damping according to equation 3.8, while part (b) shows the comparison between estimated and defined ('actual') values of damping. The two different approaches to estimate equivalent viscous damping ratio using logarithmic decrement, as described previously, were used. Logarithmic decrement method estimated damping ratios that are as much as 50% different from the actual values in the nonlinear systems.

A parametric study of the damping ratio estimated through equation 3.8 was done considering different elastic variable-stiffness systems and two different types of damping, similar to the examples of Figure 3-14 and Figure 3-15. A few load-versus-displacement curves of the systems considered are plotted in Figure 3-16 (a). For sake of clarity not all the load versus displacement curves were plotted in Figure 3-16 (a). The secant stiffness of each system is defined by the parameter α as $k(x) = 4\pi^2 \cdot |x|^{-\alpha}$, with $\alpha = 0$ corresponding to the constant-stiffness system. Values of α between 0 and 0.5 were considered. Damping ratio was defined as $\zeta = 0.01$ for constant damping cases and as $\zeta(x) = 0.01 + 0.01 \cdot x$

for the cases where equivalent viscous damping ratio varies linearly with displacement. Only initial-to-current peak type of logarithmic decrement estimation was used. Forty peaks ($n = 40$) of the responses were used for estimation according to equation 3.8.

Figure 3-16 (b) shows the ratio between estimated and actual damping values after n cycles. It is seen that higher stiffness nonlinearities (larger values of α) in the system lead to larger overestimation of damping. Figure 3-16 (c) shows the ratio of estimated damping using the first and second peaks ζ_1 and using first and last peak ζ_N . It is observed that estimation of damping is almost independent of pair of peaks used for computation in the case of a system with constant damping. On the other hand, variability of estimated damping is almost 30% in all cases when nonlinear damping exists. These observations illustrate uncertainties in estimating damping by logarithmic decrement when the most general case, i.e. nonlinear stiffness and nonlinear damping are present in the structure.

3.8. Energy method to estimate damping from free vibration response

Consider the free vibration displacement response of an elastic SDOF system as shown in Figure 3-17 (a). At each peak of the displacement response the potential energy is maximized and the kinetic energy is zero. The energy equilibrium between peaks i and $i+1$ can be written as

$$PE_i = PE_{i+1} + DE_i \quad \text{Eq. 3.9}$$

where PE_i and PE_{i+1} are the potential energy at peaks i and $i+1$ and DE_i is the dissipated energy between those peaks. The potential energy terms are estimated as the area under the curve of restoring force P versus displacement x shown in Figure 3-17 (b)

$$PE_i = \int_0^{x_i} P(x) dx \quad \text{Eq. 3.10}$$

where x_i is the i -th peak displacement. The dissipated energy formulated for the equivalent viscous damper can be written as

$$DE_i = \int_{t_i}^{t_{i+1}} c \dot{x}^2 dt \quad \text{Eq. 3.11}$$

where $\dot{x}$ is velocity, t_i and t_{i+1} are the instants at which the peak-displacements i and $i+1$ happen, respectively. The damping coefficient c is expressed through the mass m and the displacement-dependent damping ratio $\zeta(x)$ and circular frequency $\omega(x)$, as $c = 2 \cdot \zeta(x) \cdot \omega(x) \cdot m$. The restoring force can be expressed by using equation 3.7 and using circular frequency as $P(x) = \omega(x)^2 \cdot x \cdot m$. Equation 3.9 can be rewritten by using these previous formulations and equations 3.10 and 3.11 as

$$\int_{x_{i+1}}^{x_i} \omega(x)^2 \cdot x \cdot dx = \int_{t_i}^{t_{i+1}} 2 \cdot \zeta(x) \cdot \omega(x) \cdot \dot{x}(t)^2 \cdot dt \quad \text{Eq. 3.12}$$

Using estimated circular frequency ω from section 3.6 and the given response of the system (velocity and displacement), equivalent viscous damping ratio can be solved from equation 3.12. This approach is called here as the energy method to estimate damping.

Damping estimation on the system of Figure 3-6 was done by using the energy method. Figure 3-18 (a) shows the free vibration response and the peak displacements used for computing the energy. Figure 3-18 (b) illustrates the energy balance of equation 3.12 by using the properties defined for the system. Figure 3-18 (c) shows the damping estimated by using the given response

displacement and velocity. Deviation of estimated damping from actual value is attributed to the discontinuity of the Hilbert transform at the ends of the response data (as explained in section 3.4.1). Because the system considered is linear results obtained are identical to those given by the logarithmic decrement method.

Damping was estimated in elastic systems with nonlinear stiffness and nonlinear damping through the energy method. Linear displacement-dependent damping ratio was defined as $\zeta(x) = \zeta_o + r \cdot |x|$. A mass-normalized free-vibration version of equation 3.6, i.e. $\ddot{x} + 2\zeta(x)\omega(x)\dot{x} + \omega(x)^2 x = 0$, permits to replicate the response by using estimated values of frequency ω and damping ζ . Because there are two parameters needed to define damping (ζ_o, r) , an additional constraint was imposed on the solution through the energy method. Besides the energy balance of equation 3.12, condition of identical displacement between the actual and simulated response was used.

Figure 3-19 (a) shows the free vibration response and peak displacements used in energy computations of a system with nonlinear stiffness ($\alpha = 0.20$, similar to systems of Figure 3-16). Figure 3-19 (b) shows a comparison of the damping estimates by the energy method and logarithmic decrement to actual values. Figure 3-19 (c) shows the comparison between the actual response and the simulated response obtained using estimated frequency and damping. Deviations from actual values were observed as 15% and 2% on the logarithmic decrement and energy method, respectively. An example of a system with higher stiffness-nonlinearity ($\alpha = 0.50$) is shown in Figure 3-20. In this case, deviations from actual values were observed as 40% and 15% on the logarithmic decrement and energy method, respectively. Regardless of local deviations of damping estimated by the energy method, there is a clear trend over the cycles of the response. In the last two examples a linear-displacement-dependent damping

was used as the type of damping defined for the system. Reliability of damping ratio measurement was evidenced by the consistent trend of the damping curve.

In the previous examples damping ratio was estimated through the energy method by assuming a linear variation of damping with displacement. This assumption turned out to match the actual damping defined in the system. For real applications where variation of damping with displacement is unknown, different damping functions might be tried. Although the load-displacement relationship can be extracted via the Hilbert transform, the quality of the estimations of damping of each function must be evaluated by comparison between actual and simulated response.

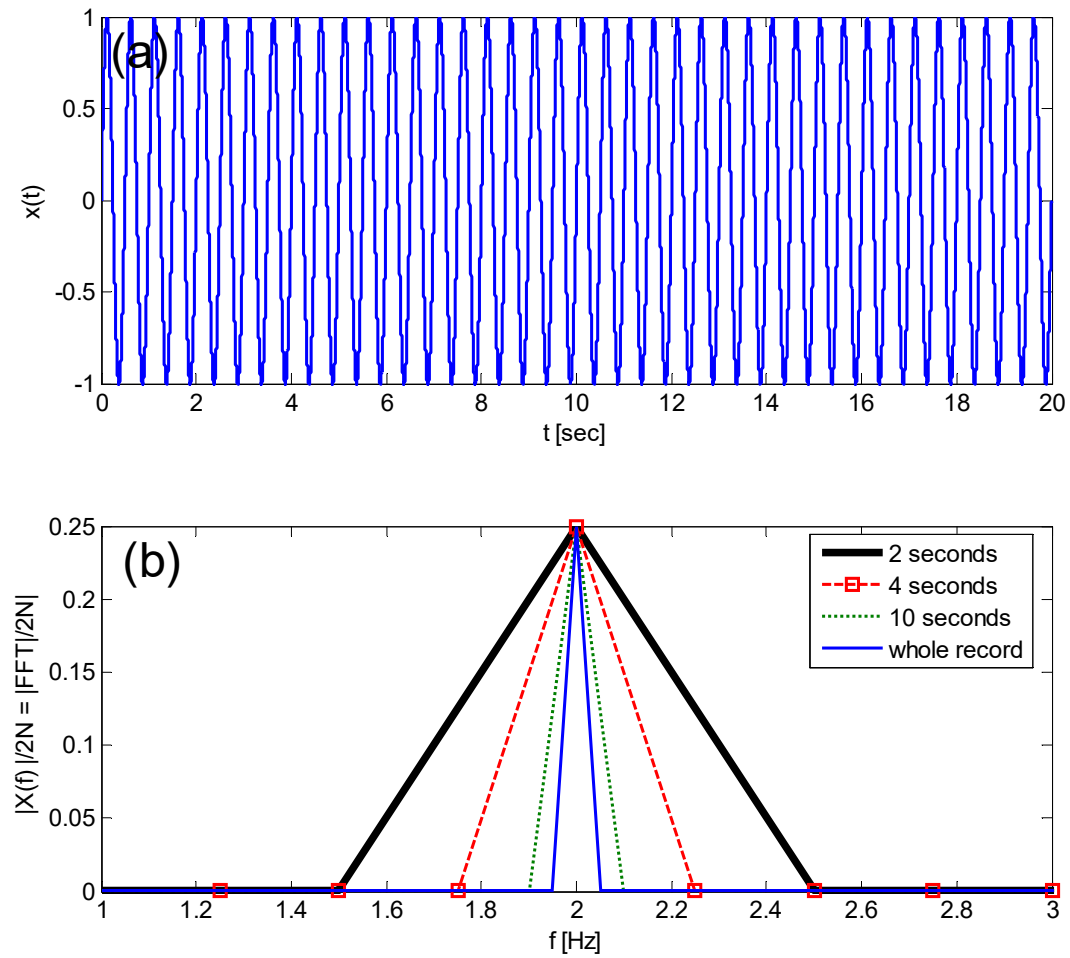


Figure 3-1 Fourier spectra of different time portions of a harmonic signal. (a) Harmonic signal. (b) Fourier spectra.

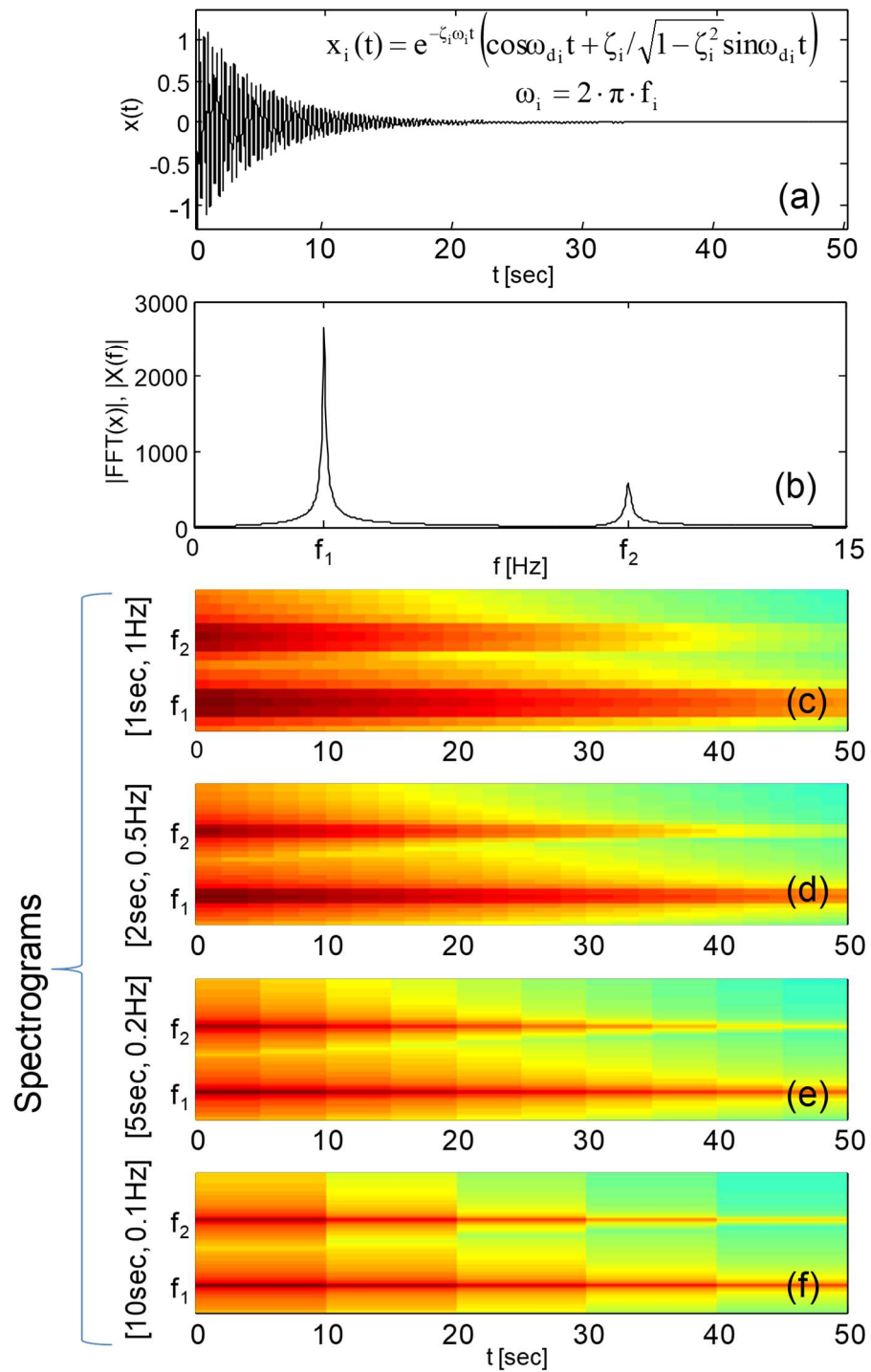


Figure 3-2 Free vibration response of 2-DOF linear-elastic system. (a) original response. (b) Fourier spectrum of the whole response. (c) through (f) Different window-time Spectrograms.

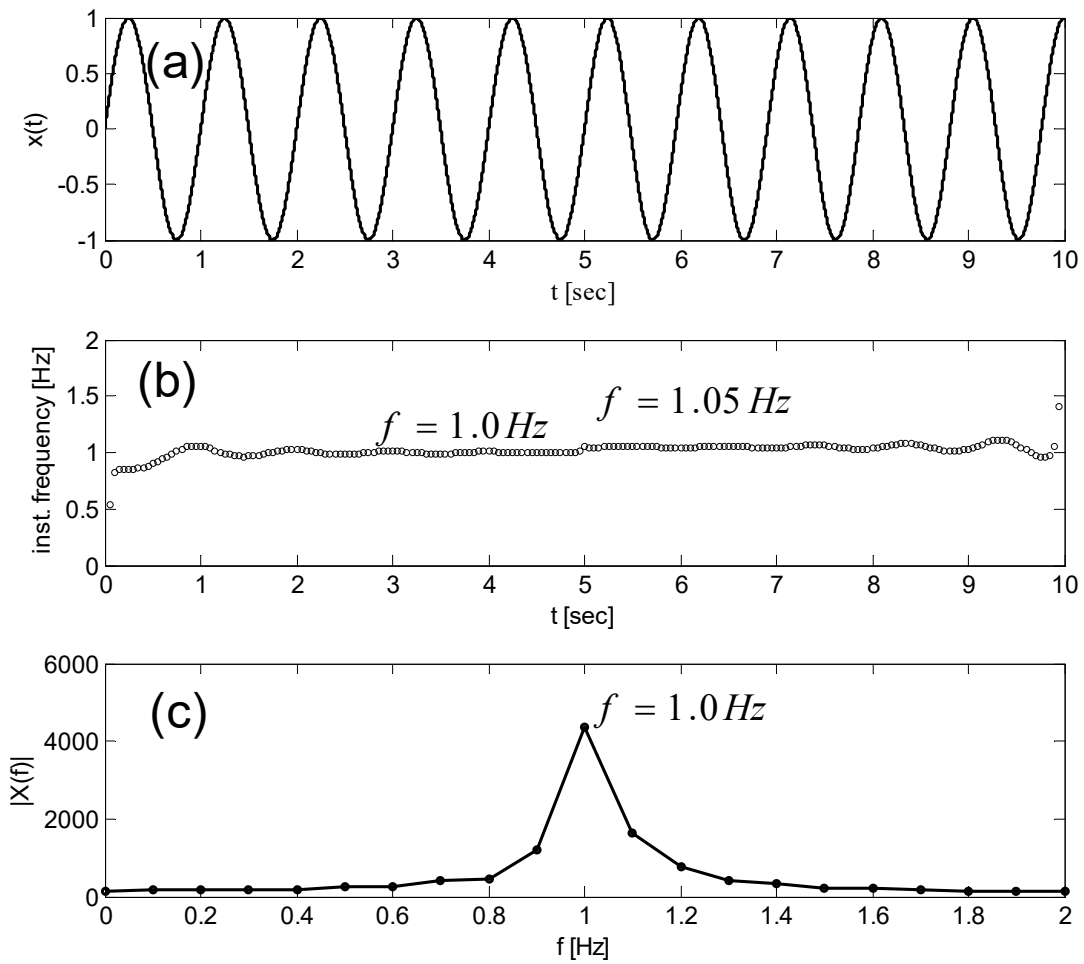


Figure 3-3 Example 1: Harmonic signal with $f = 1.0$ Hz for $t \leq 5.0$ sec and $f = 1.05$ Hz for $t > 5.0$ sec. (a) Original signal. (b) Instantaneous frequency. (c) Fourier spectrum of the signal.

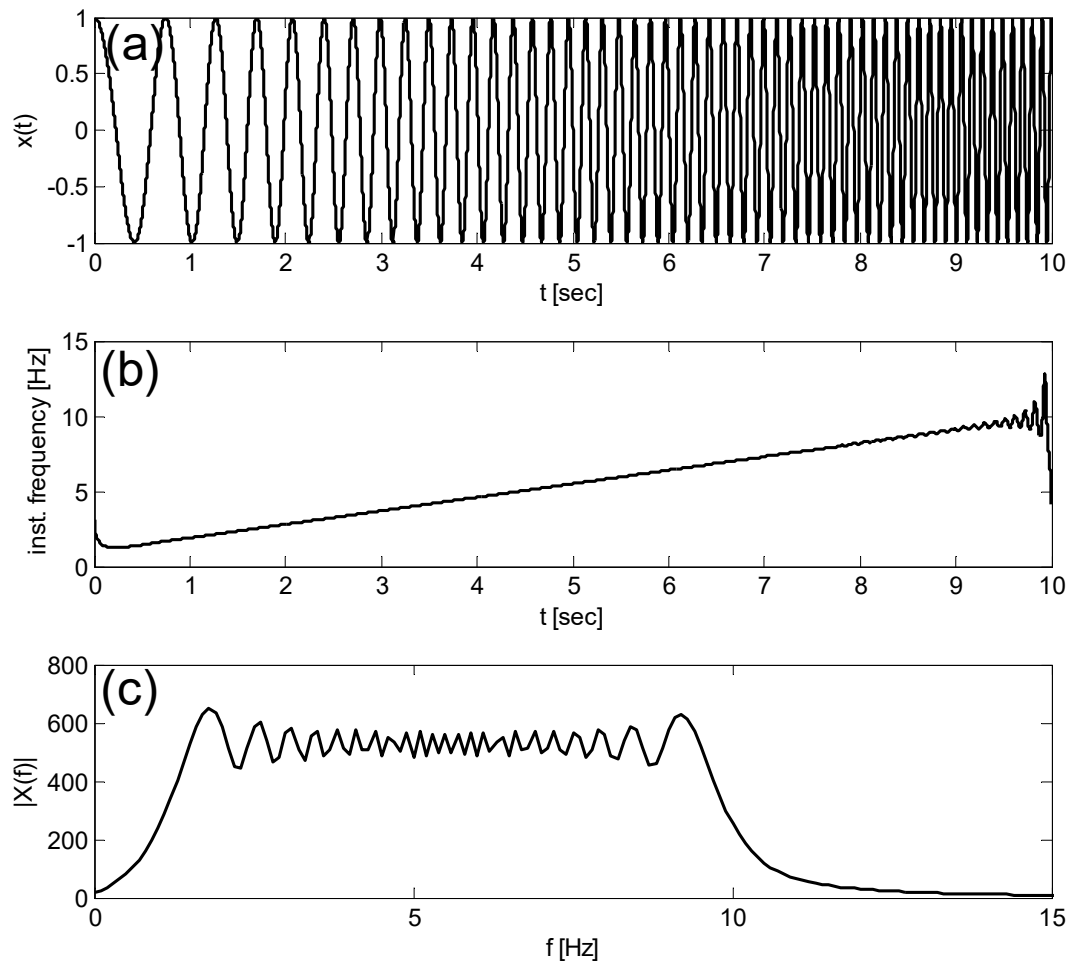


Figure 3-4 Example 2: Chirp signal, $f = 1.0$ Hz at $t = 0$ and $f = 10$ Hz at $t = 10$ sec. (a) Original signal. (b) Instantaneous frequency. (c) Fourier spectrum of the signal.

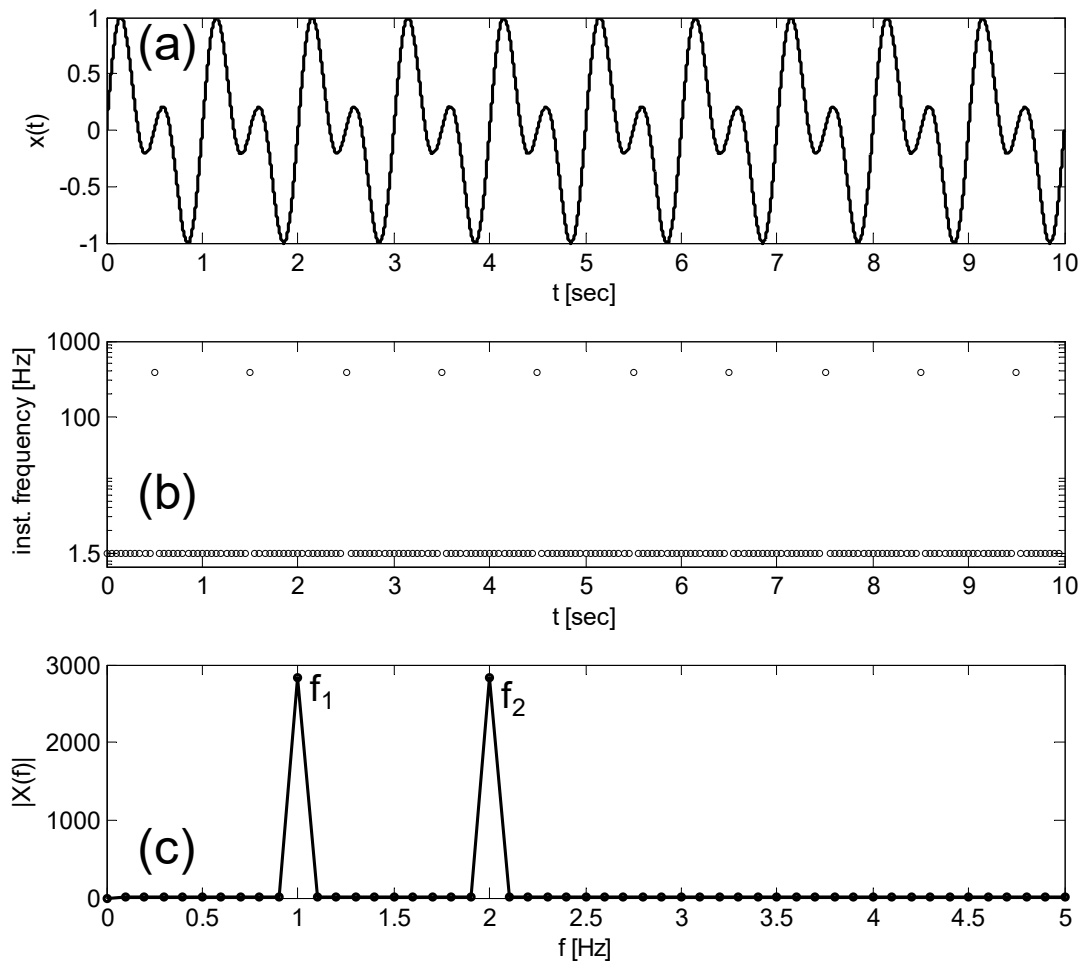


Figure 3-5 Example 3: Two-component harmonic signal, $f_1 = 1.0$ Hz and $f_2 = 2$ Hz. (a) original signal. (b) Instantaneous frequency. (c) Fourier spectrum of the signal.

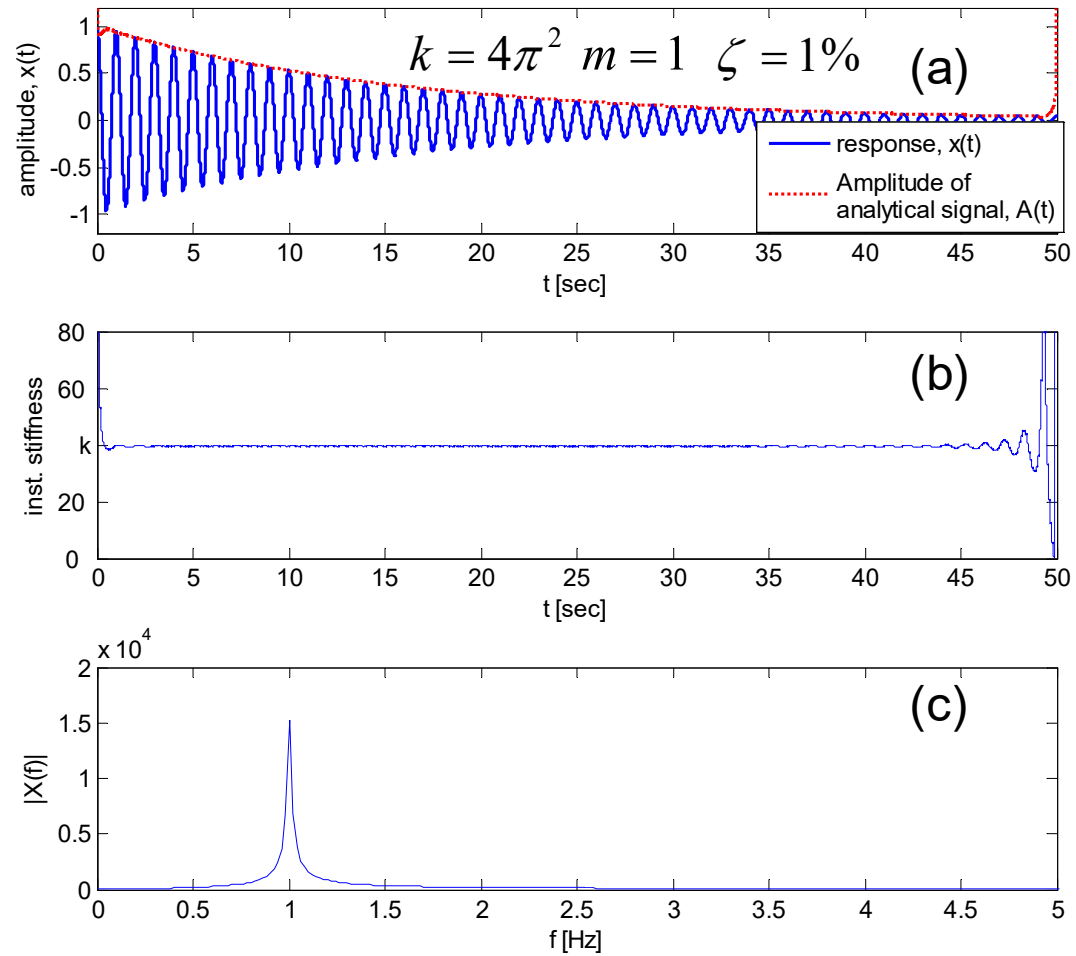


Figure 3-6 Free vibration response of a linear system. (a) Response. (b) Instantaneous frequency. (c) Fourier spectrum of the response.

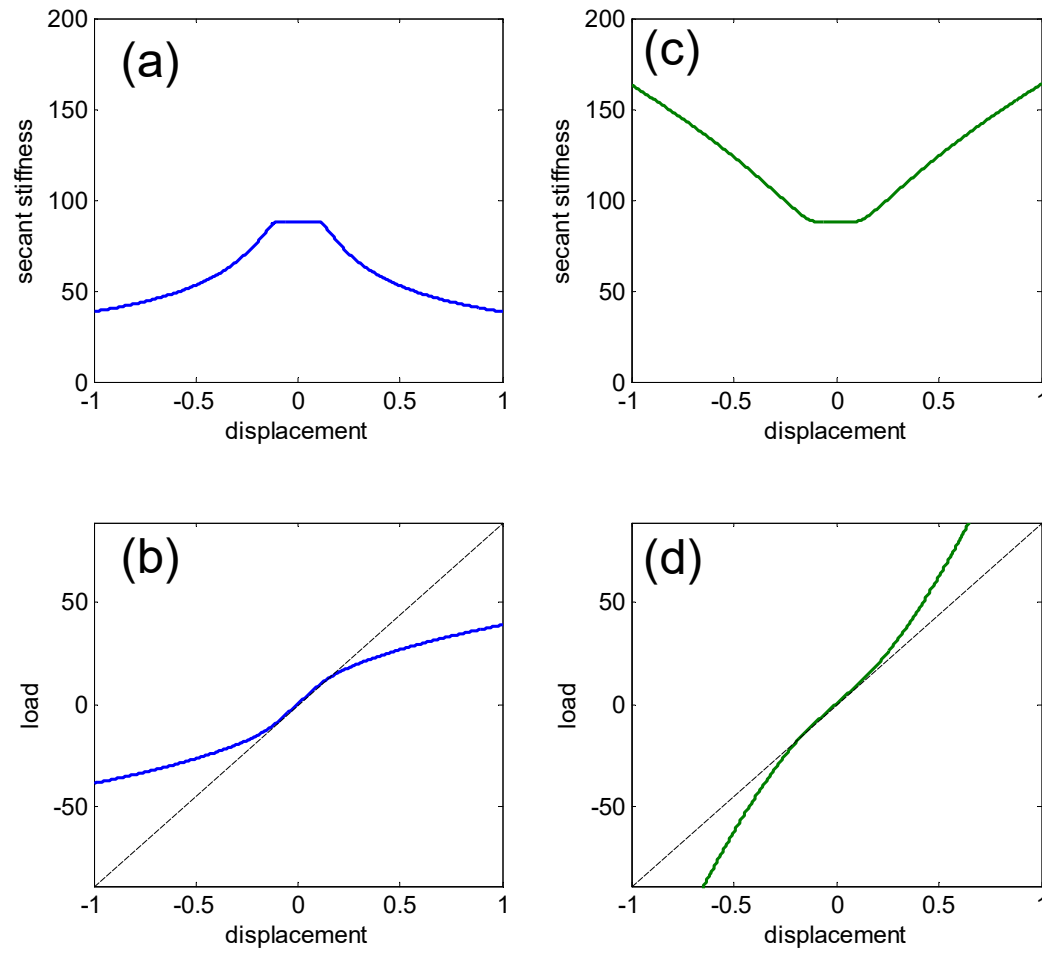


Figure 3-7 Nonlinear elastic systems. (a) and (b) nonlinear softening system. (c) and (d) nonlinear hardening system.

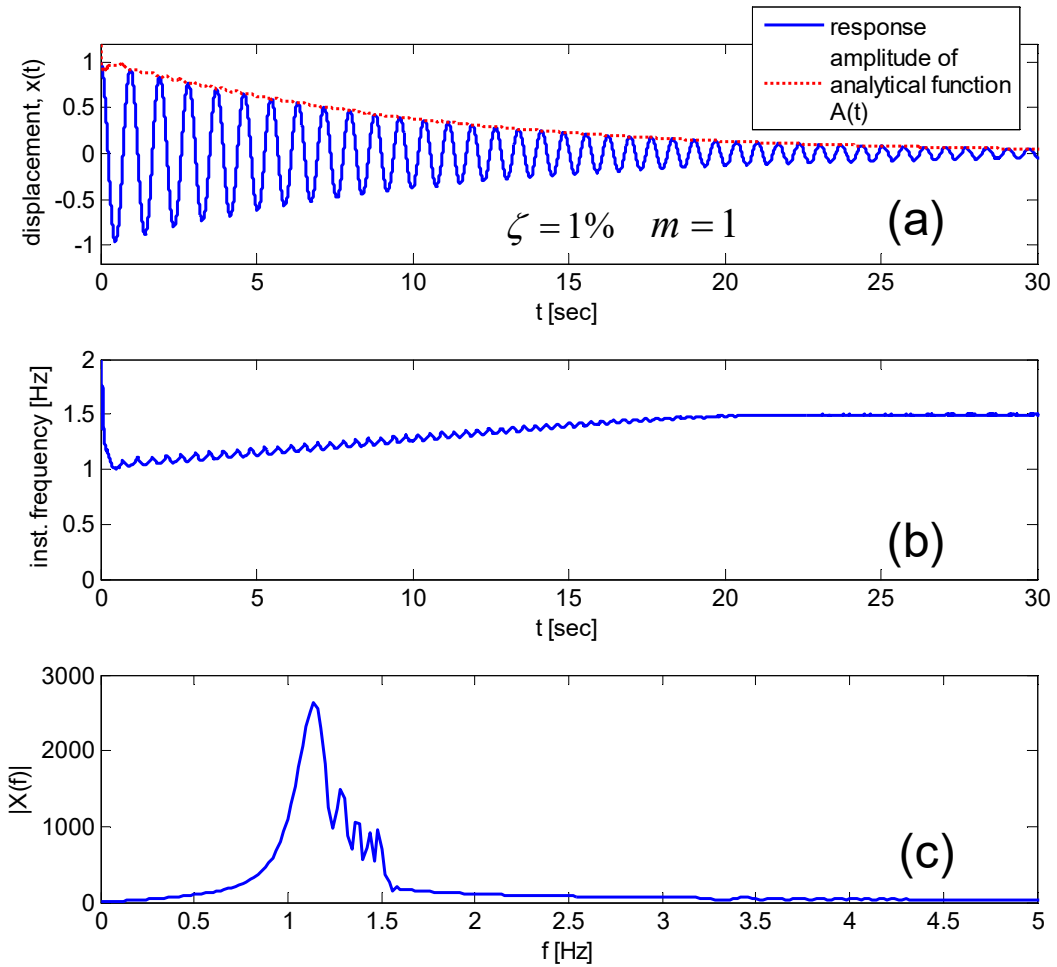


Figure 3-8 Free vibration response of a nonlinear softening system. (a) Displacement response. (b) Instantaneous frequency. (c) Fourier spectrum of displacement response.

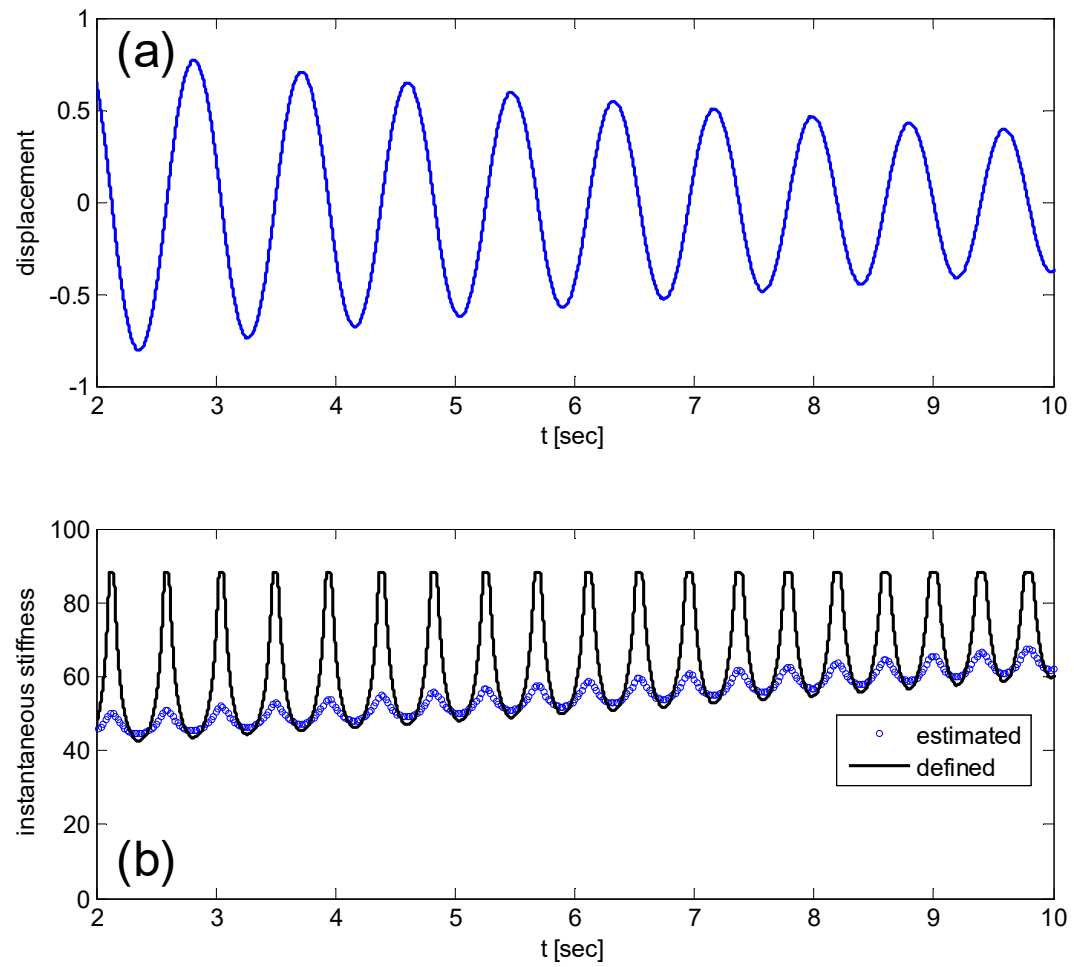


Figure 3-9 Comparison of estimated and defined stiffness of a nonlinear softening system. (a) displacement response.
(b) instantaneous stiffness.

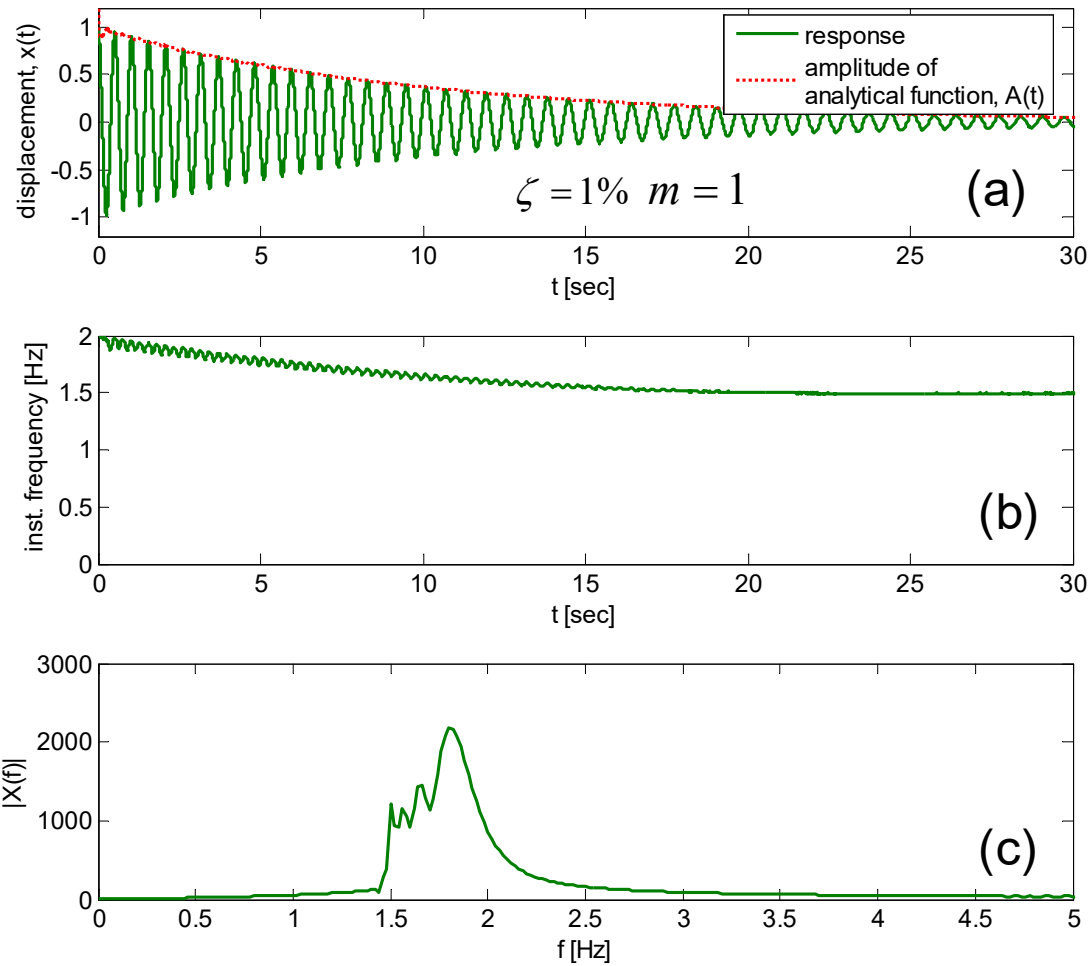


Figure 3-10 Free vibration response of a nonlinear hardening system. (a) Displacement response. (b) Instantaneous frequency. (c) Fourier spectrum of the displacement response.

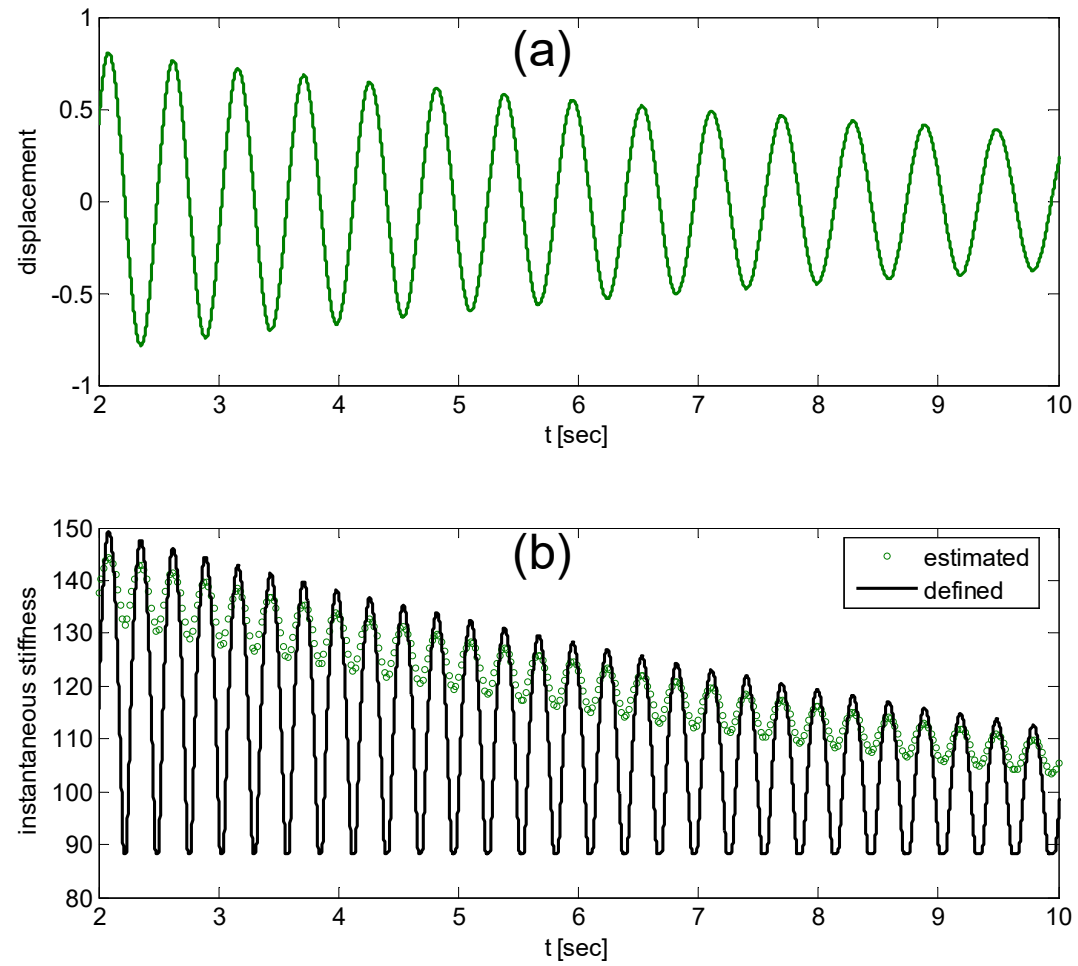


Figure 3-11 Comparison of estimated and defined stiffness of a nonlinear hardening system. (a) Displacement response. (b) Instantaneous stiffness.

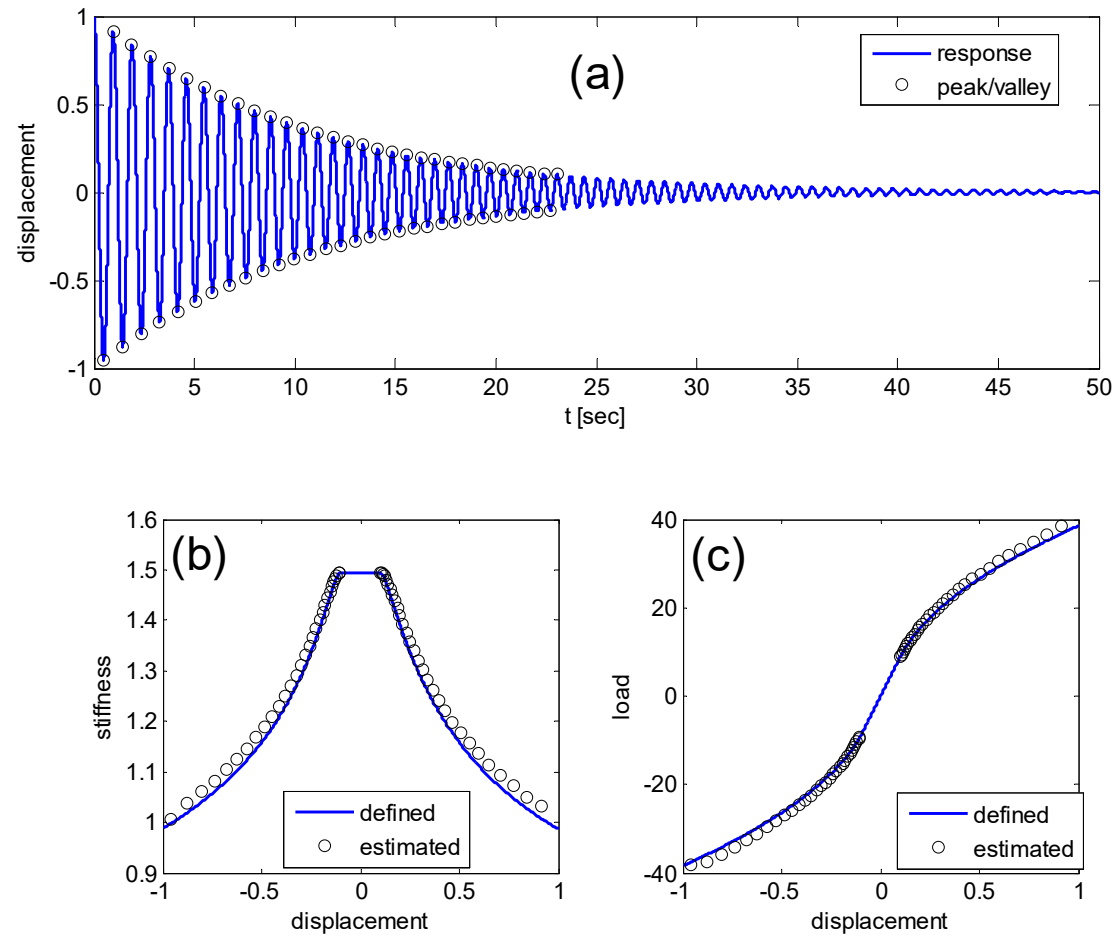


Figure 3-12 Extraction of the load-displacement curve of a nonlinear softening system. (a) Given displacement response. (b) Secant stiffness vs. displacement curve. (c) Load vs. displacement curve.

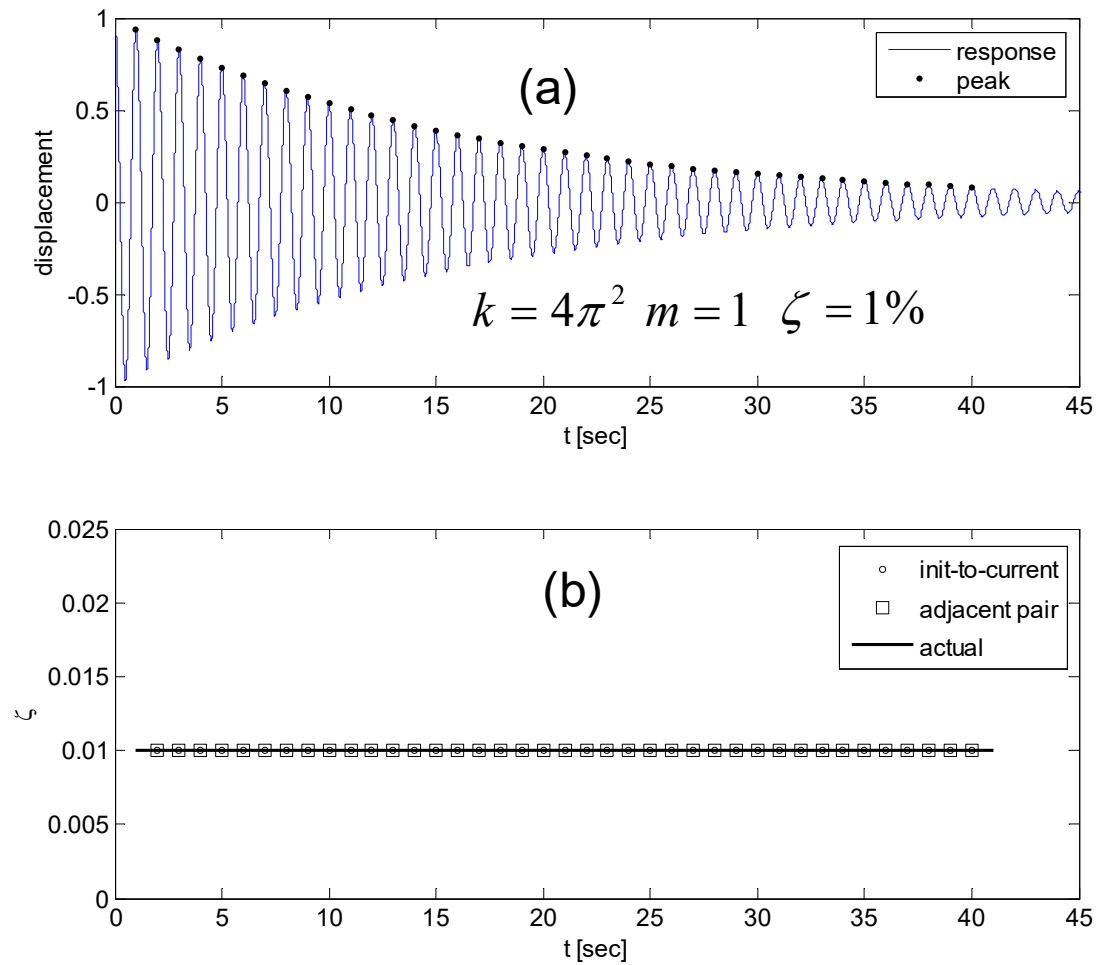


Figure 3-13 Logarithmic decrement method estimate for equivalent viscous damping of a linear system with constant damping ratio. (a) displacement response. (b) Equivalent viscous damping ratio.

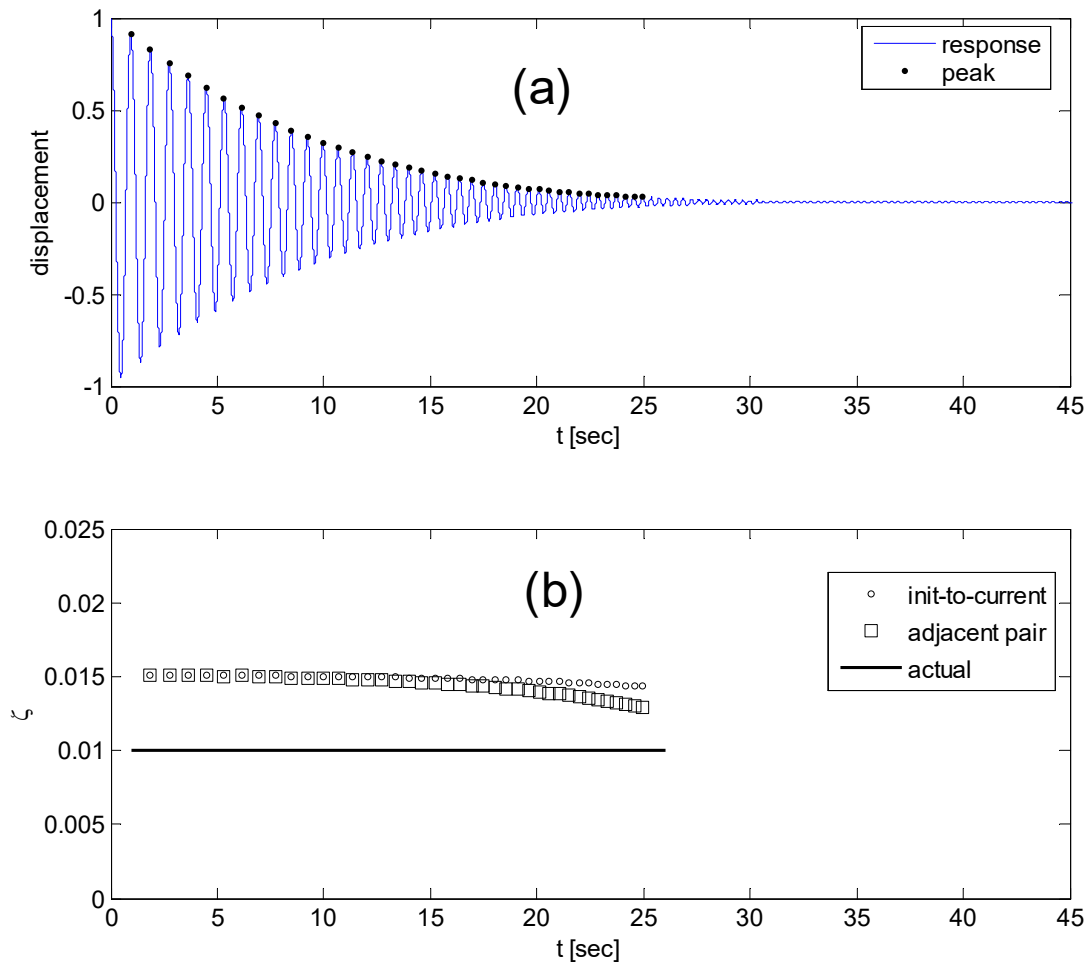


Figure 3-14 Logarithmic decrement damping estimation for a nonlinear elastic system with constant damping ratio.
 (a) Displacement response. (b) Equivalent viscous damping ratio.

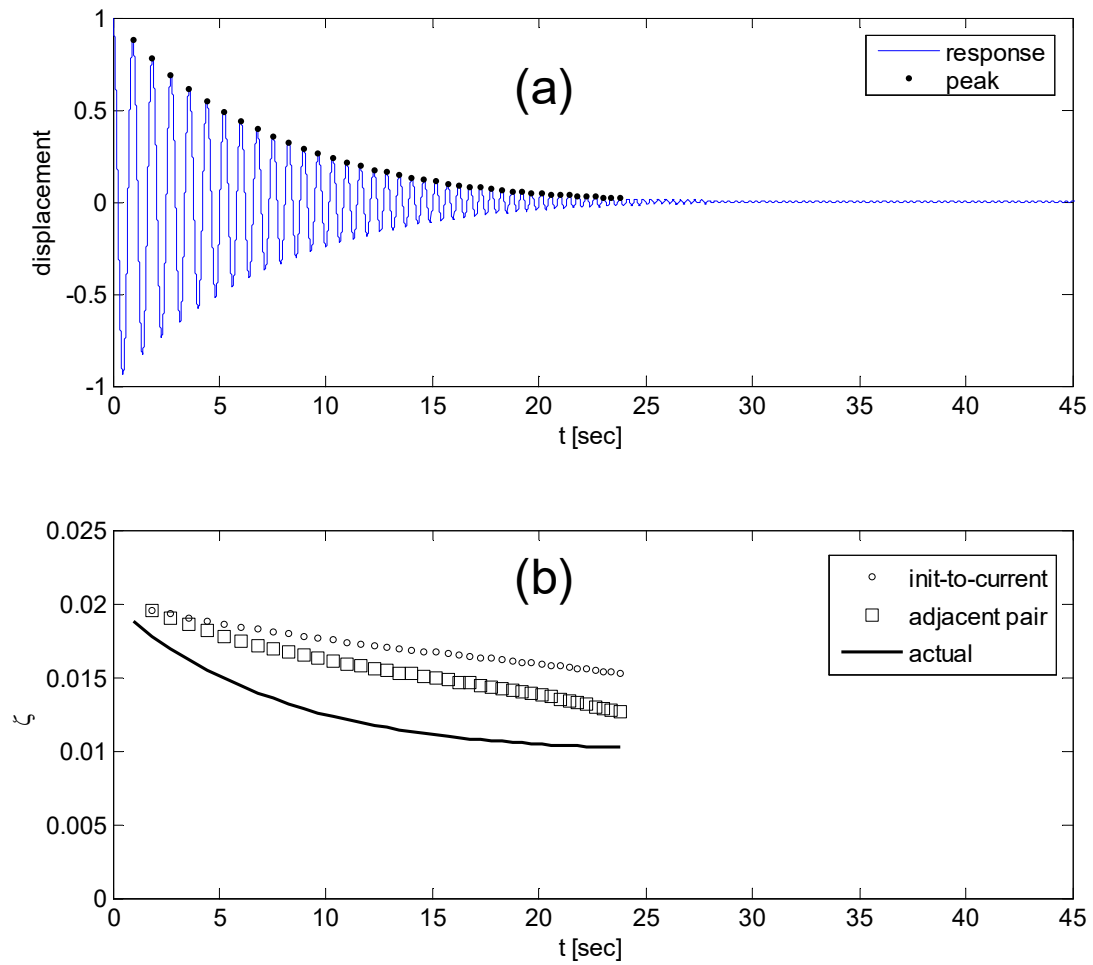


Figure 3-15 Logarithmic decrement damping estimation for a nonlinear elastic system with variable damping.
 (a) Displacement response. (b) Equivalent viscous damping ratio.

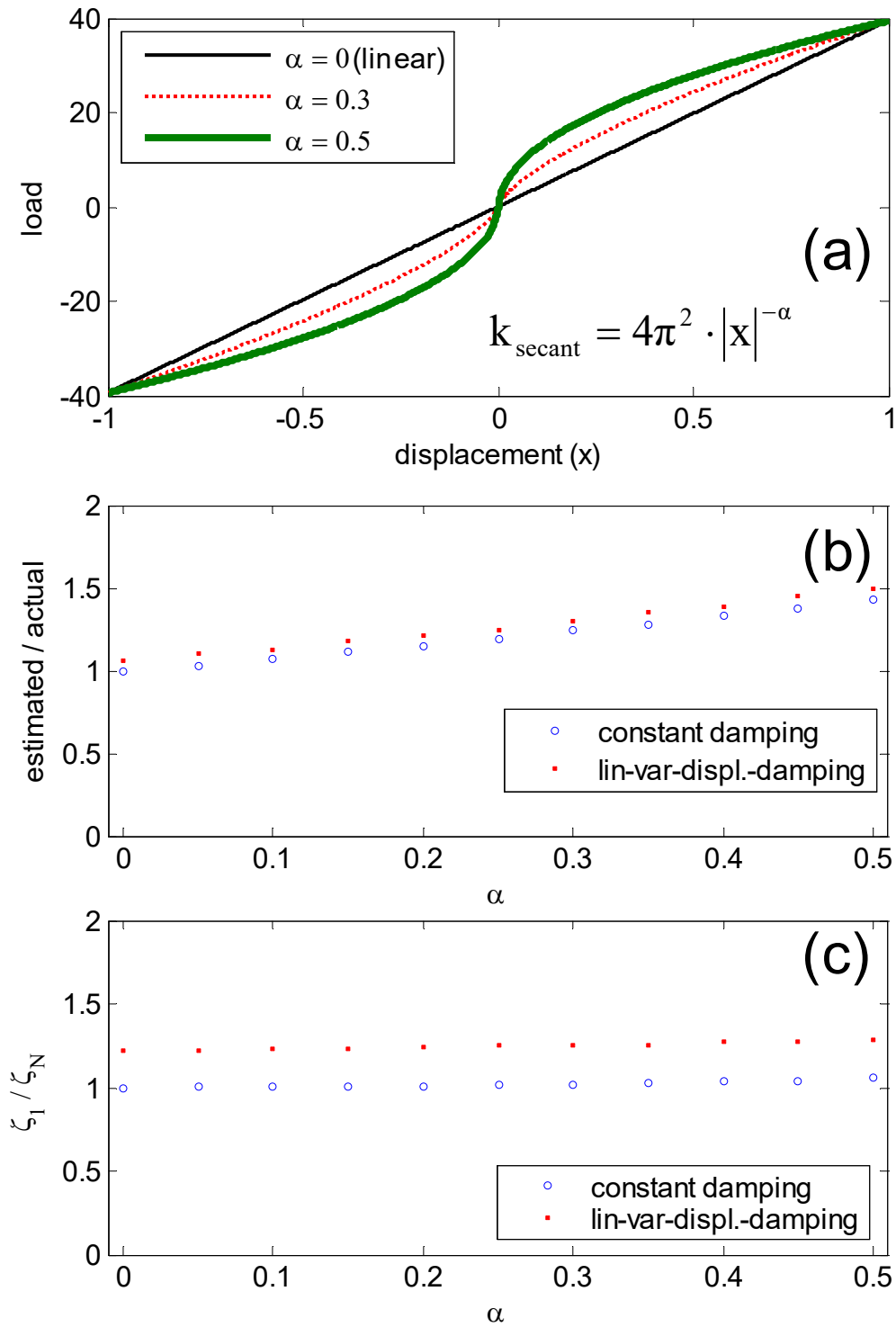


Figure 3-16 Estimation of damping through the logarithmic decrement on different nonlinear-elastic systems. (a) Load vs. displacement curves. (b) Ratio between estimated and actual values. (c) Variability of damping ratio estimates.

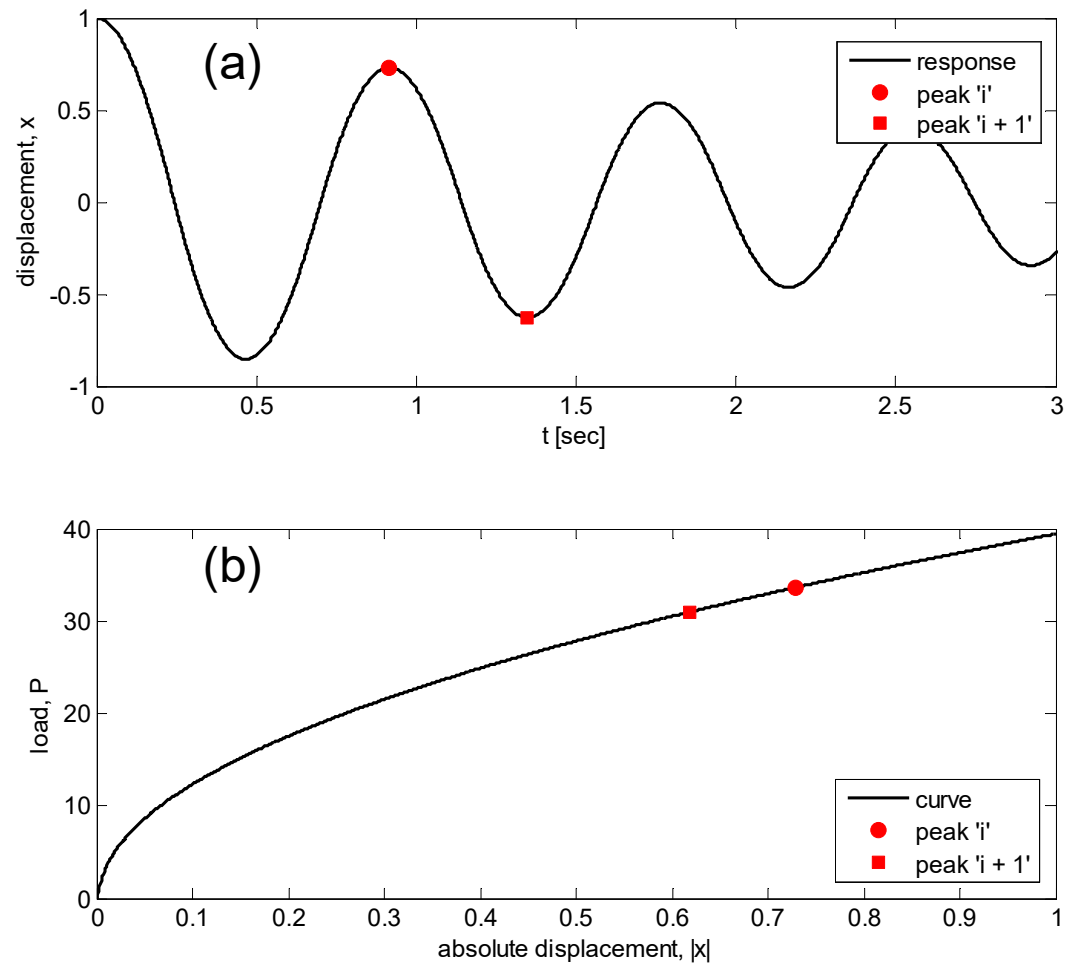


Figure 3-17 Peak points on free vibration response and load-displacement curve. (a) Peaks of the displacement response. (b) Load values corresponding to the displacement peaks.

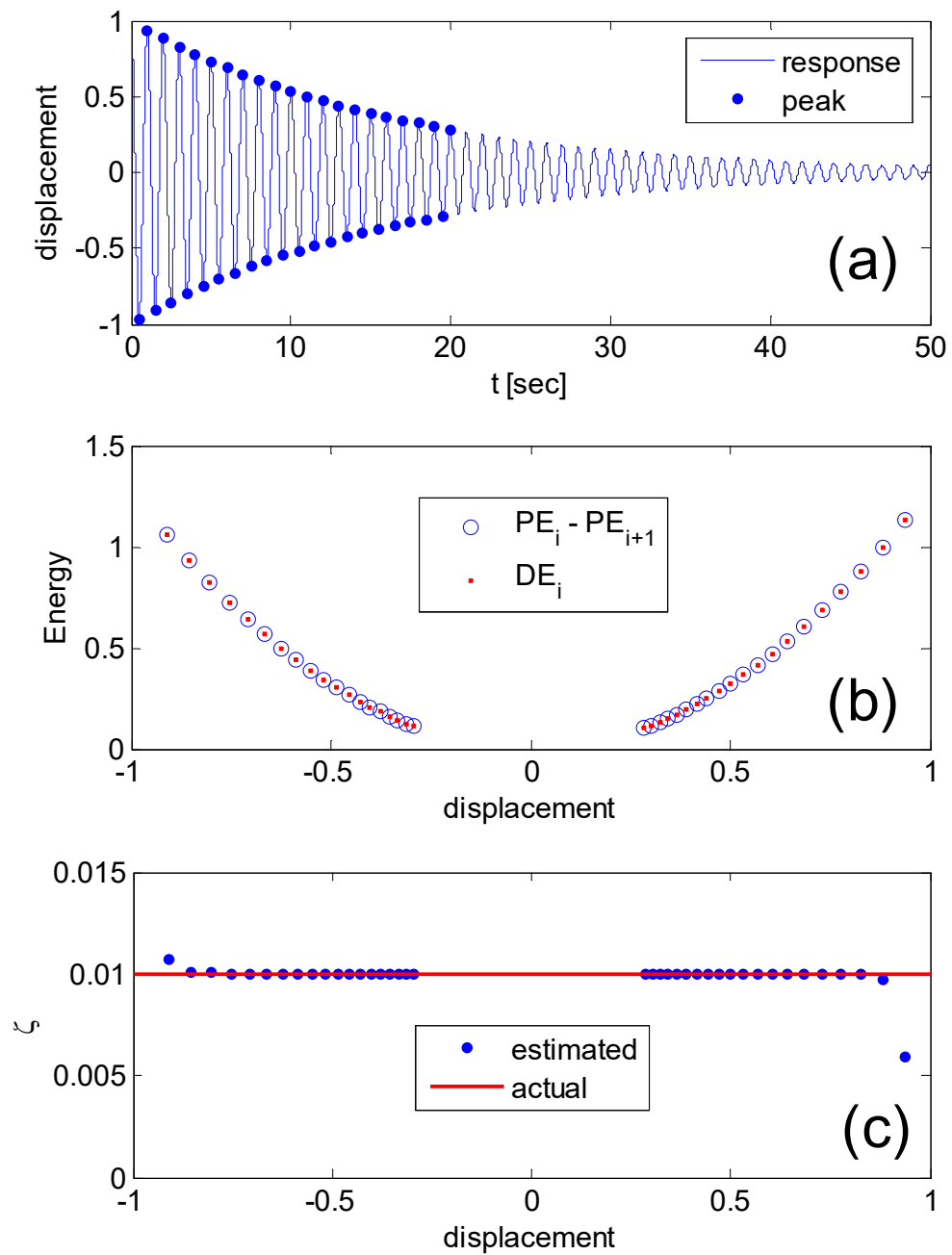


Figure 3-18 Estimation of viscous damping through the energy method in a linear system. (a) Displacement response. (b) Mechanical energy of the system. (c) Equivalent viscous damping ratio.

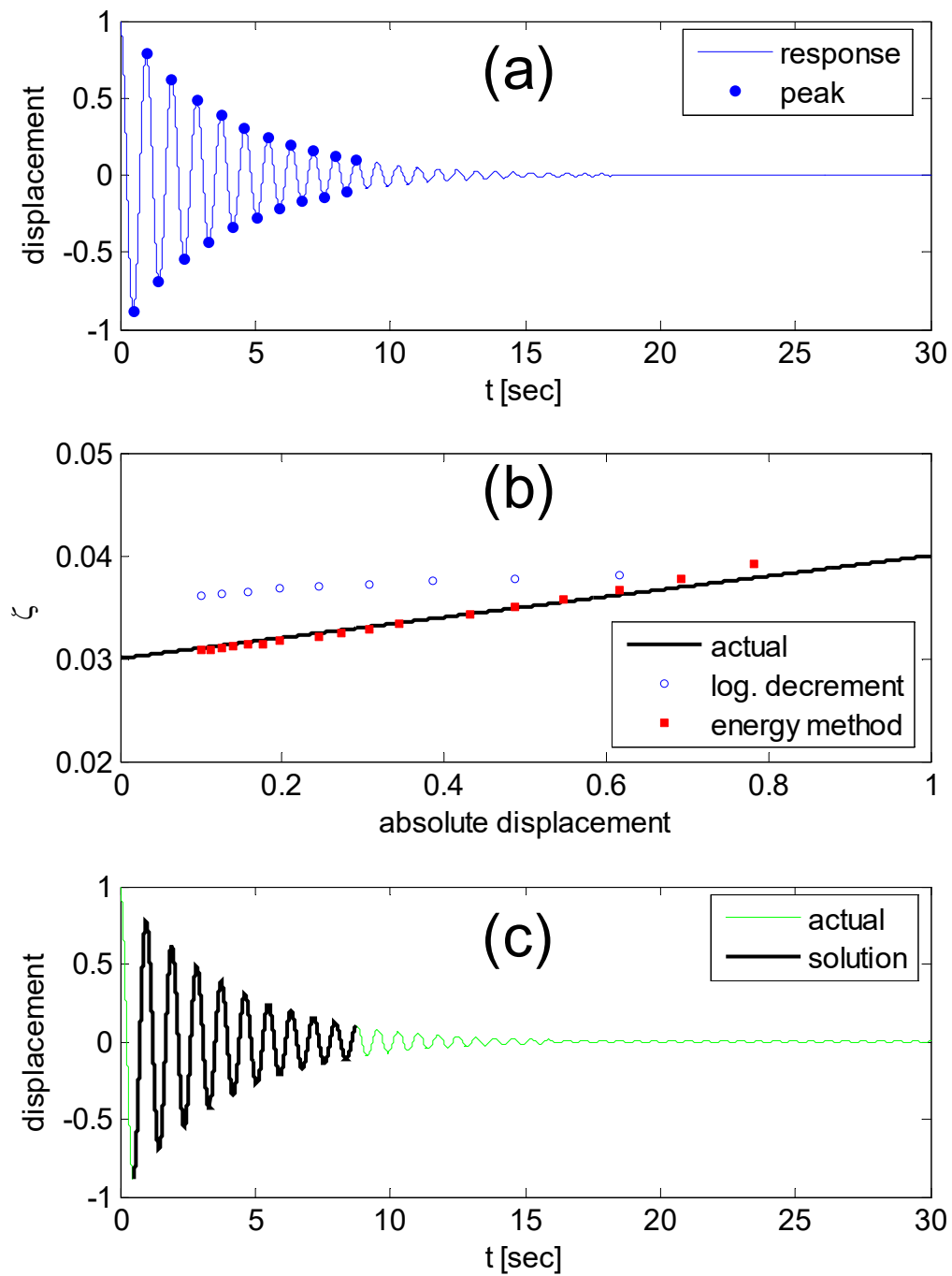


Figure 3-19 Estimation of viscous damping of an elastic system with nonlinear stiffness ($\alpha = 0.20$) and nonlinear damping. (a) Displacement response. (b) Equivalent viscous damping ratio. (c) Comparison of actual and simulated response (solution).

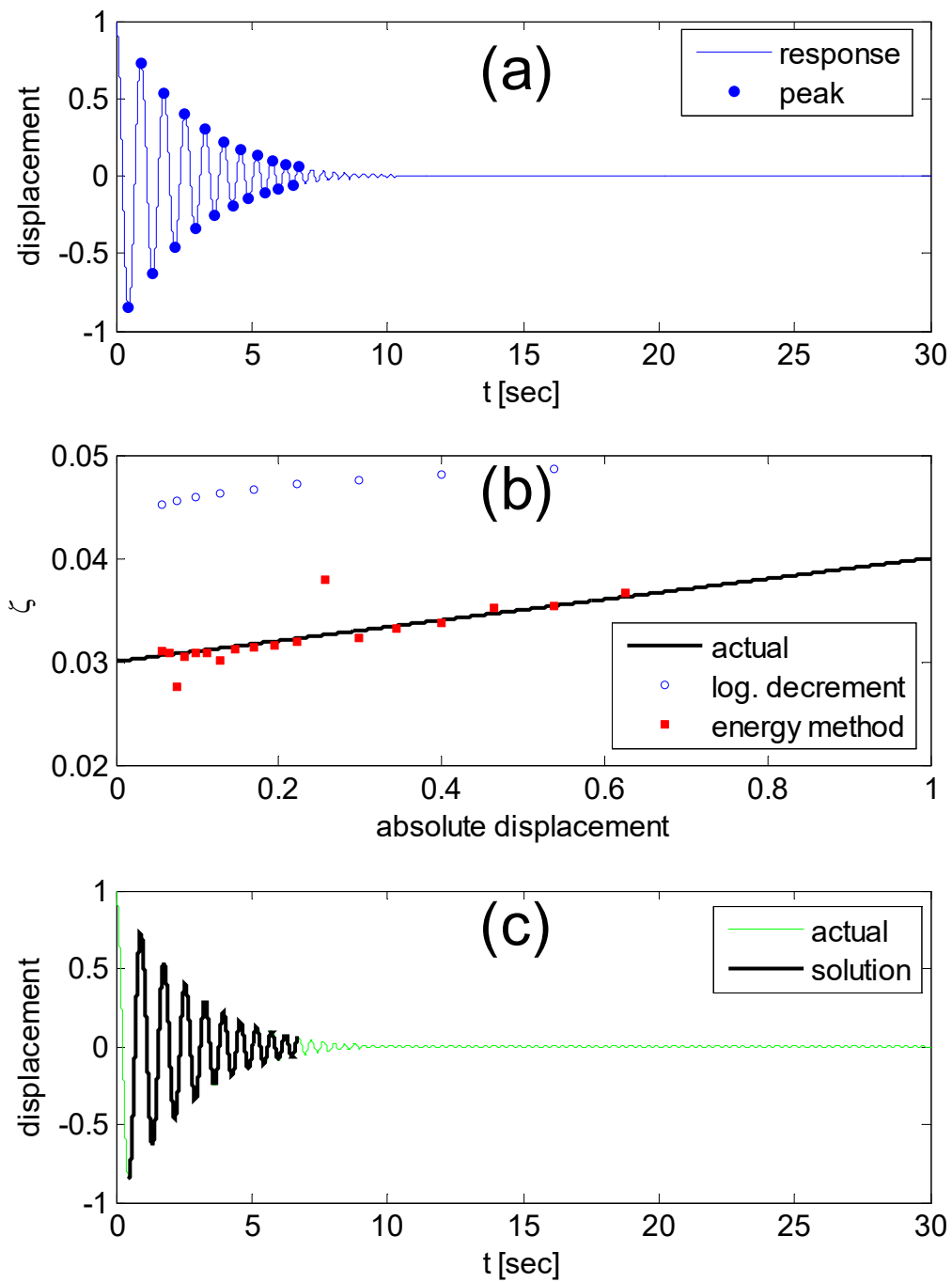


Figure 3-20 Estimation of viscous damping of an elastic system with nonlinear stiffness ($\alpha = 0.50$) and nonlinear damping. (a) Displacement response. (b) Equivalent viscous damping ratio. (c) Comparison of actual and simulated response (solution).

CHAPTER 4. EXPERIMENTAL PROGRAM

4.1. Introduction

Two different types of reinforced concrete structures were tested in the Robert L. and Terry L. Bowen Laboratory for Large-Scale Civil Engineering Research at Purdue University: one full-scale-3-story flat-plate building and six small-scale cantilever beams. Natural frequencies, damping ratios and mode shapes were measured at small-amplitude vibrations after the specimens were subjected to pseudo-static lateral loads resulting in different lateral distortions. The dynamic properties were measured at small displacement levels bounded by a limit corresponding to 0.14% drift ratio in the full-scale structure and 0.03% drift ratio in the small-scale beams. Drift demand in the pseudo-static loading cycles was increased progressively until drift capacity and strength were reached. Frequencies and mode shapes were estimated by means of free, forced and ambient vibration tests. Damping ratios were estimated from free and forced vibrations.

4.2. Full-scale 3-story flat-plate building

The test specimen was a full-scale two-span by one-span three-story reinforced concrete flat-plate structure consisting of six columns spaced at 20 ft in each direction supporting a 7 in. thick slab at each story. Figure 4-1(a), (b) and (c) show a general view of the building, north and east elevation, respectively. Each footing was post-tensioned to the 33 in. thick laboratory strong floor with four 1-3/8 in. Dywidag Threadbars®, each post-tensioned to 100 kips. Superimposed load of 13.3 psf was added at each floor by means forty 55-gallons barrels filled with water.

The structure was pseudo-statically loaded by applying lateral forces at each floor along the longer (north-south) direction of the plan. The applied loads followed a triangular distribution over the height of the specimen (Fick 2008), namely, followed a 3:2:1 ratio at the roof, second floor, and first floor slab levels. Figure 4-2 summarizes the four pseudo-static loading cycles in the form of base shear ratio, i.e. base shear force as a percentage of total weight, versus roof drift ratio. The structure failed by developing a punching-shear failure at roof drift ratio of 3.0% (pulled towards south). The failure is illustrated in Figure 4-3. Dynamic tests were performed at three stages of the pseudo-static test: (a) before cycle 1 (i.e., as-built condition), (b) after cycle 3 (i.e. after 1.5% roof drift ratio), and (c) after cycle 4 (i.e., after 3% roof drift ratio). A sensor data sampling rate of 2,000 samples/sec was used in all tests. Table 4-1 describes the experimental program in the full-scale 3-story flat-plate building. Locations and properties of the sensors used in the dynamic tests are given in Appendix A.

4.2.1. Free vibrations

Free-vibration tests were made by use of a quick-release mechanism similarly to tests done by Arboleda (2006). A pseudo-static load applied at a given floor was rapidly removed to let the system vibrate freely. Figure 4-4 shows the set-up used for the free vibration test when the initial pull was at the roof level. A steel cable was connected to the building and the quick-release mechanism. Tension in the cable was increased to pull the structure and give it an initial displacement. Once the LVDT and dial gauge indicated that the target initial displacement had been reached, the quick-release mechanism was activated to disengage the pull cable and let the building vibrate freely. Maximum initial displacement at the roof level was set to 0.02 in (0.006% overall drift ratio) for as-built condition tests (before cycle 1), and to 0.25 in (0.07% drift ratio) and 0.50 in (0.14% drift ratio) for tests after cycles 3 and 4, respectively. Figure 4-5 (a) shows a typical displacement response from the loading portion (pseudo-static part) to the end of vibrations after releasing the structure (dynamic part). The time axis of the Figure

4-5 (a) was stretched for the dynamic part to show the free vibration response. Figure 4-5 (b) show the load versus displacement curve measured during the pseudo-static part of the tests monitored by the LVDT and the dial gauge.

Figure 4-6 shows Fourier spectra of accelerations in the NS direction recorded at each slab level when the structure was pulled and released at each slab level. Each column of graphs corresponds to a given stage of the pseudo-static test: as-built condition, after 1.5% overall drift ratio and after 3.0% overall drift ratio. Each row of graphs corresponds to tests of pull-release performed at the same slab level. Initial displacement Δ given to the structure at each slab-level before allowing free vibration is indicated on the associated Fourier spectrum. Smaller displacements in the as-built condition were used to preserve the original condition of the structure. The largest contribution of fundamental mode was seen when both pull-release and recording was done at the roof level. Second and third modes were better observed when pull-release was done at first and second slab-level.

Identification of fundamental mode was done using pull-release and response recorded at the roof level. Figure 4-7 (a), (b) and (c) show the free vibration displacement response records for an identical initial displacement of 0.02 in at the roof level in the as-built, after cycle 3 and after cycle 4 conditions, respectively. Figure 4-7 (d), (e) and (f) show the Fourier spectra of the records. The decay envelope of a linear system with constant viscous damping $e^{-\zeta\omega_n t}$ is also shown in the displacement record figures. Table 4-2 summarizes the free vibration tests of the NS direction for the three stages of the pseudo-static test. Data were recorded for 60 seconds in each test. Load 'P' in the cable before releasing the structure is shown. Natural frequency 'f₁' is determined by the largest peak of the Fourier spectrum of the free vibration record. Mode shape of the idealized lumped-mass model of Figure 4-8 (a) is obtained by normalizing amplitude peaks of the Fourier spectrum of records on each floor level as

illustrated Figure 4-8 (b). The damping ratio is estimated by the logarithmic decrement considering either 20 or 50 cycles ('N'). Similar tests carried out in the EW direction at the same stages of the pseudo-static test can be found in Appendix B.

Properties of second translational mode in the NS direction obtained by pull-release of second-slab level are shown in Table 4-3. In the as-built condition, such properties were obtained from tests done before superimposed live load described in section 4.2 was placed. Those tests in the building were performed by Arboleda (2006). Natural frequency ' f_2 ' is determined by the peak in the Fourier spectrum shown in Figure 4-9 (a). Mode shape ϕ of the idealized 3-degree-of-freedom lumped-mass model of Figure 4-8 (a) is determined by normalizing the acceleration records at each slab level, as shown in Figure 4-9 (b). Second mode response was isolated by using a band-pass Butterworth filter with cut-off frequencies of $f_2 \pm 2Hz$. Figure 4-10 (a) shows the total first slab-level acceleration and the isolated second mode response in the as-built condition. The equivalent viscous damping ratio ' ζ_2 ' values reported in Table 4-3 estimated through the logarithmic decrement method on the isolated second-mode response of first slab-level is illustrated in Figure 4-10 (b).

Properties of the third translational mode in the NS direction obtained from pull-release test on the first slab-level are shown in Table 4-4. Similar to the approach described previously for the second mode, natural frequency ' f_3 ' and mode shape ϕ are estimated by the Fourier spectrum and the normalized acceleration amplitudes of Figure 4-11 (a) and (b), respectively. The third mode response was isolated by using a band-pass Butterworth filter with cut-off frequencies of $f_3 \pm 2Hz$. Figure 4-12 (a) shows the total first slab-level acceleration and the isolated third mode response from free vibrations in the as-built condition. The equivalent viscous damping ratio values ' ζ_3 ' of the third translational mode

reported in Table 4-4 are obtained through the logarithmic decrement method applied to the isolated third mode as shown in Figure 4-12 (b).

4.2.2. Forced and ambient vibrations

Forced vibration tests were done by use of a linear actuator placed on the roof level of the building. Figure 4-13 shows the set-up for the forced vibration tests. Harmonic dynamic force was transmitted to the building when the linear actuator reacted against the roof slab-level. An accelerometer placed on top of the cart allowed estimation of the dynamic force. The dynamic force computed using the cart accelerations allowed verification of load measured by the load cell of the actuator. Harmonic excitations with frequencies in the vicinity of the natural frequency of interest were used. Force-input and output of the building were recorded for 200 seconds after steady-state response was observed.

Dynamic parameters were estimated by obtaining Transfer Functions (TFs) via forced vibration tests. A TF is defined as the ratio between the output and the input amplitudes of the structure for a given frequency. When the input is defined as the applied force and the output as the displacement response, magnitude and phase angle of the TF for a linear single degree of freedom system are given by

$$|TF(\Omega)| = \frac{1/k}{\sqrt{[1 - (\Omega/\omega)^2]^2 + [2 \cdot \zeta \cdot \Omega/\omega]^2}} \quad \text{Eq. 4.1}$$

$$\phi(\Omega) = \tan^{-1} \left(\frac{2\zeta(\Omega/\omega)}{1 - (\Omega/\omega)^2} \right) \quad \text{Eq. 4.2}$$

where Ω is the excitation-frequency, ω is the fundamental mode circular frequency of the system, ζ is the equivalent viscous damping ratio and k is the stiffness.

Two different types of TFs were obtained from forced vibrations on the full-scale structure. They are named here as “non-constant” and “constant” displacement response TFs respectively. In the first one, only the frequency of the actuator is changed between the different forced vibration tests. In these “non-constant” tests the displacement response increases as the excitation frequency gets closer to the natural frequency of the structure. This type of testing is common in practice. In the second type of TF, the stroke of the actuator is modified to keep the peak displacement response reached in forced vibration tests identical at all frequencies of the tests. Figure 4-14 shows a non-constant-displacement response TF of the building on the as-built condition, obtained for the fundamental translation mode. Figure 4-14 (b) shows the measured magnitude and phase angle of the TF. Two frequencies, one lower and another higher than the natural frequency of the structure (f_n), are labeled as f_A and f_B in Figure 4-14 (b). Figure 4-14 (a) and (c) illustrate estimation of the TFs for frequencies f_A and f_B respectively.

Figure 4-15 shows a comparison of the two types of TFs obtained in the three stages of the pseudo-static test for the fundamental mode. Actuator force and roof-level displacement were used as input and output respectively. Data points of minimum and maximum roof-level displacement of the non-constant-displacement response TF are indicated. For the constant-displacement response TFs, data from dynamic tests with roof displacement of 0.01 in were used. Best fit obtained using the least-square method on the constant-displacement response TF (given in equation 4.1), by proper selection of ω , k and ζ is also plotted. Constant-displacement response TFs for the fundamental translation mode for after-1.5% and after-3.0% conditions can be seen in Figure

4-16 and Figure 4-17, respectively. Best fit curves based on the least-square method, were obtained by finding the SDOF model with TFs given by equations 4.1 and 4.2, that result in the closest match to the experimental data.

Fundamental translational mode shape of the NS direction was estimated following two procedures. The first approach is based on normalized slab-floor accelerations observed during forced vibrations. Figure 4-18 (a) shows the 3-degree-of-freedom lumped-mass model under forced vibrations, when the excitation frequency Ω is the closest to the fundamental frequency ω_1 . An example of the corresponding input force and output accelerations at each slab-level are depicted in Figure 4-18 (b). The fundamental mode shape ϕ is estimated by normalizing the acceleration magnitudes measured at each level (a_1, a_2, a_3) shown in Figure 4-18 (b). Figure 4-18 (c) shows the mode shape obtained by normalization of acceleration magnitudes with respect to the roof level (a_1). The second approach used to estimate the fundamental mode shape is based on the restoring force measured at each story of the building. Figure 4-19 (a) illustrates the building when the excitation frequency Ω is the closest to the fundamental frequency ω_1 . The dynamic equation of the lumped mass m_3 and the restoring force of the third story are also shown R_3 in Figure 4-19 (a). Restoring force versus inter-story displacement for each story has been plotted in Figure 4-19 (b). Story stiffnesses are computed as the average slope of the restoring force vs. inter-story displacement. The stiffness matrix K built using the measured story stiffnesses illustrated in Figure 4-19 (b) and the corresponding inertia matrix M of the building considering only translational degrees of freedom are shown in Figure 4-19 (c). Fundamental mode shape is obtained by finding the eigenvectors of the problem ($|K - \omega^2 M| = 0$) shown in Figure 4-19 (c). Good agreement between the two approaches was observed.

A summary of dynamic parameters obtained from forced and ambient vibrations for the fundamental mode can be given in Table 4-5. Only constant-displacement response type of TFs was used. The natural frequency ' f_1 ', the equivalent stiffness at the roof level ' K_{roof} ', and the equivalent viscous damping ratio ' ζ_1 ', were estimated from best-fit of equations 4.1 and 4.2 to the experimental data. The natural frequency values from ambient vibrations were identified by the largest peak of the corresponding Fourier spectrum.

Second translational mode of the NS direction was investigated via forced vibrations as illustrated in Figure 4-20. Due to the fact that very low level vibrations were taking place displacements could not be measured accurately. Instead of the displacement data, accelerations recorded by accelerometers were used (accelerometers A1 through A3 in Table A-1). The displacement inferred from the smallest acceleration measured by these accelerometers at 7 Hz is 0.0002 in. A comparison of displacements directly measured with those inferred from accelerations can be seen in Appendix C. Inability to monitor displacements during tests for the investigation of the second translational mode did not allow constant-displacement response test approach. Figure 4-20 (a) illustrates the response of the lumped-mass model of the structure when it is excited by a harmonic force at a frequency Ω close to the second translational mode frequency ω_2 of the structure, i.e., when only minimal participation by other modes exists. The analytical expression for the TF between acceleration (output) at the roof level a_3 and the load (input) at the roof level is also shown in Figure 4-20 (a). Maximum and minimum displacements computed from accelerations are shown in Figure 4-20 (b), (c) and (d), to illustrate the deviation of tests from a constant-displacement response type of TF. Figure 4-20 (b) shows the displacement-response curves measured for the as-built condition before and after superimposed load (as described in section 4.2) was placed in the structure. TFs could not be obtained on the as-built condition because of low signal-to-noise-ratio in the forcing record. Under the assumption of constant excitation load

at all of the frequencies swept, natural frequency was identified by the peak of the curves of Figure 4-20 (b) and damping computed by the half-power method (Clough and Penzien 1975). Displacement-response curves before and after water barrels were installed are shown in Figure 4-20 (b) to illustrate the effect of superimposed load on the second mode. Unfortunately, because the natural frequency of barrels were very close the second natural frequency of the building, barrels acted as tuned-mass dampers and almost eliminated second mode response from forced vibrations, rendering the results unfit for comparison. More illustration of interaction of barrels with second mode of structure is given in Appendix D. Figure 4-20 (c) and (d) show TFs of the building after 1.5% and 3.0% overall drift ratio, respectively. TFs were measured by using force and floor acceleration at the roof level as input and output, respectively. Best-fits using least-square method on TF given in Figure 4-20 (a), by proper selection of ω_2 , k and ζ_2 are plotted in Figure 4-20 (c) and (d).

Third translational mode of the NS direction was investigated via forced vibrations illustrated as Figure 4-21. As experienced during the second mode tests, very low level vibrations took place during these tests and displacements could not be measured accurately. Therefore, accelerations recorded by accelerometers were used in the analyses (accelerometers A1 through A3 in Table A-1). The displacement inferred from the smallest acceleration measured by these accelerometers at 10 Hz is 0.0001 in. The largest acceleration magnitudes were measured at first slab-level a_1 and therefore used as the output response for estimation of the TFs. Figure 4-21 (a) depicts the force input and acceleration response of the building and shows the analytical expression of the TF used for the third mode. Maximum and minimum displacements computed from accelerations are shown in Figure 4-21 (b), (c) and (d) to illustrate the deviation of tests from a constant-displacement response type of TF. Figure 4-21 (b) shows the displacement-response curve estimated from measured accelerations in the as-built condition. Because of low signal-to-noise-ratio in the

forcing record the TF corresponding to the third mode could not be obtained in the as-built condition. The frequency and damping from forced vibrations in the as-built condition were estimated as the peak of the displacement-response curve and through the half-power method applied to the curve of Figure 4-21 (a). TFs measured in the conditions after-1.5% and after-3.0% drift ratios using acceleration (output) at the first slab-level a_1 and input force applied at the roof-level P_o , are shown in Figure 4-21 (c) and (d), respectively. Best-fits using the least-square method on the TF of Figure 4-21 (a), by proper selection of ω_3 , k_{31} and ζ_3 are also plotted in Figure 4-21 (c) and (d).

Summary of properties for second and third mode are given in Table 4-6 and Table 4-7, respectively. The mode shapes were estimated normalizing the acceleration amplitudes similar to the procedure used for the fundamental mode illustrated in Figure 4-18. The values reported for the as-built condition of second mode correspond to those obtained before superimposed load was applied. The results from forced vibration tests in the EW direction after 3.0% overall drift ratio can be found in Appendix B.

4.3. Small-scale cantilever beams

Six 28-in long reinforced concrete cantilever beams with square cross-sections of 4 in were tested to study variation of dynamic properties after pseudo-static loading cycles of increasing amplitudes. Figure 4-22 (a) and (b) show the dimensions and steel reinforcing sizes of the specimens. Figure 4-22 (c) shows a general view of one of the specimens. The foundation-beam was post-tensioned to the 33-in strong-floor of the laboratory to provide the restrain at the base of the beam specimen. Two batches of concrete mix were used to cast three specimens at a time. Mechanical properties measured for the concrete and steel reinforcement are given in Appendix E. An average concrete compressive strength of 3,700 psi was measured from cylinder compression tests made on

the day of the start of the tests. Average modulus of elasticity of concrete measured after 28 days was 3,450 ksi (ASTM_C-469). Average yield stresses of 58 ksi and 42 ksi were obtained for the longitudinal and transverse steel reinforcements, respectively. Deformed bars were used for the longitudinal reinforcement of the beam-specimen. Transverse reinforcement was used to easy construction of specimens and to provide extra shear strength in addition of the capacity of the concrete, to guarantee full development of the flexural capacity of the beam.

The specimens were subjected to pseudo-static loading cycles of different peak displacement levels. Table 4-8 shows the displacement amplitudes used in the tests for each specimen. The maximum displacement corresponded to an overall drift ratio of 13.2% in specimen 6. Dynamic properties were measured in the as-built condition and after pseudo-static cycles of different amplitude displacements. Each pseudo-static test with displacement amplitude equal or smaller than 1.5 in consisted of two cycles of the given amplitude. Pseudo-static tests with displacement amplitudes larger than 1.5 in consisted of one cycle instead of two cycles. In specimens 3 and 4, dynamic properties were also measured after the first pseudo-static cycle of a given amplitude displacement. Figure 4-23 shows the load-displacement history for the pseudo-static loading cycles for each specimen. Flexural strength measured in the specimens was within 10% of the strength estimated as $M_n = A_s * f_y * d$. The evolution of cracks due to pseudo-static loading is illustrated for the specimen with the largest number of pseudo-static cycles (specimen 6) in Figure 4-24. Crack surveys for the other specimens are shown in Appendix E. Table 4-9 describes the general set of dynamic tests performed in the specimens. Only fundamental translational mode properties of the small-scale beams were investigated. Properties and location of sensors used in the small-scale beams can be found in Appendix E.

4.3.1. Free vibrations

Free vibration tests in the small-scale beams were carried out following a procedure similar to the one described in section 4.2.1 for the full-scale structure. Figure 4-25 shows the test set-up. The load was applied at the free end of the beam using a steel cable connected to the specimen and a quick-release mechanism. The specimen was pulled by increasing the tension in the cable until a dial gauge indicated that a target displacement had been reached. The dial gauge was disconnected from the system before activating the quick-release mechanism to allow free vibrations of the specimen. To avoid the large amount of damping introduced by internal friction in its core, no LVDT was attached to the beam during free vibrations. A comparison of free vibration tests with and without an LVDT attached to the specimen can be found in Appendix E.

Figure 4-26 shows the time and frequency response of free vibrations of specimen 5 when released from an initial displacement of 0.006 in measured at the tip of the beam. Responses from the as-built condition to the condition after the last pseudo-static test from Table 4-8 are shown from top to bottom. The natural frequency identified by the largest peak in the Fourier spectrum and the equivalent viscous damping ratio computed through the logarithmic decrements are shown in Figure 4-26. Similar frequency and damping estimations with different initial displacements for specimen 4 are shown in Figure 4-27. The envelope of the response for a linear system with constant viscous damping, which is in the form of $e^{-\zeta\omega_n t}$, is also drawn in Figure 4-27. Frequency f and equivalent viscous damping ratio ζ estimated from free vibrations for each specimen are shown on Table 4-10 through Table 4-15. In the case of Specimen 3 (Table 4-12) and Specimen 4 (Table 4-13), properties are estimated from tests performed after the second pseudo-static cycles at a given amplitude. The load 'P' on the pull cable before releasing the beam and the number of cycles 'N' considered for damping estimation are given in the tables. Fundamental mode shape ϕ is estimated normalizing acceleration amplitudes (a_1, a_2, a_3, a_4) over the

length of the beam as illustrated in Figure 4-28. Table 4-16 shows the mode shape estimated before and after each pseudo-static loading cycle on specimens 4, 5 and 6. The natural frequency and the damping ratio values before and after each pseudo-static loading cycle are plotted in Figure 4-29 and Figure 4-30, respectively.

4.3.2. Forced and ambient vibrations

Forced vibration tests in the small-scale beams were done using an electro-dynamic modal exciter (mass-shaker). Figure 4-31 shows the test set-up. The mass-shaker was anchored to a beam extension using four pre-tensioned steel bolts. The extension beam was connected to a fixed stand anchored to the strong floor of the laboratory. A steel rod connected to the tip of the extension beam was used to prevent vibrations of the beam. The mass-shaker was connected to the specimen through its stinger. Input-force and output-acceleration were measured by use of a piezo-resistive force-sensor and accelerometer. The dial gauge shown in Figure 4-31 was used to monitor vibrations of the mass-shaker. Inertial forces due to vibration of the mass-shaker were estimated as $f_{\text{shaker}} = \text{mass}_{\text{shaker}} * \Omega^2 * d_{\text{gauge}}$ (where Ω is the excitation frequency and d_{gauge} the displacement measured by the dial gauge). Maximum inertial forces due to vibrations of the mass-shaker were found to be less than 3% of the inertial force in the specimen. Only constant-displacement response tests were made and associated TFs were obtained. Because no LVDTs were used to avoid the artificial damping that would have been introduced into the system, displacements could not be monitored directly. However, data from accelerometer were used to estimate the displacement from recorded acceleration as $d = - \text{accel.} / \Omega^2$. Accuracy of such procedure was carefully verified and is illustrated in Appendix E. Verification of sinusoidal input excitation during forced vibration tests is also illustrated in appendix E.

Figure 4-32 shows the TFs measured from specimen 5 before and after each of the pseudo-static loading cycles described in the Table 4-8. Tests carried out at beam tip displacement amplitudes of 0.001 in, 0.003 in, 0.006 in and 0.009 in, are labeled as 1, 2, 3 and 4, respectively. Best-fit analytical models (equation 4.1) were used to estimate the properties of each specimen at each stage. Similar TFs obtained for specimen 6 are shown in Appendix E. Table 4-17 through Table 4-22 present the frequency ' f ' stiffness ' K_{top} ' and the equivalent viscous damping ratio ' ζ ' estimated for specimens 1 through 6, obtained by fitting the analytical curve on the experimental TFs using the least-square error method. (A comparison of estimates of frequency and damping from the best-fit method to those estimates by obtaining the load versus displacement curve from forced vibrations is given in Appendix E). In the case of specimens 3 (Table 4-19) and 4 (Table 4-20), properties are estimated from tests performed after the second pseudo-static cycle at each amplitude. Results from each TF on those tables are identified by their corresponding beam tip displacement amplitude. Mode shapes reported for specimens 4 through 6 were obtained normalizing the acceleration response amplitudes along the beam, similar to the procedure described for the full-scale building in section 4.2.2 and illustrated in Figure 4-18. The fundamental frequency identified by the largest peak of the Fourier spectrum of ambient vibrations was estimated for specimens 5 and 6. The equivalent viscous damping ratio from ambient vibrations in those specimens was estimated through the logarithmic decrement using "free vibration" portions of the records as described in Appendix E. The natural frequency and the damping ratio values before and after each pseudo-static loading cycle are plotted in Figure 4-33 and Figure 4-34, respectively.

Table 4-1 Experimental program on the full-scale structure

TEST		Description
Dynamic	Free vibration	Set of pull-release tests on each slab-level NS* direction. $\Delta_{\max} = 0.02$ in roof. Set of pull-release tests on roof-level EW direction. $\Delta_{\max} = 0.02$ in roof.
	Forced vibration	NS dir. roof-level excitation by use of actuator. $\Delta_{\max} = 0.01$ in roof
	Ambient vibration	1-hour record of ambient vibrations
Pseudo-Static (cycles 1, 2 and 3)		Series of pseudo-static loading cycle. $\Delta_{\max} = 5.4$ in roof (1.5% overall drift)
Dynamic	Free vibration	Set of pull-release tests on each slab-level NS direction. $\Delta_{\max} = 0.25$ in roof. Set of pull-release tests on roof-level EW direction. $\Delta_{\max} = 0.04$ in roof.
	Forced vibration	NS dir. roof-level excitation by use of actuator. $\Delta_{\max} = 0.10$ in roof
	Ambient vibration	1-hour record of ambient vibrations
Pseudo-Static (cycle 4)		Series of pseudo-static loading cycle. $\Delta_{\max} = 10.8$ in roof (3.0% overall drift)
Dynamic	Free vibration	Set of pull-release tests on each slab-level NS direction. $\Delta_{\max} = 0.50$ in roof. Set of pull-release tests on roof-level EW direction. $\Delta_{\max} = 0.08$ in roof.
	Forced vibration	NS dir. roof-level excitation by use of actuator. $\Delta_{\max} = 0.10$ in roof EW dir. roof-level excitation by use of actuator. $\Delta_{\max} = 0.07$ in roof
	Ambient vibration	1-hour record of ambient vibrations

* NS direction is along the longer direction of the plan: right-to-left direction of Figure 4-1 (b)

Table 4-2 Summary of free vibration tests with pull-release at roof level

	Test	Δ_{roof} [in]	P [kip]	f_1 [Hz]	Mode Shape 1			Damping	
					ϕ_1	ϕ_2	ϕ_3	N (cycles)	ζ_1 [%]
As-built	1-1	0.020	1.8	1.92	0.25	0.66	1.00	50	0.8
	1-2	0.021	1.8	1.92	0.25	0.66	1.00	50	0.8
	1-3	0.020	1.8	1.91	0.25	0.65	1.00	50	0.8
After 1.5% overall drift	2-1	0.020	0.8	1.26	0.25	0.66	1.00	20	2.4
	2-2	0.020	0.8	1.26	0.27	0.67	1.00	20	2.4
	2-3	0.021	0.8	1.27	0.27	0.67	1.00	20	2.4
	2-4	0.040	1.6	1.24	0.26	0.66	1.00	20	2.5
	2-5	0.040	1.5	1.23	0.27	0.67	1.00	20	2.3
	2-6	0.040	1.5	1.23	0.27	0.67	1.00	20	2.3
	2-7	0.100	3.3	1.17	0.27	0.67	1.00	20	3.1
	2-8	0.101	3.2	1.16	0.27	0.67	1.00	20	2.9
	2-9	0.101	3.1	1.15	0.27	0.67	1.00	20	2.8
	2-10	0.250	5.9	1.02	0.27	0.67	1.00	20	3.6
	2-11	0.251	5.9	1.01	0.26	0.66	1.00	20	3.5
	2-12	0.250	5.9	1.01	0.27	0.67	1.00	20	3.2
After 3.0% overall drift	3-1	0.016	0.4	0.99	0.30	0.67	1.00	20	2.4
	3-2	0.020	0.4	1.00	0.29	0.69	1.00	20	2.4
	3-3	0.020	0.5	0.99	0.29	0.67	1.00	20	2.4
	3-4	0.041	0.9	0.96	0.29	0.67	1.00	20	2.6
	3-5	0.041	0.9	0.96	0.29	0.67	1.00	20	2.7
	3-6	0.042	0.9	0.96	0.29	0.66	1.00	20	2.6
	3-7	0.101	2.0	0.93	0.29	0.67	1.00	20	3.0
	3-8	0.102	2.0	0.93	0.29	0.67	1.00	20	3.0
	3-9	0.101	2.0	0.93	0.29	0.67	1.00	20	3.0
	3-10	0.252	4.4	0.88	0.29	0.67	1.00	20	3.5
	3-11	0.251	4.3	0.86	0.29	0.66	1.00	20	3.6
	3-12	0.252	4.3	0.86	0.29	0.67	1.00	20	3.8
	3-13	0.401	6.3	0.81	0.29	0.66	1.00	20	3.9
	3-14	0.401	6.1	0.83	0.29	0.66	1.00	20	4.0
	3-15	0.404	6.1	0.81	0.28	0.66	1.00	20	3.6
	3-16	0.492	7.2	0.79	0.28	0.66	1.00	20	4.1
	3-17	0.503	7.2	0.79	0.28	0.66	1.00	20	4.1
	3-18	0.500	7.1	0.79	0.28	0.66	1.00	20	4.2

Table 4-3 Summary of free vibration tests with pull-release at second slab-level

	Test	$\Delta_{2nd\text{-slab}}$ [in]	P [kip]	f_2 [Hz]	Mode Shape 2			Damping	
					ϕ_1	ϕ_2	ϕ_3	N (cycles)	ζ_2 [%]
After 1.5% overall drift	As-built								
	1-1	0.009	1.8	7.6	1.00	0.89	-0.88	50	0.8
	1-2	0.010	1.9	7.6	1.00	0.96	-0.81	50	0.8
	1-3	0.009	1.7	7.6	1.00	0.96	-0.81	50	0.9
	2-1	0.131	7.0	4.3	1.00	0.87	-0.86	20	2.9
	2-2	0.131	7.0	4.3	1.00	0.86	-0.87	20	3.2
	2-3	0.131	7.0	4.3	1.00	0.86	-0.87	20	3.1
	After 3.0% overall drift								
	3-1	0.100	4.4	4.0	1.00	0.75	-0.88	20	2.1
	3-2	0.101	4.3	4.0	1.00	0.74	-0.87	20	2.2
	3-3	0.101	4.3	4.0	1.00	0.74	-0.86	20	2.2
	3-4	0.181	6.5	3.92	1.00	0.74	-0.85	20	2.6
	3-5	0.170	7.0	3.92	1.00	0.75	-0.86	20	2.7
	3-6	0.170	6.8	3.89	1.00	0.74	-0.86	20	2.7

Table 4-4 Summary of free vibration tests with pull-release at first slab-level

	Test	$\Delta_{1st\text{-slab}}$ [in]	P [kip]	f_3 [Hz]	Mode Shape 3			Damping	
					ϕ_1	ϕ_2	ϕ_3	N (cycles)	ζ_3 [%]
After 1.5% overall drift	As-built								
	1-1	0.004	3.1	15.4	1.00	-0.89	0.33	20	1.6
	1-2	0.004	2.9	15.5	1.00	-0.84	0.32	20	1.6
	1-3	0.003	2.4	15.5	1.00	-0.84	0.31	20	1.6
	2-1	0.005	2.9	12.1	1.00	-0.76	0.31	15	2.8
	2-2	0.005	2.3	12.4	1.00	-0.78	0.31	15	3.4
	2-3	0.005	2.3	12.5	1.00	-0.76	0.32	14	3.5
	2-4	0.020	6.1	11.7	1.00	-0.76	0.30	15	3.3
	2-5	0.021	5.9	11.7	1.00	-0.85	0.34	15	3.0
	2-6	0.020	5.9	11.6	1.00	-0.76	0.28	15	3.6
	After 3.0% overall drift								
	3-1	0.005	1.1	12.4	1.00	-0.75	0.46	4	9.0
	3-2	0.005	1.1	12.4	1.00	-0.79	0.31	5	4.5
	3-3	0.005	1.1	12.4	1.00	-0.70	0.31	6	6.7
	3-4	0.020	3.9	11.1	1.00	-0.86	0.29	6	8.4
	3-5	0.020	3.6	11.2	1.00	-0.87	0.32	7	6.6
	3-6	0.020	3.7	11.2	1.00	-0.89	0.31	5	7.1
	3-7	0.030	5.2	11.1	1.00	-0.90	0.31	4	11.7
	3-8	0.030	5.3	11.1	1.00	0.88	0.33	5	9.9
	3-9	0.030	5.1	11.1	1.00	-0.90	0.31	4	11.0

Table 4-5 Forced and ambient vibrations summary for fundamental mode

	Mode Shape 1						
	Δ roof [in]	f_1 [Hz]	K_{roof} [kip/in]	ζ_1 [%]	ϕ_1	ϕ_2	ϕ_3
As-built	< 0.01(ambient)	2.00	-	-	-	-	-
	0.01	1.91	79.8	0.8	0.26	0.66	1.00
After 1.5% overall drift	< 0.01(ambient)	1.48	-	-	-	-	-
	0.01	1.17	37.7	2.1	0.28	0.65	1.00
	0.02	1.15	36.1	2.0	0.27	0.66	1.00
	0.04	1.06	32.3	2.1	0.29	0.67	1.00
	0.10	1.02	30.9	2.0	0.28	0.67	1.00
After 3.0% overall drift	< 0.01(ambient)	1.08	-	-	-	-	-
	0.01	0.96	21.1	2.2	0.28	0.66	1.00
	0.02	0.92	18.2	2.6	0.29	0.66	1.00
	0.04	0.89	18.5	2.5	0.29	0.66	1.00
	0.10	0.84	17.4	2.5	0.29	0.67	1.00

Table 4-6 Forced vibrations summary for second mode

	Δ roof [in]		f_2 [Hz]	K_{roof} [kip/in]	ζ_2 [%]	Mode Shape 2		
	max	min				ϕ_1	ϕ_2	ϕ_3
As-built	0.002	0.0002	7.3	-	0.5	1.00	0.90	-0.93
After 1.5% overall drift	0.001	0.002	4.7	1,100	3.0	1.00	0.84	-0.91
After 3.0% overall drift	0.009	0.001	3.9	900	2.9	1.00	0.75	-0.86

Table 4-7 Forced vibrations summary for third mode

	Δ 1st-floor [in]		f_3 [Hz]	K_{31} [kip/in]	ζ_3 [%]	Mode Shape 3		
	max	min				ϕ_1	ϕ_2	ϕ_3
As-built	0.0005	0.0001	15.5	-	1.1	1.00	-0.89	0.53
After 1.5% overall drift	0.0006	0.0003	11.8	760	1.8	1.00	-1.06	1.35
After 3.0% overall drift	0.0010	0.0005	11.3	290	2.8	1.00	-1.12	1.03

Table 4-8 Pseudo-static tests on small-scale specimens

Cycle-amplitude		Specimen					
beam-top displacement [in]	drift [%]	1	2	3	4	5	6
0.10	0.4	X	X	X	X	X	X
0.30	1.1	X	X	X	X	X	X
0.50	1.8	X	X	X	X	X	X
0.75	2.7	X	X	X	X	X	X
1.00	3.6	X	X	X	X	X	X
1.50	5.4	X	X			X	X
2.00	7.1				X	X	X
2.50	8.9					X	X
2.8 or 3.7	10.0 or 13.2				X	X	X

Table 4-9 Dynamic tests on the small-scale beams

Test	Description
Free vibration	Pull-release tests with top of beam initial displacements of 0.003", 0.006" and 0.009". 10,000sps
Forced vibration	Set of frequency-steps with top of beam displacement amplitudes of 0.001", 0.003", 0.006" and 0.009". Data recorded for 100 seconds at 2,000sps
Ambient vibration	1-hour records of ambient vibration at 200sps

Table 4-10 Specimen 1 free vibration summary

	Test	Δ top [in]	P [lbf]	f [Hz]	Damping	
					N (cycles)	ζ [%]
As-built	1-1	0.003	20	74.3	30	1.5
after 0.10 in	2-1	0.003	11	54.4	30	2.0
	2-2	0.003	11	53.7	30	2.5
	2-3	0.006	17	50.1	30	2.6
	2-4	0.006	18	48.2	30	2.9
	2-5	0.006	18	47.8	30	3.3
after 0.30 in	3-1	0.004	12	42.9	30	2.1
	3-2	0.003	9	41.2	30	2.4
	3-3	0.003	9	42.0	30	2.3
	3-4	0.006	14	41.7	30	2.4
	3-5	0.006	13	41.6	30	2.3
	3-6	0.006	14	41.2	30	2.4
after 0.50 in	4-1	0.003	7	40.0	30	2.1
	4-2	0.003	7	40.1	30	2.1
	4-3	0.003	7	39.9	30	2.2
	4-4	0.006	13	39.9	30	2.1
	4-5	0.006	11	39.9	30	2.2
	4-6	0.006	11	41.1	30	2.2
after 0.75 in	5-1	0.003	4	33.1	30	1.5
	5-2	0.003	5	32.7	30	1.7
	5-3	0.003	4	32.6	30	1.7
	5-4	0.006	7	32.5	30	1.7
	5-5	0.006	7	32.7	30	1.7
	5-6	0.006	8	32.3	30	1.7
after 1.0 in	6-1	0.003	4	32.2	30	1.5
	6-2	0.003	4	32.0	30	1.6
	6-3	0.003	4	32.3	30	1.6
	6-4	0.006	8	32.2	30	1.5
	6-5	0.006	8	32.5	30	1.5
	6-6	0.006	7	32.5	30	1.4
after 1.5 in	7-1	0.003	8	32.6	30	1.2
	7-2	0.003	5	32.8	30	1.0
	7-3	0.003	5	32.8	30	1.1
	7-4	0.006	9	32.5	30	1.3
	7-5	0.006	9	32.4	30	1.4
	7-6	0.006	8	32.5	30	1.2

Table 4-11 Specimen 2 free vibration summary

	Test	Δ top [in]	P [lbf]	f [Hz]	Damping	
					N (cycles)	ζ [%]
as-built condition	1-1	0.003	17	75.6	30	1.1
	1-2	0.003	17	75.5	30	1.1
	1-3	0.003	17	75.6	30	1.1
	1-4	0.006	36	73.7	30	1.4
	1-5	0.006	37	73.8	30	1.4
	1-6	0.006	32	73.7	30	1.4
after 0.10 in	2-1	0.003	9	51.1	30	1.7
	2-2	0.003	8	52.8	30	1.6
	2-3	0.003	8	50.7	30	1.9
	2-4	0.006	15	49.4	30	2.2
	2-5	0.006	15	48.8	30	2.2
	2-6	0.006	16	49.5	30	2.2
after 0.30 in	3-1	0.003	6	45.6	30	1.9
	3-2	0.003	6	46.0	30	1.8
	3-3	0.003	6	46.3	30	1.9
	3-4	0.006	11	42.6	30	2.2
	3-5	0.006	11	42.2	30	2.2
	3-6	0.006	10	43.4	30	2.0
	3-7	0.009	15	41.7	30	2.3
	3-8	0.009	15	41.7	30	2.3
	3-9	0.009	14	40.8	30	2.3
after 0.50 in	4-1	0.003	5	39.3	30	1.6
	4-2	0.003	5	38.8	30	1.6
	4-3	0.003	5	39.1	30	1.7
	4-4	0.006	9	37.8	30	1.7
	4-5	0.006	9	37.1	30	1.8
	4-6	0.006	9	38.0	30	1.7
	4-7	0.009	12	37.0	30	1.9
	4-8	0.009	12	36.7	30	2.0
	4-9	0.009	12	37.0	30	1.9

Table 4-11 (cont'd)

	Test	Δ top [in]	P [lbf]	f [Hz]	Damping	
					N (cycles)	ζ [%]
after 0.75 in	5-1	0.003	4	34.7	30	1.3
	5-2	0.003	4	34.8	30	1.2
	5-3	0.003	4	34.2	30	1.4
	5-4	0.006	8	34.0	30	1.6
	5-5	0.006	7	33.9	30	1.6
	5-6	0.006	8	34.3	30	1.4
	5-7	0.009	10	33.2	30	1.8
	5-8	0.009	10	33.5	30	1.6
	5-9	0.009	10	33.6	30	1.8
after 1.0 in	6-1	0.003	6	35.0	30	1.4
	6-2	0.003	4	34.1	30	1.3
	6-3	0.003	4	34.4	30	1.2
	6-4	0.006	8	34.0	30	1.4
	6-5	0.006	7	33.6	30	1.5
	6-6	0.006	8	33.7	30	1.5
	6-7	0.009	11	33.1	30	1.6
	6-8	0.009	11	33.3	30	1.6
	6-9	0.009	12	33.3	30	1.6
after 1.5 in	7-1	0.003	4	33.1	30	1.1
	7-2	0.003	3	32.8	30	1.3
	7-3	0.003	3	33.5	30	1.1
	7-4	0.006	8	32.6	30	1.5
	7-5	0.006	8	32.9	30	1.3
	7-6	0.006	8	33.1	30	1.2
	7-7	0.009	11	32.6	30	1.5
	7-8	0.009	11	32.6	30	1.5
	7-9	0.009	11	32.5	30	1.5

Table 4-12 Specimen 3 free vibration summary (properties estimated from tests after second cycles)

	Test	Δ top [in]	P [lbf]	f [Hz]	Damping	
					N (cycles)	ζ [%]
as-built condition	1-1	0.003	16	66.7	40	1.3
	1-2	0.003	17	65.8	40	1.3
	1-3	0.003	18	65.7	40	1.3
	1-4	0.006	29	63.5	40	1.5
	1-5	0.006	30	62.8	40	1.6
	1-6	0.006	28	62.6	40	1.6
after 0.10 in	2-1	0.003	8	51.3	40	1.4
	2-2	0.003	9	51.6	40	1.4
	2-3	0.003	9	51.1	40	1.5
	2-4	0.006	15	50.4	40	1.7
	2-5	0.006	15	49.7	40	1.7
	2-6	0.006	15	49.8	40	1.7
	2-7	0.009	20	46.5	40	2.0
	2-8	0.009	19	46.4	40	2.0
	2-9	0.009	20	46.4	40	1.9
after 0.30 in	3-1	0.003	7	45.0	40	1.4
	3-2	0.003	7	44.6	40	1.4
	3-3	0.003	6	44.7	40	1.4
	3-4	0.006	10	42.6	40	1.7
	3-5	0.006	10	42.8	40	1.7
	3-6	0.006	10	42.6	40	1.7
	3-7	0.009	13	40.6	40	1.9
	3-8	0.009	13	40.6	40	1.9
	3-9	0.009	12	40.3	40	1.9
after 0.50 in	4-1	0.003	5	37.3	40	1.6
	4-2	0.003	5	38.1	40	1.4
	4-3	0.003	5	37.4	40	1.5
	4-4	0.006	7	35.4	40	1.6
	4-5	0.006	7	36.4	40	1.6
	4-6	0.006	7	36.2	40	1.6
	4-7	0.009	9	32.9	40	1.7
	4-8	0.009	9	32.9	40	1.8
	4-9	0.009	9	32.7	40	1.8

Table 4-12 (cont'd)

		Damping				
	Test	Δ top [in]	P [lbf]	f [Hz]	N (cycles)	ζ [%]
after 0.75 in	5-1	0.003	4	35.1	40	1.2
	5-2	0.003	4	35.9	40	1.4
	5-3	0.003	5	34.9	40	1.4
	5-4	0.006	7	34.8	40	1.5
	5-5	0.006	7	34.7	40	1.5
	5-6	0.006	7	34.0	40	1.5
	5-7	0.009	9	34.0	40	1.6
	5-8	0.009	9	32.6	40	1.6
	5-9	0.009	9	32.8	40	1.6
after 1.0 in	6-1	0.003	4	35.2	40	1.1
	6-2	0.003	5	35.3	40	1.2
	6-3	0.003	5	34.9	40	1.1
	6-4	0.006	7	34.9	40	1.2
	6-5	0.006	7	34.5	40	1.3
	6-6	0.006	8	34.5	40	1.3
	6-7	0.009	9	34.6	40	1.3
	6-8	0.009	9	34.5	40	1.3
	6-9	0.009	10	34.0	40	1.4

Table 4-13 Specimen 4 free vibration summary (properties estimated from tests after second cycles)

	Test	Δ top [in]	P [lbf]	f [Hz]	Damping	
					N (cycles)	ζ [%]
as-built condition	1-1	0.003	24	82.9	40	0.9
	1-2	0.003	25	82.8	40	0.9
	1-3	0.003	26	82.9	40	0.9
	1-4	0.006	47	81.7	40	1.0
	1-5	0.006	49	81.5	40	1.0
	1-6	0.006	50	81.3	40	1.1
after 0.10 in	2-1	0.003	9	50.6	40	1.5
	2-2	0.003	8	49.9	40	1.5
	2-3	0.003	8	49.9	40	1.5
	2-4	0.006	14	42.2	40	2.2
	2-5	0.006	14	43.5	40	3.0
	2-6	0.006	13	44.2	40	2.5
after 0.30 in	3-1	0.003	5	41.7	40	1.6
	3-2	0.003	5	35.5	40	1.3
	3-3	0.003	6	41.3	40	1.4
	3-4	0.006	8	37.3	40	2.0
	3-5	0.006	9	33.1	40	2.2
	3-6	0.006	9	32.8	40	2.6
	3-7	0.009	12	37.3	40	2.1
	3-8	0.009	12	37.1	40	2.2
	3-9	0.009	12	37.2	40	2.3
after 0.50 in	4-1	0.003	3	37.2	40	1.2
	4-2	0.004	4	35.4	40	1.5
	4-3	0.003	4	35.7	40	1.6
	4-4	0.006	5	34.8	40	1.8
	4-5	0.006	6	32.2	40	1.8
	4-6	0.006	6	34.5	40	1.8
	4-7	0.009	7	32.2	40	1.9
	4-8	0.009	8	32.3	40	2.0
	4-9	0.009	8	32.2	40	2.0

Table 4-13 (cont'd)

	Test	Δ top [in]	P [lbf]	f [Hz]	Damping	
					N (cycles)	ζ [%]
after 0.75 in	5-1	0.003	4	36.7	40	1.4
	5-2	0.003	4	35.4	40	1.2
	5-3	0.003	4	35.5	40	1.3
	5-4	0.006	6	34.9	40	1.5
	5-5	0.006	7	35.7	40	1.4
	5-6	0.006	7	34.8	40	1.5
	5-7	0.009	9	34.6	40	1.6
	5-8	0.009	9	34.2	40	1.6
	5-9	0.009	9	34.1	40	1.7
after 1.0 in	6-1	0.003	4	35.2	40	1.1
	6-2	0.003	4	35.2	40	1.2
	6-3	0.003	4	34.8	40	1.3
	6-4	0.007	5	34.8	40	1.3
	6-5	0.007	7	34.3	40	1.3
	6-6	0.008	7	34.3	40	1.4
	6-7	0.009	9	33.6	40	1.6
	6-8	0.009	9	32.2	40	1.7
	6-9	0.009	9	33.3	40	1.4
after 2.0 in	8-1	0.003	5	34.4	40	0.9
	8-2	0.003	5	34.4	40	0.9
	8-3	0.003	5	34.5	40	0.9
	8-4	0.006	8	34.1	40	1.1
	8-5	0.006	9	34.0	40	1.1
	8-6	0.006	9	34.0	40	1.1
	8-7	0.009	11	33.8	40	1.2
	8-8	0.009	11	33.8	40	1.2
	8-9	0.009	11	33.7	40	1.2
after 2.8 in	10-1	0.003	4	31.3	40	1.0
	10-2	0.003	4	31.3	40	1.0
	10-3	0.003	4	31.1	40	1.1
	10-4	0.006	7	31.6	40	0.6
	10-5	0.006	7	31.1	40	1.1
	10-6	0.006	7	30.9	40	1.1
	10-7	0.009	10	30.5	40	1.3
	10-8	0.009	10	30.6	40	1.2
	10-9	0.009	10	30.4	40	1.3

Table 4-14 Specimen 5 free vibration summary

	Test	Δ top [in]	P [lbf]	f [Hz]	Damping	
					N (cycles)	ζ [%]
as-built condition	1-1	0.003	17	72.3	40	1.1
	1-2	0.003	19	72.0	40	1.2
	1-3	0.003	19	71.8	40	1.3
	1-4	0.006	33	68.4	40	1.6
	1-5	0.006	34	67.7	40	1.6
	1-6	0.006	34	66.3	40	1.7
after 0.10 in	2-1	0.003	10	53.3	40	1.6
	2-2	0.003	10	49.2	40	1.6
	2-3	0.003	10	52.1	40	1.7
	2-4	0.006	17	48.5	40	1.8
	2-5	0.006	16	48.0	40	1.9
	2-6	0.006	17	46.6	40	2.0
after 0.30 in	3-1	0.003	6	45.2	40	1.7
	3-2	0.003	6	44.1	40	1.6
	3-3	0.003	6	44.8	40	1.7
	3-4	0.006	11	39.8	40	1.9
	3-5	0.006	11	39.8	40	2.0
	3-6	0.006	11	39.8	40	1.9
	3-7	0.009	15	37.1	40	2.4
	3-8	0.009	14	37.1	40	2.4
	3-9	0.009	15	37.2	40	2.3
after 0.50 in	4-1	0.003	5	38.7	40	1.5
	4-2	0.003	5	39.1	40	1.5
	4-3	0.003	5	40.1	40	1.4
	4-4	0.006	8	36.9	40	1.6
	4-5	0.006	8	36.4	40	1.7
	4-6	0.006	9	35.4	40	1.7
	4-7	0.009	10	35.5	40	1.8
	4-8	0.009	10	33.7	40	1.9
	4-9	0.009	10	37.0	40	1.8
after 0.75 in	5-1	0.003	5	37.6	40	1.1
	5-2	0.003	5	37.6	40	1.2
	5-3	0.003	5	37.5	40	1.2
	5-4	0.006	9	36.9	40	1.3
	5-5	0.006	9	36.9	40	1.3
	5-6	0.006	9	36.6	40	1.4
	5-7	0.009	11	36.9	40	1.4
	5-8	0.009	12	35.5	40	1.5
	5-9	0.009	11	36.5	40	1.4

Table 4-14 (cont'd)

	Test	Δ top [in]	P [lbf]	f [Hz]	Damping	
					N (cycles)	ζ [%]
after 1.0 in	6-1	0.003	6	36.7	40	1.1
	6-2	0.003	5	36.1	40	1.2
	6-3	0.003	5	36.3	40	1.1
	6-4	0.006	9	35.6	40	1.3
	6-5	0.006	9	35.9	40	1.2
	6-6	0.006	9	35.7	40	1.3
	6-7	0.009	12	35.7	40	1.3
	6-8	0.009	11	35.3	40	1.4
	6-9	0.009	11	35.7	40	1.2
after 1.5 in	7-1	0.003	5	34.4	40	1.2
	7-2	0.003	5	34.5	40	1.1
	7-3	0.003	5	34.4	40	1.1
	7-4	0.006	8	34.2	40	1.2
	7-5	0.006	8	34.2	40	1.2
	7-6	0.006	8	33.9	40	1.3
	7-7	0.009	12	34.1	40	1.3
	7-8	0.009	11	33.6	40	1.3
	7-9	0.009	11	33.3	40	1.4
after 2.0 in	8-1	0.003	5	34.2	40	0.9
	8-2	0.003	5	34.1	40	0.9
	8-3	0.003	6	34.1	40	1.0
	8-4	0.006	9	33.7	40	1.1
	8-5	0.006	9	33.6	40	1.2
	8-6	0.006	9	33.5	40	1.2
	8-7	0.009	12	33.4	40	1.3
	8-8	0.009	12	33.4	40	1.3
	8-9	0.009	12	33.4	40	1.3
after 2.5 in	9-1	0.003	5	32.5	40	1.1
	9-2	0.003	4	32.5	40	1.1
	9-3	0.003	5	32.6	40	1.0
	9-4	0.006	8	32.2	40	1.2
	9-5	0.006	8	32.2	40	1.2
	9-6	0.006	8	32.3	40	1.2
	9-7	0.009	11	31.7	40	1.4
	9-8	0.009	11	31.8	40	1.3
	9-9	0.009	11	31.8	40	1.3
after 2.8 in	10-1	0.003	3	28.4	40	1.0
	10-2	0.003	3	28.3	40	1.0
	10-3	0.003	3	28.2	40	1.1
	10-4	0.006	6	28.0	40	1.1
	10-5	0.006	6	28.1	40	1.2
	10-6	0.006	6	28.0	40	1.1
	10-7	0.009	9	27.7	40	1.2
	10-8	0.009	9	27.7	40	1.3
	10-9	0.009	9	27.7	40	1.3

Table 4-15 Specimen 6 free vibration summary

	Test	Δ top [in]	P [lbf]	f [Hz]	Damping	
					N (cycles)	ζ [%]
as-built	1-1	0.003	26	81.9	40	0.9
	1-2	0.003	26	81.6	40	0.9
	1-3	0.003	28	81.1	40	1.0
after 0.10 in	2-1	0.003	10	50.3	40	1.6
	2-2	0.003	10	50.4	40	1.7
	2-3	0.003	10	49.9	40	1.7
	2-4	0.006	17	47.8	40	1.9
	2-5	0.006	17	47.9	40	1.9
	2-6	0.006	16	47.7	40	1.9
after 0.30 in	3-1	0.003	7	46.6	40	1.6
	3-2	0.003	8	45.6	40	1.6
	3-3	0.003	8	45.7	40	1.5
	3-4	0.006	14	43.1	40	1.8
	3-5	0.006	13	42.2	40	1.8
	3-6	0.006	13	42.3	40	1.8
	3-7	0.009	18	41.5	40	1.9
	3-8	0.009	17	41.1	40	1.9
	3-9	0.009	17	41.0	40	1.9
after 0.50 in	4-1	0.003	6	41.0	40	1.6
	4-2	0.003	6	41.6	40	1.2
	4-3	0.003	6	40.7	40	1.4
	4-4	0.006	11	39.2	40	1.5
	4-5	0.006	11	39.3	40	1.6
	4-6	0.006	11	38.7	40	1.6
	4-7	0.009	15	37.2	40	1.7
	4-8	0.009	14	36.8	40	1.7
	4-9	0.009	14	36.3	40	1.8
after 0.75 in	5-1	0.003	6	37.6	40	1.1
	5-2	0.003	6	37.7	40	1.2
	5-3	0.003	6	37.6	40	1.1
	5-4	0.006	10	36.6	40	1.3
	5-5	0.006	10	36.6	40	1.3
	5-6	0.006	10	36.4	40	1.3
	5-7	0.009	13	36.5	40	1.2
	5-8	0.009	13	35.8	40	1.5
	5-9	0.009	13	35.7	40	1.5

Table 4-15 (cont'd)

		Damping				
	Test	Δ top [in]	P [lbf]	f [Hz]	N (cycles)	ζ [%]
after 1.0 in	6-1	0.003	5	35.6	40	0.9
	6-2	0.003	5	35.5	40	1.0
	6-3	0.003	5	35.8	40	1.0
	6-4	0.006	9	34.7	40	1.2
	6-5	0.006	9	35.0	40	1.1
	6-6	0.006	9	34.8	40	1.2
	6-7	0.009	13	34.7	40	1.3
	6-8	0.009	12	34.5	40	1.3
	6-9	0.009	12	34.4	40	1.3
after 1.5 in	7-1	0.003	5	32.2	40	1.0
	7-2	0.003	4	32.6	40	0.9
	7-3	0.003	5	32.3	40	0.9
	7-4	0.006	8	31.9	40	1.1
	7-5	0.006	8	31.7	40	1.1
	7-6	0.006	8	31.9	40	1.1
	7-7	0.009	12	31.7	40	1.2
	7-8	0.009	12	31.5	40	1.3
	7-9	0.009	11	31.5	40	1.2
after 2.0 in	8-1	0.003	3	31.0	40	1.0
	8-2	0.003	4	31.2	40	0.9
	8-3	0.003	4	31.0	40	1.0
	8-4	0.006	7	30.6	40	1.1
	8-5	0.006	7	30.5	40	1.2
	8-6	0.006	7	30.8	40	1.1
	8-7	0.009	11	30.3	40	1.2
	8-8	0.009	12	30.3	40	1.3
	8-9	0.009	10	30.4	40	1.2
after 2.5 in	9-1	0.003	4	29.7	40	1.0
	9-2	0.003	4	29.8	40	0.9
	9-3	0.003	3	29.8	40	0.9
	9-4	0.006	7	29.4	40	1.1
	9-5	0.006	7	29.3	40	1.1
	9-6	0.006	7	29.5	40	1.1
	9-7	0.009	10	29.1	40	1.2
	9-8	0.009	11	29.0	40	1.2
	9-9	0.009	11	29.1	40	1.2
after 3.7 in	10-1	0.003	4	27.3	40	1.0
	10-2	0.003	4	27.3	40	0.9
	10-3	0.003	4	27.4	40	0.9
	10-4	0.006	6	27.0	40	1.1
	10-5	0.006	6	27.0	40	1.1
	10-6	0.006	6	26.9	40	1.1
	10-7	0.009	9	26.7	40	1.2
	10-8	0.009	9	26.5	40	1.2
	10-9	0.009	9	26.5	40	1.2

Table 4-16 Mode shape from free vibrations for specimens 4, 5 and 6

		initial top of beam displacement [in]											
		0.003				0.006				0.009			
		ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_1	ϕ_2	ϕ_3	ϕ_4
Specimen 4	Pseudo-static cycle												
	as-built	-	0.28	0.70	1.00	-	0.28	0.70	1.00	-	-	-	-
	after 0.10 in	-	0.34	0.75	1.00		0.36	0.74	1.00	-	-	-	-
	after 0.30 in	0.08	0.34	0.75	1.00	0.09	0.36	0.75	1.00	0.09	0.35	0.76	1.00
	after 0.50 in	0.09	0.35	0.75	1.00	0.10	0.36	0.76	1.00	0.10	0.37	0.76	1.00
	after 0.75 in	0.06	0.33	0.75	1.00	0.06	0.33	0.75	1.00	0.07	0.34	0.75	1.00
	after 1.0 in	0.05	0.33	0.74	1.00	0.06	0.34	0.75	1.00	0.06	0.34	0.75	1.00
	after 2.0 in	0.07	0.34	0.75	1.00	0.07	0.35	0.75	1.00	0.07	0.35	0.75	1.00
	after 2.8 in	0.12	0.35	0.75	1.00	0.12	0.35	0.75	1.00	0.13	0.35	0.75	1.00
Specimen 5	as-built	0.06	0.31	0.73	1.00	0.06	0.32	0.74	1.00	-	-	-	-
	after 0.10 in	0.11	0.34	0.76	1.00	0.07	0.35	0.76	1.00	-	-	-	-
	after 0.30 in	0.07	0.33	0.75	1.00	0.07	0.34	0.75	1.00	0.07	0.34	0.75	1.00
	after 0.50 in	0.07	-	0.75	1.00	0.07	-	0.76	1.00	0.07	-	0.75	1.00
	after 0.75 in	0.05	0.31	0.70	1.00	0.05	0.31	0.70	1.00	0.06	0.32	0.70	1.00
	after 1.0 in	0.05	0.32	0.70	1.00	0.05	0.32	0.70	1.00	0.05	0.32	0.70	1.00
	after 1.5 in	0.06	0.34	0.71	1.00	0.06	0.34	0.71	1.00	0.06	0.35	0.72	1.00
	after 2.0 in	0.08	0.35	0.72	1.00	0.08	0.35	0.72	1.00	0.08	0.35	0.72	1.00
	after 2.5 in	0.10	0.36	0.72	1.00	0.10	0.36	0.72	1.00	0.10	0.36	0.72	1.00
	after 2.8 in	0.01	0.35	0.71	1.00	0.01	0.35	0.71	1.00	0.01	0.35	0.71	1.00

Table 4-16 (cont'd)

	Pseudo-static cycle	initial top of beam displacement [in]											
		0.003				0.006				0.009			
		ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_1	ϕ_2	ϕ_3	ϕ_4
Specimen 6	as-built	0.05	0.26	0.65	1.00	-	-	-	-	-	-	-	-
	after 0.10in	0.08	0.32	0.69	1.00	0.08	0.33	0.69	1.00	-	-	-	-
	after 0.30in	0.08	0.31	0.68	1.00	0.08	0.32	0.68	1.00	0.08	0.32	0.69	1.00
	after 0.50in	0.07	0.31	0.68	1.00	0.08	0.32	0.69	1.00	0.08	0.33	0.69	1.00
	after 0.75in	0.07	0.31	0.69	1.00	0.07	0.32	0.69	1.00	0.07	0.32	0.69	1.00
	after 1.0in	0.07	0.32	0.69	1.00	0.07	0.32	0.69	1.00	0.07	0.33	0.69	1.00
	after 1.5in	0.10	0.36	0.71	1.00	0.10	0.36	0.71	1.00	0.10	0.36	0.71	1.00
	after 2.0in	0.12	0.38	0.72	1.00	0.13	0.39	0.72	1.00	0.13	0.39	0.72	1.00
	after 2.5in	0.15	0.40	0.73	1.00	0.15	0.41	0.73	1.00	0.15	0.41	0.73	1.00
	after 3.7in	0.13	0.37	0.71	1.00	0.14	0.37	0.71	1.00	0.14	0.37	0.72	1.00

Table 4-17 Specimen 1 forced vibration summary

	Δ top [in]	f [Hz]	K top [lbf/in]	ζ [%]
as-built	0.003	77.8	7,938	0.8
	0.006	75.1	7,554	1.0
	0.009	72.9	7,258	1.1
after 0.10 in	0.003	44.5	2,733	2.5
	0.006	41.1	2,358	3.1
after 0.30 in	0.003	39.3	2,168	2.3
	0.006	37.0	1,869	2.5
after 0.50 in	0.003	35.0	1,711	2.2
	0.006	33.2	1,597	3.0
after 0.75 in	0.003	32.5	1,563	1.5
	0.006	30.6	1,348	2.9
after 1.0 in	0.003	31.4	1,609	1.1
	0.006	30.4	1,359	2.2
after 1.5 in	0.003	30.1	1,227	1.6
	0.006	27.8	1,153	4.4

Table 4-18 Specimen 2 forced vibration summary

	Δ top [in]	f [Hz]	K top [lbf/in]	ζ [%]
as-built	0.001	73.4	6,697	0.9
	0.003	71.7	6,608	1.2
after 0.10 in	0.003	46.2	2,898	2.9
	0.006	42.6	2,522	3.7
after 0.30 in	0.001	42.3	2,340	2.1
	0.003	39.4	2,062	3.1
	0.006	36.6	1,841	3.6
after 0.50 in	0.001	37.9	2,043	1.4
	0.003	35.4	1,801	2.1
	0.006	32.5	1,481	3.7
after 0.75 in	0.001	34.4	1,738	0.9
	0.003	32.5	1,549	1.8
	0.006	30.1	1,296	4.1
	0.008	28.5	1,193	5.1
after 1.0 in	0.001	33.6	1,633	0.9
	0.003	32.3	1,505	1.5
	0.006	30.0	1,311	3.6
	0.008	28.5	1,234	5.5
after 1.5 in	0.001	32.8	1,634	0.7
	0.003	31.6	1,490	1.3
	0.006	29.3	1,206	3.2
	0.008	28.5	1,193	3.8

Table 4-19 Specimen 3 forced vibration summary (properties estimated from tests after second cycles)

	Δ top [in]	f [Hz]	K top [lbf/in]	ζ [%]
as-built	0.001	64.2	5,924	1.1
	0.003	59.2	5,101	1.6
after 0.10 in	0.001	46.6	3,097	3.8
	0.003	42.5	2,869	4.9
	0.006	38.0	2,513	6.1
	0.009	34.8	1,602	10.6
after 0.30 in	0.001	41.8	2,587	2.2
	0.003	36.6	1,929	5.3
	0.006	31.5	1,331	11.2
	0.009	27.1	1,123	13.7
after 0.50 in	0.001	36.2	2,200	1.7
	0.003	29.8	1,441	8.0
	0.006	25.6	1,031	12.7
	0.009	22.5	792	15.2
after 0.75 in	0.001	34.2	1,662	1.9
	0.003	29.1	1,400	6.8
	0.006	23.7	988	13.8
	0.009	21.0	898	12.6
after 1.0 in	0.001	34.0	1,740	1.2
	0.003	29.4	1,313	7.6
	0.006	25.7	1,088	11.4
	0.009	22.3	821	13.7

Table 4-20 Specimen 4 forced vibration summary (properties estimated from tests after second cycles)

	Δ top [in]	f [Hz]	K top [lbf/in]	ζ [%]	Mode Shape 1			
					ϕ_1	ϕ_2	ϕ_3	ϕ_4
as-built	0.001	81.9	9,395	0.7	-	0.28	0.70	1.00
	0.003	80.3	8,318	0.9	-	0.28	0.70	1.00
after 0.10 in	0.001	46.7	4,654	2.9	-	0.34	0.75	1.00
	0.003	42.8	3,819	5.6	-	0.36	0.75	1.00
	0.006	40.1	3,463	7.1	-	0.37	0.76	1.00
after 0.30 in	0.001	38.2	2,255	3.3	0.08	0.34	0.75	1.00
	0.003	34.3	2,198	4.7	0.09	0.36	0.76	1.00
	0.006	30.9	1,780	7.5	0.10	0.37	0.76	1.00
	0.009	27.9	1,073	14.1	0.10	0.38	0.76	1.00
after 0.50 in	0.001	34.3	1,736	3.1	0.09	0.35	0.76	1.00
	0.003	29.8	1,324	11.0	0.11	0.38	0.76	1.00
	0.006	25.7	1,081	15.7	0.11	0.39	0.77	1.00
	0.009	22.4	704	22.7	0.12	0.40	0.77	1.00
after 0.75 in	0.001	34.6	1,885	1.3	0.06	0.33	0.75	1.00
	0.003	30.8	1,363	5.8	0.07	0.35	0.75	1.00
	0.006	26.7	1,202	9.5	0.08	0.37	0.76	1.00
	0.009	23.9	943	13.9	0.09	0.38	0.76	1.00
after 1.0 in	0.001	33.5	2,065	1.5	0.06	0.33	0.75	1.00
	0.003	30.8	1,388	4.4	0.06	0.35	0.75	1.00
	0.006	26.9	1,148	8.1	0.07	0.37	0.76	1.00
	0.009	24.4	958	10.6	0.08	0.38	0.77	1.00
after 2.0 in	0.001	34.2	1,807	0.9	0.07	0.34	0.75	1.00
	0.003	32.5	1,689	1.8	0.07	0.35	0.75	1.00
	0.006	29.8	1,383	5.0	0.08	0.36	0.76	1.00
	0.009	27.0	1,220	7.1	0.08	0.37	0.77	1.00
after 2.8 in	0.001	30.2	1,365	0.9	0.13	0.35	0.75	1.00
	0.003	28.7	1,304	1.7	0.14	0.36	0.76	1.00
	0.006	26.9	1,071	4.2	0.15	0.37	0.77	1.00
	0.009	24.7	859	6.1	0.15	0.38	0.77	1.00

Values of mode shape ϕ_1 not reported are due to malfunctioning of the sensor

Table 4-21 Specimen 5 forced vibration summary

	Δ top [in]	f [Hz]	K top [lbf/in]	ζ [%]	Mode Shape 1			
					ϕ_1	ϕ_2	ϕ_3	ϕ_4
as-built	<0.001 (ambient)	75.5	-	0.6	-	-	-	-
	0.001	68.9	6,824	1.2	0.06	0.31	0.73	1.00
	0.003	62.8	5,583	1.6	0.07	0.33	0.74	1.00
after 0.10 in	<0.001 (ambient)	54.5	-	0.8	-	-	-	-
	0.001	47.9	3,612	2.4	0.07	0.35	0.76	1.00
	0.003	43.6	2,873	3.4	0.08	0.36	0.76	1.00
	0.006	40.4	2,142	5.3	0.08	0.36	0.76	1.00
after 0.30 in	<0.001 (ambient)	45.4	-	0.9	-	-	-	-
	0.001	40.8	2,150	3.9	0.06	0.31	0.70	1.00
	0.003	36.9	1,886	4.9	0.07	0.32	0.70	1.00
	0.006	33.3	1,565	6.7	0.07	0.33	0.70	1.00
	0.009	30.9	1,575	6.4	0.07	0.33	0.70	1.00
after 0.50 in	<0.001 (ambient)	40.2	-	0.8	-	-	-	-
	0.001	37.7	2,074	2.3	0.06	0.31	0.70	1.00
	0.003	33.1	1,551	6.3	0.07	0.33	0.70	1.00
	0.006	29.9	1,436	8.2	0.07	0.34	0.70	1.00
	0.009	27.3	1,476	8.2	0.08	0.35	0.70	1.00
after 0.75 in	<0.001 (ambient)	38.4	-	0.6	-	-	-	-
	0.001	36.3	2,000	1.4	0.05	0.31	0.70	1.00
	0.003	32.7	1,562	4.8	0.05	0.32	0.70	1.00
	0.006	29.4	1,266	8.5	0.06	0.33	0.70	1.00
	0.009	27.0	1,111	10.9	0.06	0.34	0.71	1.00
after 1.0 in	<0.001 (ambient)	36.9	-	0.5	-	-	-	-
	0.001	35.8	1,946	1.1	0.05	0.32	0.70	1.00
	0.003	32.9	1,628	3.6	0.05	0.32	0.71	1.00
	0.006	29.4	1,190	8.5	0.06	0.34	0.71	1.00
	0.009	26.8	1,177	10.2	0.07	0.35	0.71	1.00
after 1.5 in	<0.001 (ambient)	35.3	-	0.5	-	-	-	-
	0.001	34.4	1,916	0.9	0.06	0.34	0.71	1.00
	0.003	32.0	1,567	3.4	0.06	0.34	0.72	1.00
	0.006	28.8	1,375	6.2	0.07	0.36	0.72	1.00
	0.009	26.6	1,222	8.4	0.08	0.36	0.72	1.00
after 2.0 in	<0.001 (ambient)	34.6	-	0.4	-	-	-	-
	0.001	33.9	1,889	0.7	0.08	0.35	0.72	1.00
	0.003	31.8	1,654	2.7	0.08	0.35	0.72	1.00
	0.006	28.7	1,380	5.2	0.09	0.36	0.73	1.00
	0.009	26.9	1,224	7.6	0.09	0.37	0.73	1.00
after 2.5 in	<0.001 (ambient)	33.2	-	0.5	-	-	-	-
	0.001	32.6	1,674	0.7	0.10	0.35	0.72	1.00
	0.003	30.7	1,523	2.6	0.10	0.36	0.73	1.00
	0.006	28.4	1,323	4.4	0.11	0.37	0.73	1.00
	0.009	26.5	1,253	6.7	0.11	0.38	0.73	1.00
after 2.8 in	<0.001 (ambient)	28.7	-	0.6	-	-	-	-
	0.001	28.4	1,118	0.8	0.01	0.34	0.71	1.00
	0.003	27.3	1,146	1.5	0.01	0.35	0.71	1.00
	0.006	25.3	1,076	2.9	0.01	0.36	0.72	1.00
	0.009	23.7	1,011	4.5	0.09	0.37	0.72	1.00

Table 4-22 Specimen 6 forced vibration summary

	Δ top [in]	f [Hz]	K top [lbf/in]	ζ [%]	Mode Shape 1			
					ϕ_1	ϕ_2	ϕ_3	ϕ_4
as-built	<0.001 (ambient)	84.3	-	0.4	-	-	-	-
	0.001	80.5	8,756	0.7	0.05	0.26	0.65	1.00
	0.003	76.5	9,079	0.8	0.06	0.28	0.66	1.00
after 0.10 in	<0.001 (ambient)	52.7	-	1.1	-	-	-	-
	0.001	48.8	3,317	1.8	0.08	0.32	0.69	1.00
	0.003	45.6	3,160	1.7	0.08	0.33	0.69	1.00
	0.006	43.4	3,044	1.8	0.09	0.33	0.69	1.00
after 0.30 in	<0.001 (ambient)	46.7	-	1.0	-	-	-	-
	0.001	43.0	2,660	1.7	0.08	0.32	0.68	1.00
	0.003	39.5	2,216	2.8	0.08	0.32	0.69	1.00
	0.006	36.7	2,008	3.1	0.09	0.33	0.69	1.00
	0.009	34.4	1,804	3.8	0.09	0.33	0.69	1.00
after 0.50 in	<0.001 (ambient)	41.7	-	0.9	-	-	-	-
	0.001	38.3	2,133	1.5	0.08	0.32	0.69	1.00
	0.003	35.2	1,920	2.9	0.09	0.33	0.69	1.00
	0.006	32.2	1,707	4.2	0.09	0.34	0.70	1.00
	0.009	29.3	1,488	6.5	0.10	0.35	0.71	1.00
after 0.75 in	<0.001 (ambient)	37.9	-	0.6	-	-	-	-
	0.001	36.5	1,911	1.1	0.07	0.32	0.69	1.00
	0.003	34.2	1,707	2.0	0.07	0.32	0.69	1.00
	0.006	29.6	1,302	5.4	0.08	0.34	0.70	1.00
	0.009	27.3	1,225	6.2	0.09	0.35	0.70	1.00
after 1.0 in	<0.001 (ambient)	35.9	-	0.6	-	-	-	-
	0.001	35.2	1,891	0.8	0.07	0.32	0.69	1.00
	0.003	33.6	1,674	1.4	0.07	0.33	0.69	1.00
	0.006	30.6	1,503	3.7	0.08	0.34	0.70	1.00
	0.009	27.3	1,161	6.1	0.09	0.35	0.70	1.00
after 1.5 in	<0.001 (ambient)	32.5	-	0.5	-	-	-	-
	0.001	32.0	1,526	0.8	0.10	0.36	0.71	1.00
	0.003	30.9	1,469	1.2	0.10	0.36	0.71	1.00
	0.006	29.0	1,350	2.6	0.11	0.37	0.71	1.00
	0.009	26.6	1,173	4.9	0.11	0.38	0.72	1.00
after 2.0 in	<0.001 (ambient)	31.7	-	0.5	-	-	-	-
	0.001	30.9	1,559	0.7	0.12	0.38	0.72	1.00
	0.003	29.7	1,490	1.2	0.13	0.39	0.72	1.00
	0.006	28.4	1,347	2.1	0.14	0.40	0.73	1.00
	0.009	26.6	1,357	3.4	0.15	0.41	0.73	1.00
after 2.5 in	<0.001 (ambient)	30.4	-	0.5	-	-	-	-
	0.001	29.6	1,424	0.7	0.14	0.40	0.73	1.00
	0.003	28.6	1,419	1.0	0.15	0.42	0.73	1.00
	0.006	27.4	1,281	1.6	0.16	0.42	0.74	1.00
	0.009	26.0	1,239	2.8	0.17	0.43	0.74	1.00
after 3.7 in	<0.001 (ambient)	27.9	-	0.6	-	-	-	-
	0.001	27.3	1,076	0.8	0.13	0.36	0.71	1.00
	0.003	26.7	1,070	1.0	0.14	0.37	0.72	1.00
	0.006	25.6	1,087	1.2	0.15	0.38	0.72	1.00
	0.009	24.5	920	2.4	0.16	0.39	0.73	1.00

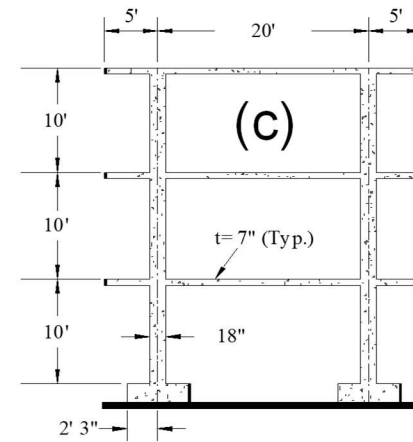
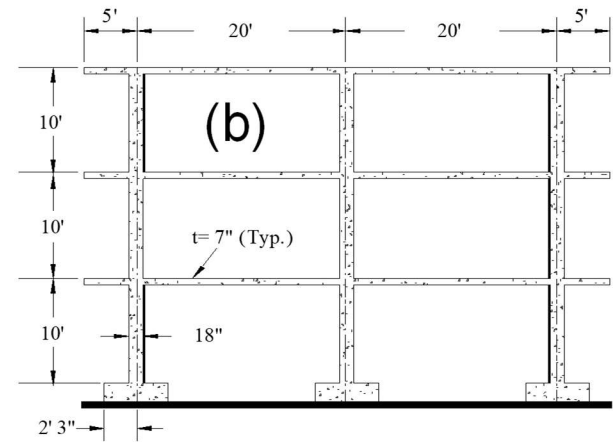


Figure 4-1 Full-scale 3-story flat-plate reinforced concrete building. (a) View from north-east. (b) North elevation. (c) East elevation.

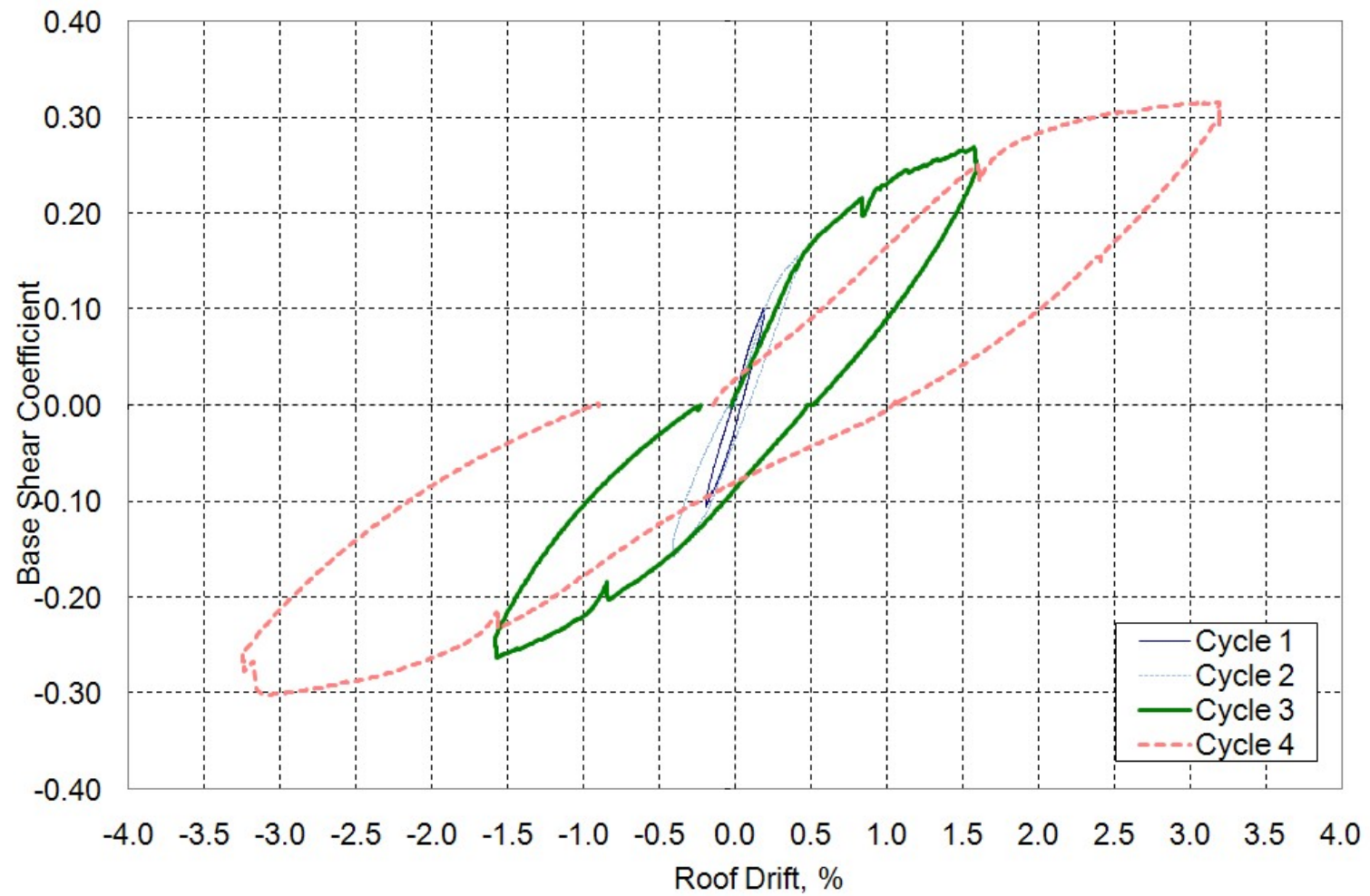


Figure 4-2 Pseudo-static loading cycles in the full-scale 3-story flat-plate building (Fick 2008).



Figure 4-3 Punching-shear failure in the full-scale 3-story flat-plate building.

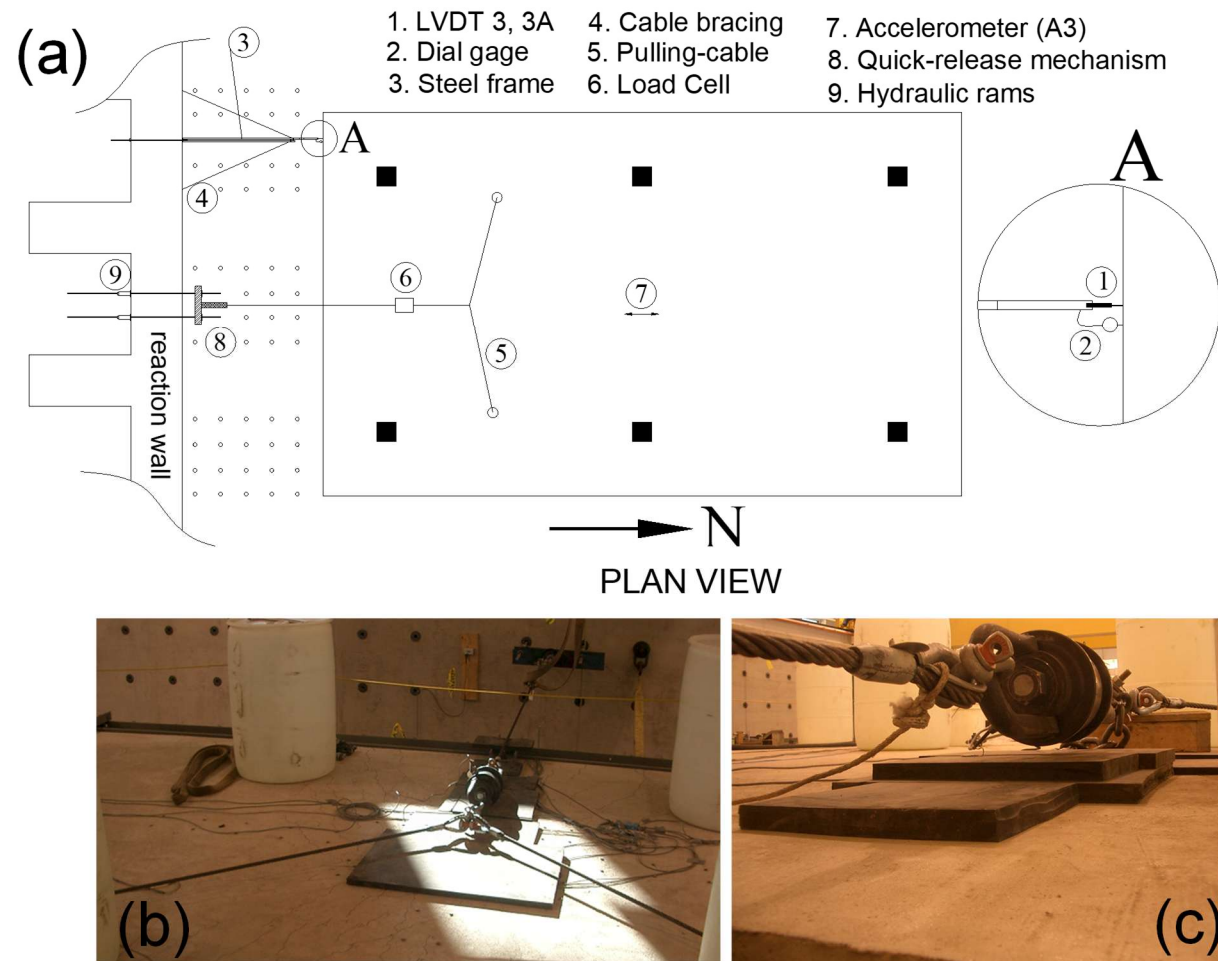


Figure 4-4 Free vibration test set-up of the full-scale 3-story flat-plate building. (a) Plan view. (b) View towards south direction. (c) Close-up of load cell.

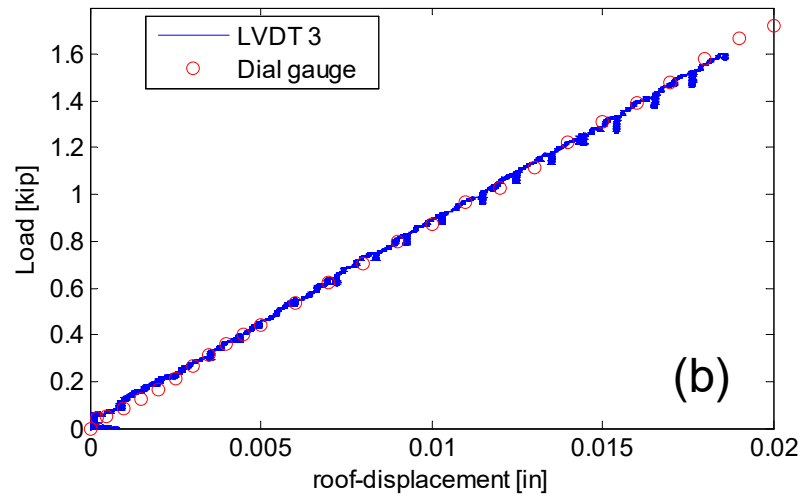
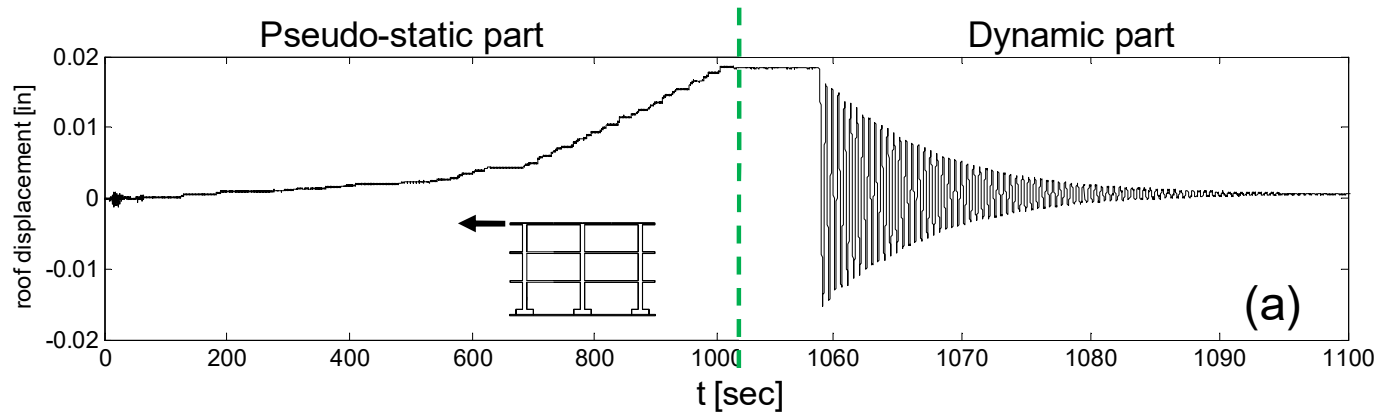


Figure 4-5 Free vibration test of the full-scale 3-story flat-plate building. (a) Displacement record.
(b) Load vs. displacement curve from pseudo-static part of test.

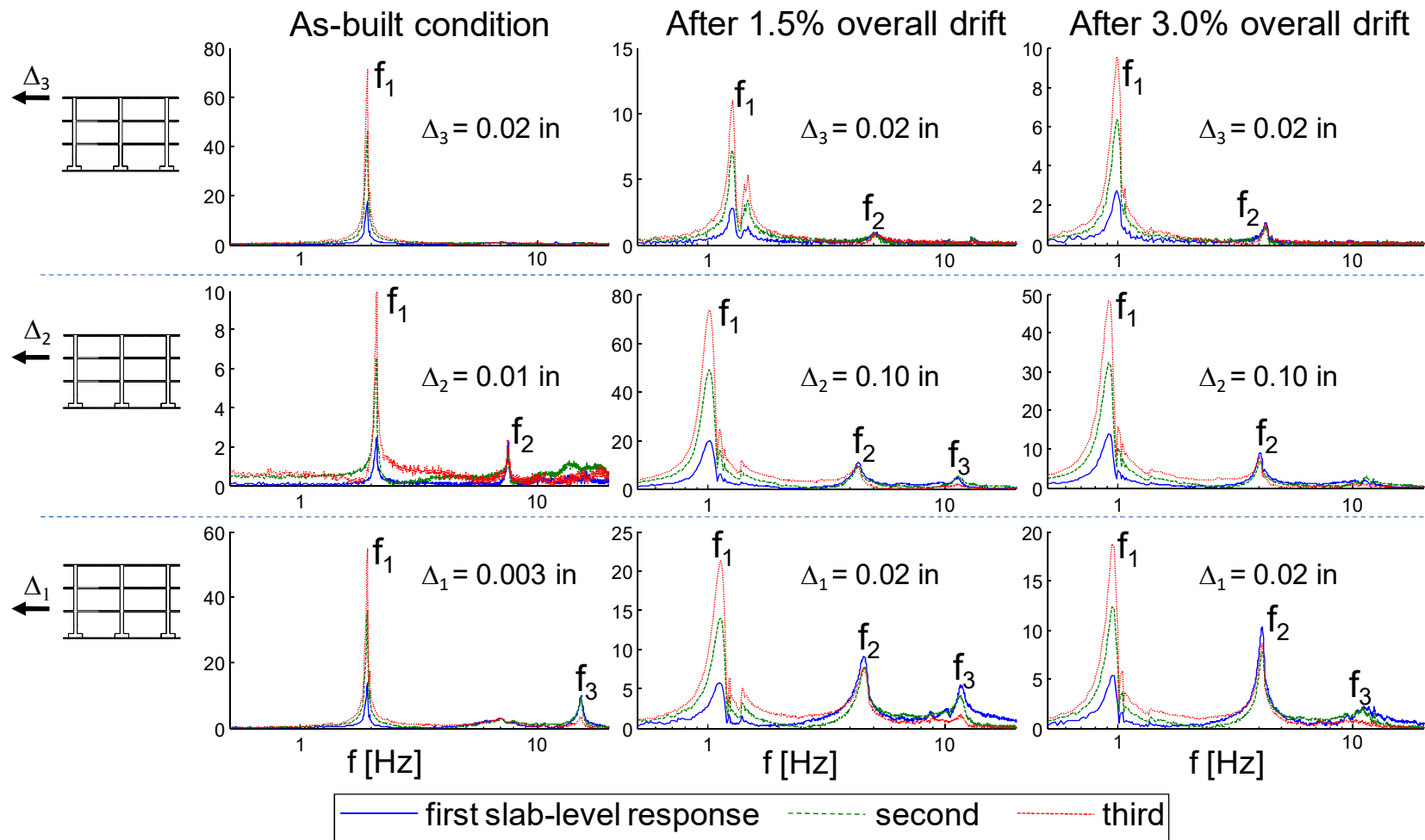


Figure 4-6 Fourier spectra of floor level records from free vibration tests with pull-release at each floor level.

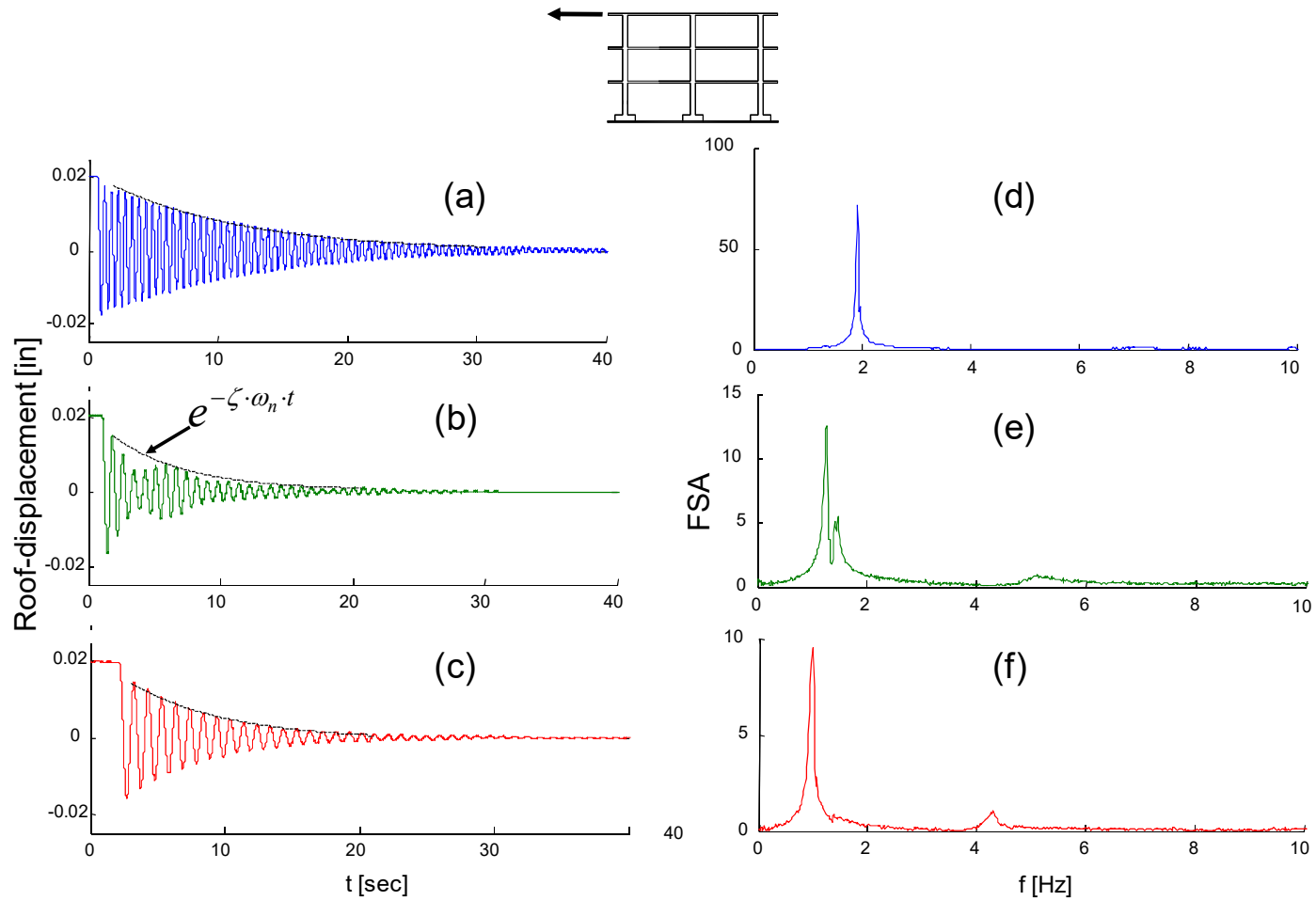


Figure 4-7 Records and Fourier spectra of free vibration displacement response obtained by pull-release at the roof level. (a) and (d) As-built condition. (b) and (e) After 1.5% drift ratio. (c) and (f) After 3.0% drift ratio.

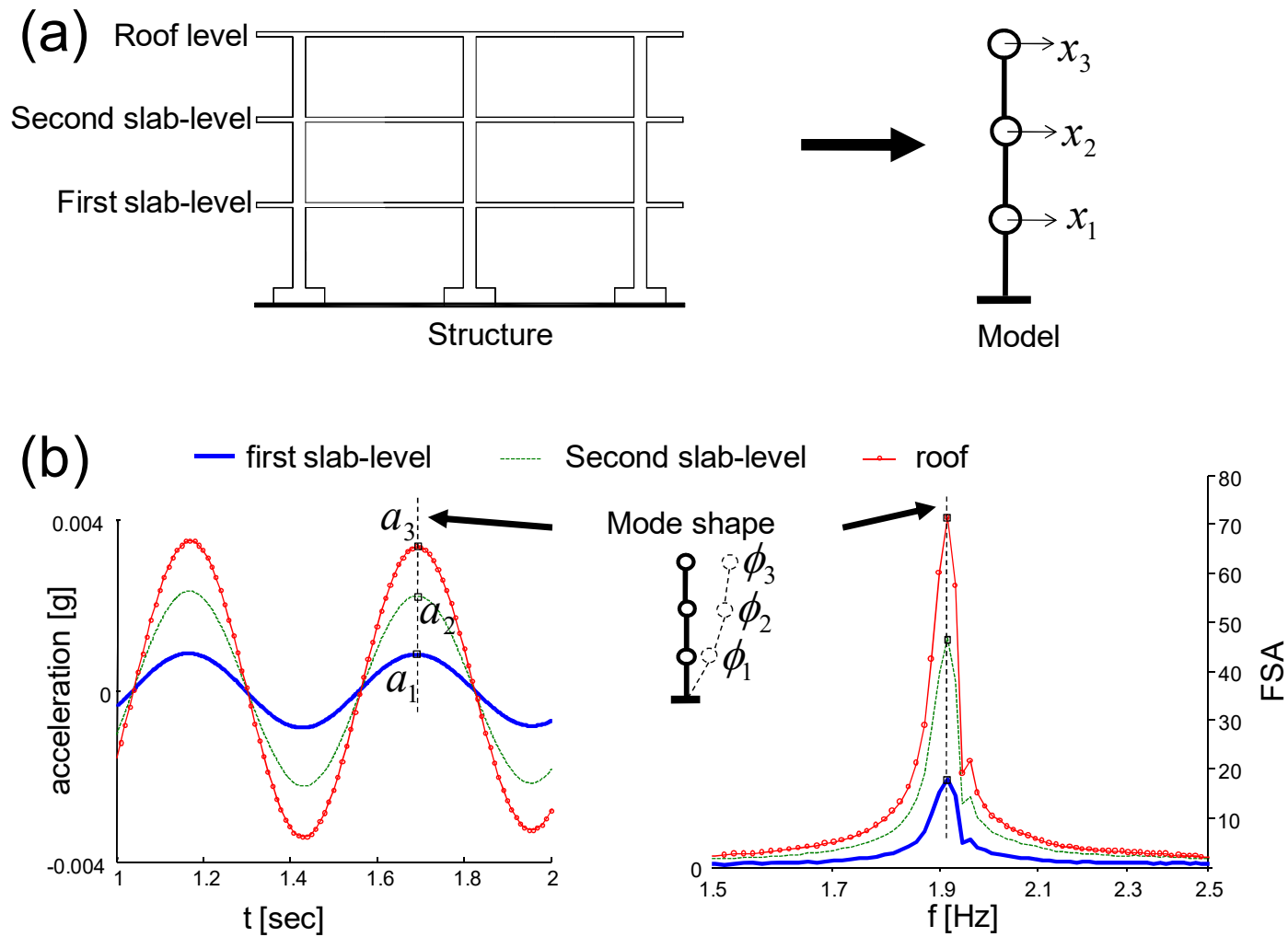


Figure 4-8 Dynamic model and fundamental mode shape. (a) The structure and its 3-DOF lumped-mass representation. (b) Mode shape from normalized acceleration records.

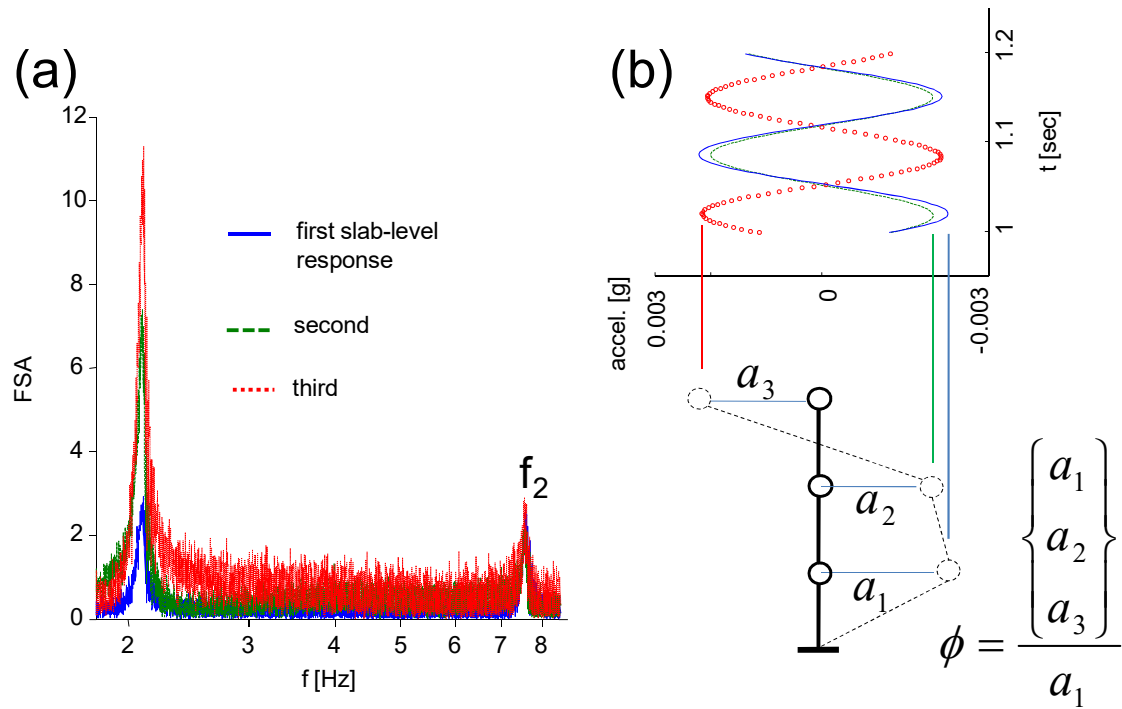


Figure 4-9 Second mode properties from free vibrations in the as-built condition.
(a) Fourier spectrum. (b) Normalized acceleration magnitudes.

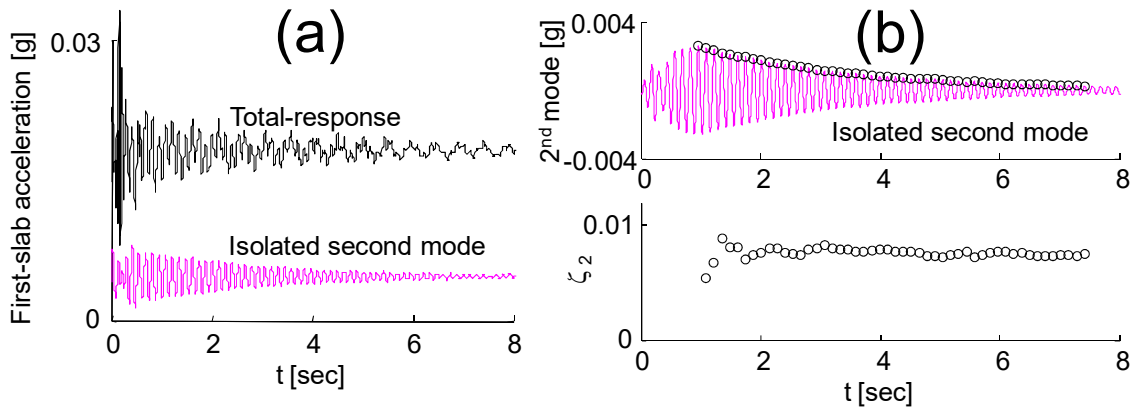


Figure 4-10 Estimation of second-mode equivalent viscous damping ratio from free vibrations in the as-built condition. (a) First slab-floor acceleration. (b) Logarithmic decrement of the isolated mode.

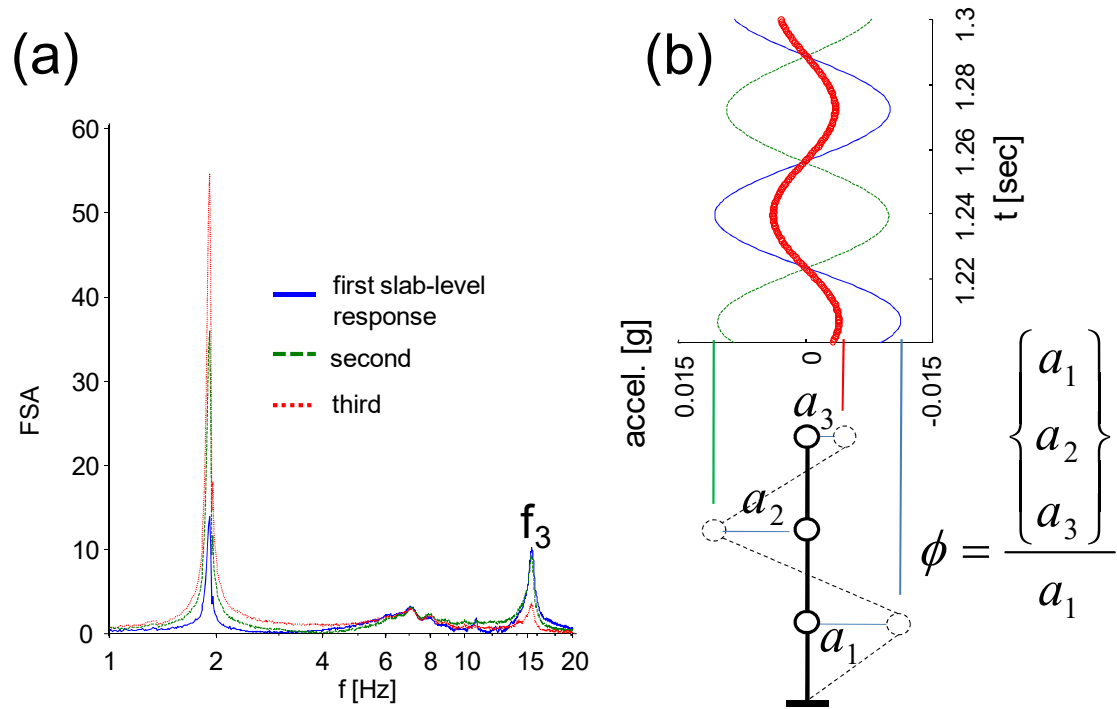


Figure 4-11 Third mode properties from free vibrations in the as-built condition.
 (a) Fourier spectrum. (b) Normalized acceleration magnitudes.

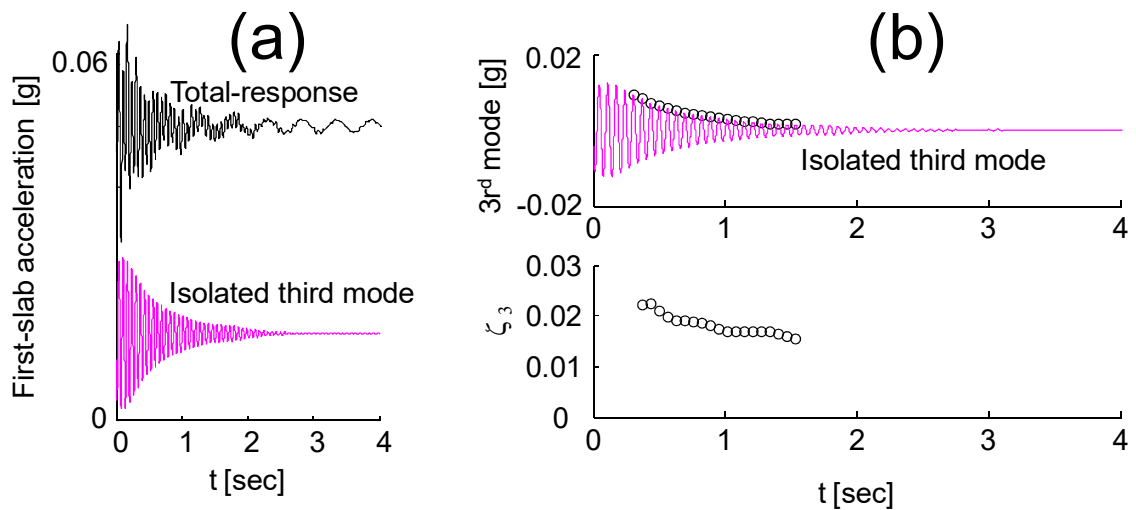


Figure 4-12 Estimation of third-mode equivalent viscous damping ratio from free vibrations in the as-built condition. (a) First slab-floor acceleration. (b) Logarithmic decrement of the isolated mode.

1. LVDT 3, 3A
2. Cart
3. Linear actuator
4. Accelerometer A3
5. Accelerometer A4

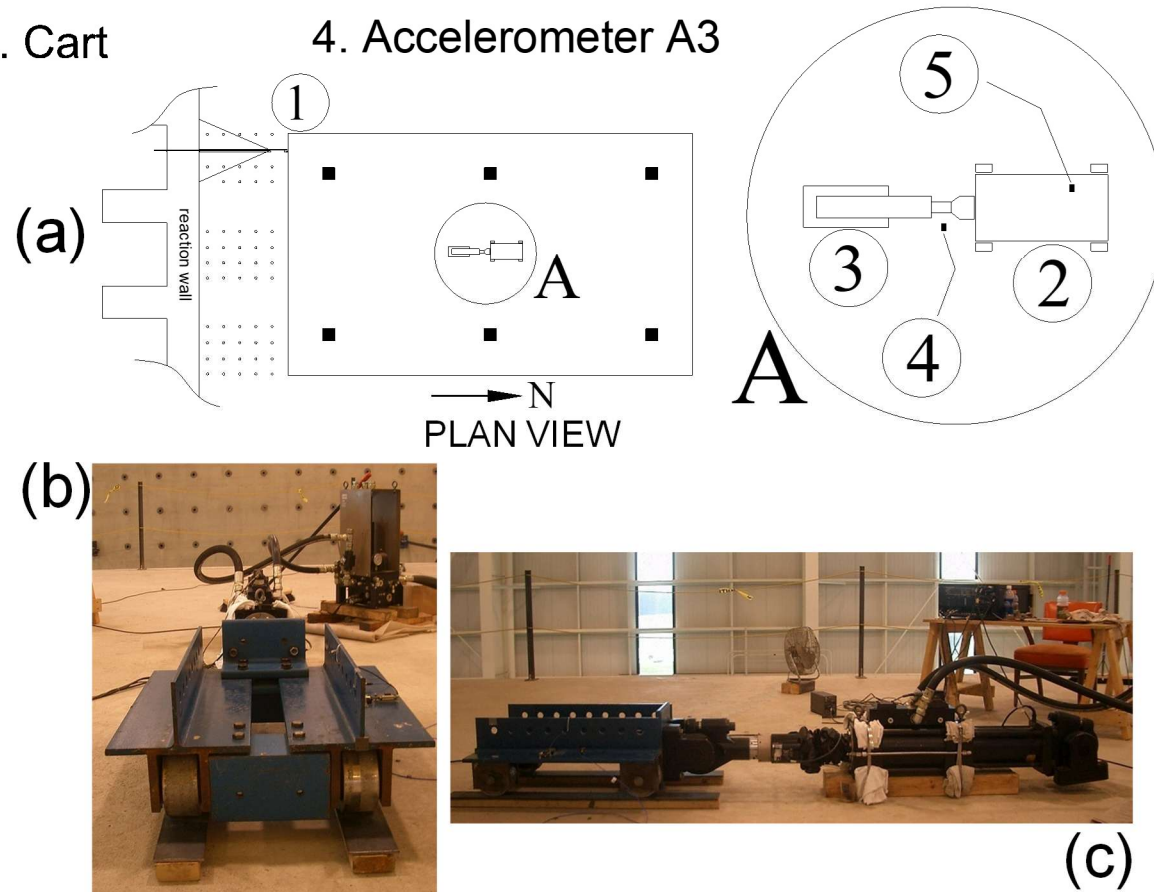


Figure 4-13 Forced vibration tests in full-scale 3-story flat-plate building. (a) Plan view. (b) View towards south direction. (c) Actuator and cart.

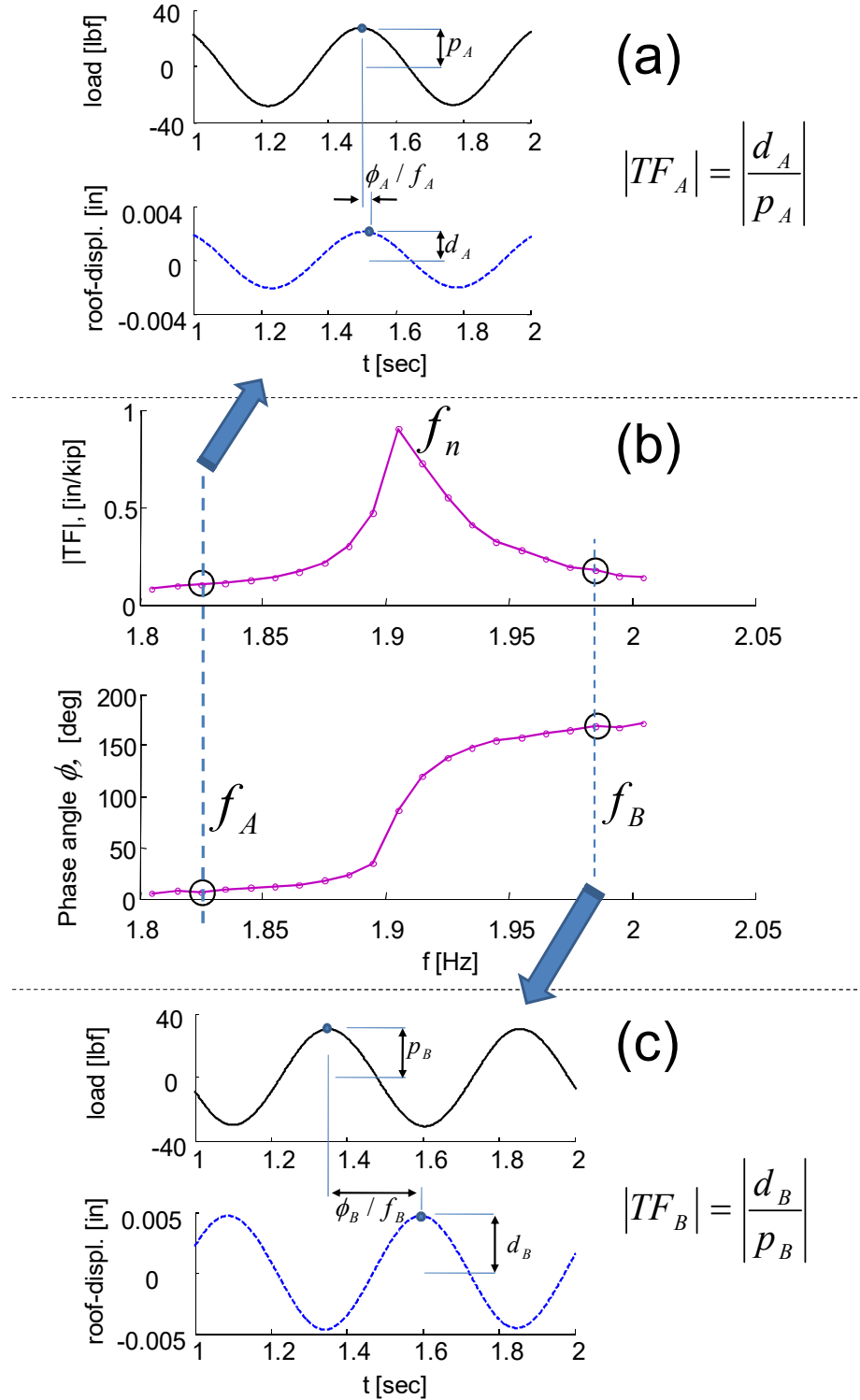


Figure 4-14 Example of non-constant peak-displacement transfer function (TF) in the as-built condition of the full-scale building. (a) Estimates for frequency f_A . (b) Magnitude and phase angle of the TF. (c) Estimates for frequency f_B .

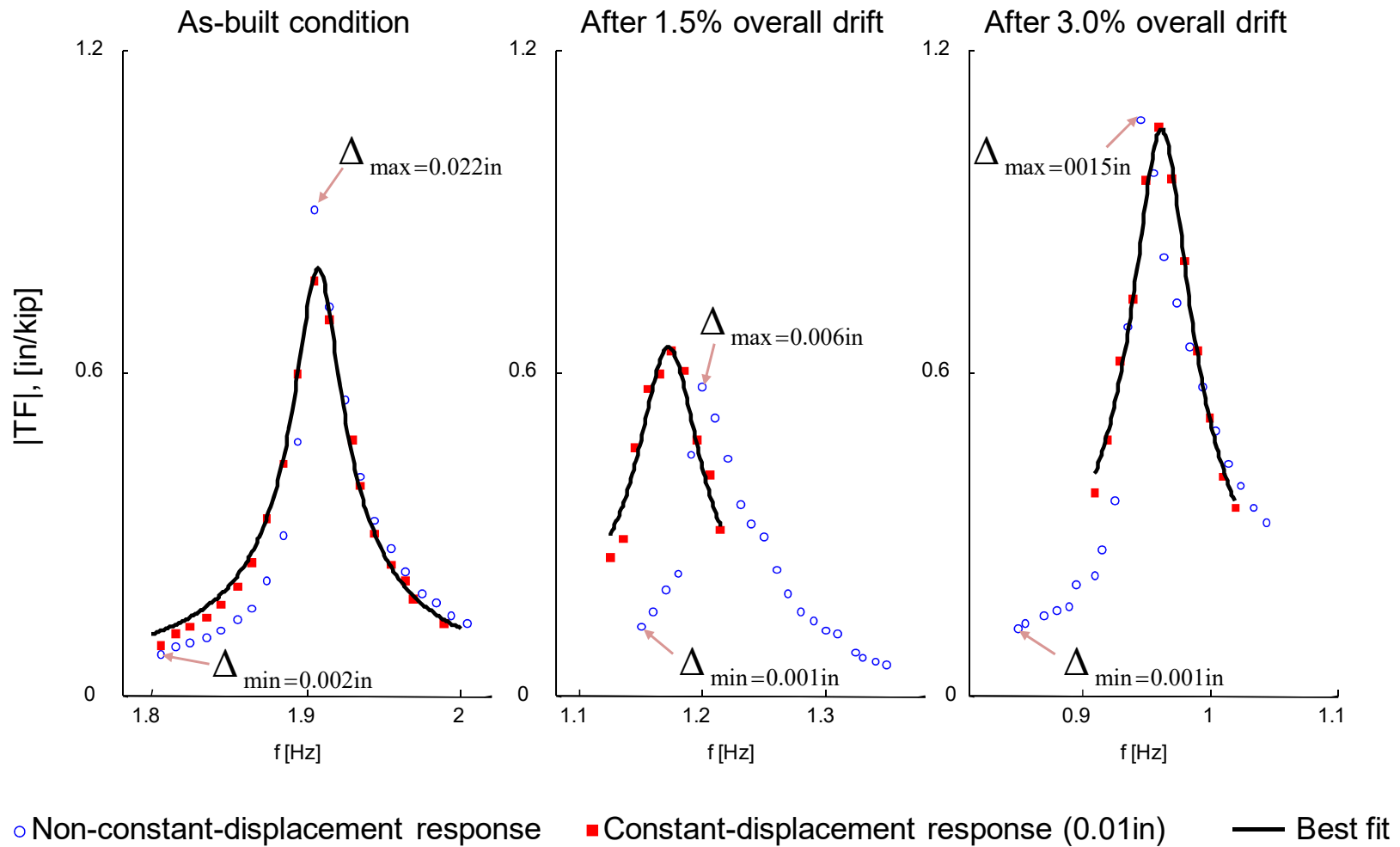


Figure 4-15 Comparison of transfer functions (TFs) obtained from forced vibration tests.

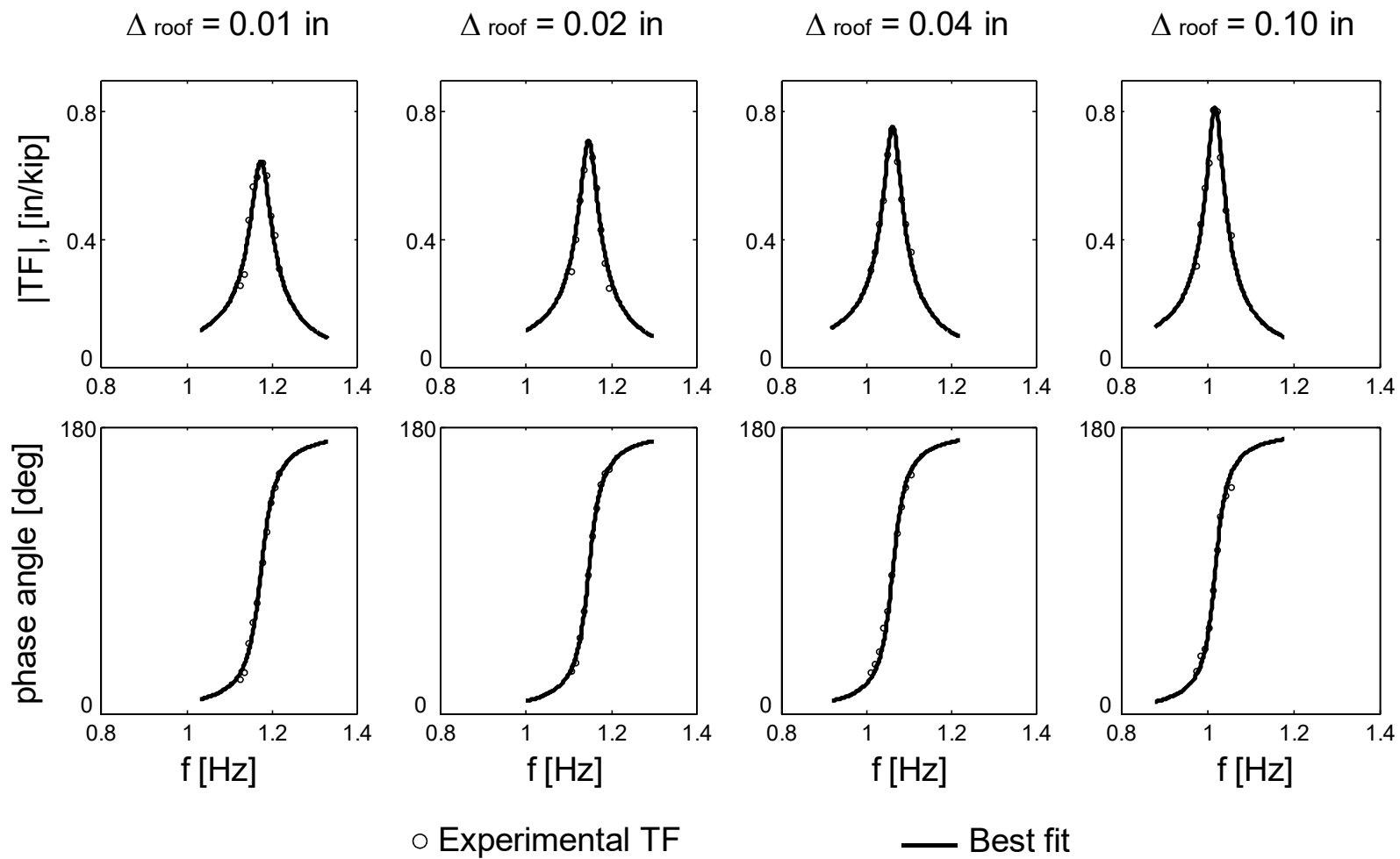


Figure 4-16 Constant-displacement response type of transfer functions after 1.5% overall drift ratio.

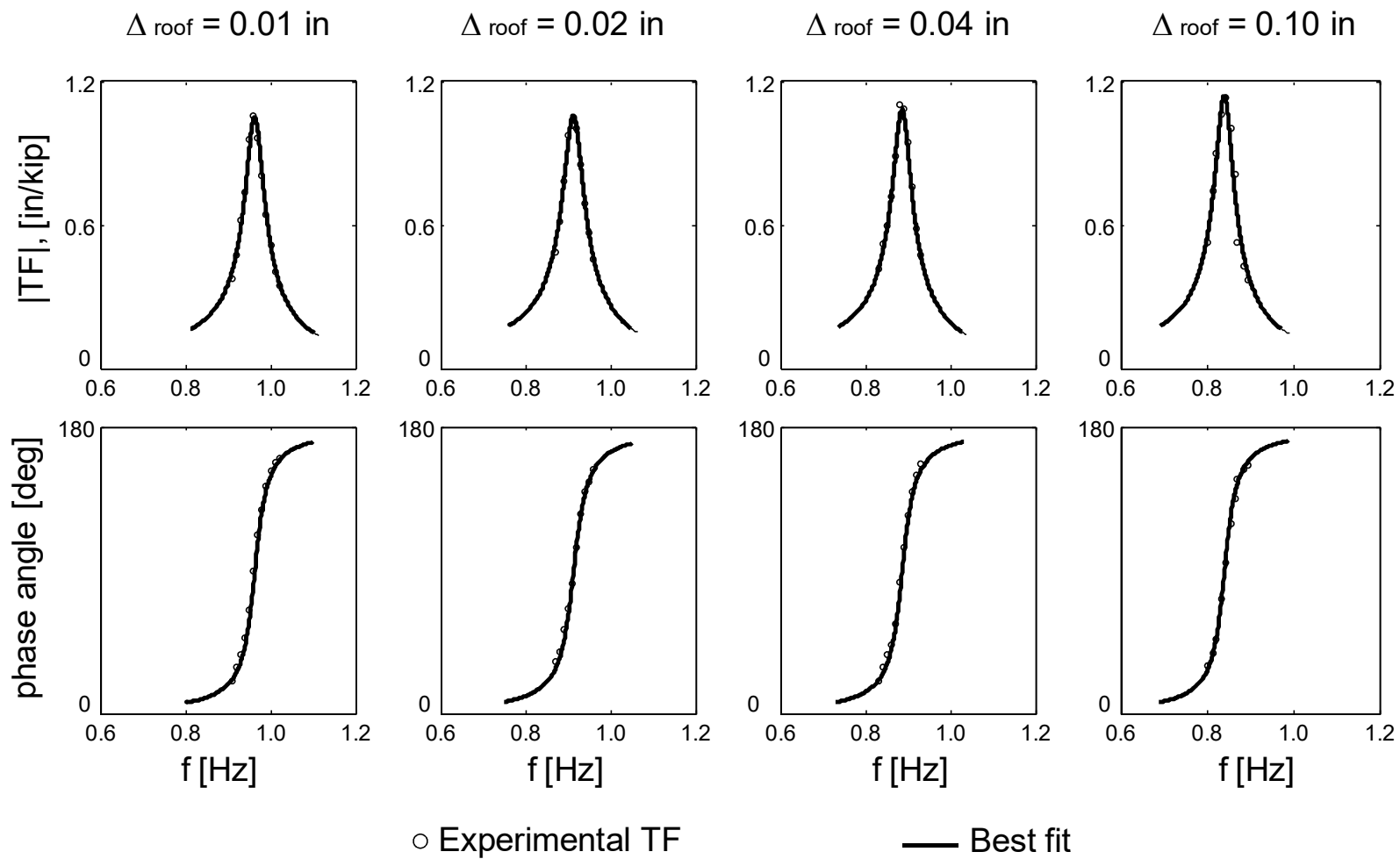


Figure 4-17 Constant-displacement response type of transfer functions after 3.0% overall drift ratio.