This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

Donation Networks in Underprivileged Communities

Andres Felipe Zambrano, *Student Member, IEEE*, Gilberto Díaz-García, *Student Member, IEEE*, Santiago Ramírez, Luis Felipe Giraldo, *Member, IEEE*, Hugo Gonzalez-Villasanti, *Member, IEEE*, Monica Tatiana Perdomo, Iván Darío Hernández, and Jesús María Godoy

***Abstract*—Individuals who have very low income and belong to an underprivileged community have a challenging task when managing their financial life, typically due to negative unpredictable events and lack of health and financial services. Cooperation strategies based on savings and credits are becoming a very important option for these communities to address such challenges and to have some level of financial stability. Since designing new cooperation strategies takes a significant amount of time and effort, computational tools have been recently introduced to simulate and evaluate communities that implement these financial cooperation schemes. In this article, based on the theory that underlies these computational tools and theoretical concepts of cooperation, we propose a new cooperation strategy based on donations that are distributed between the members of the community according to the given network topology. Through mathematical and simulation analyses, we show the scenarios where the proposed cooperation strategy, based on altruistic behavior, can potentially improve the resiliency of the community to negative unpredictable events.**

***Index Terms*—Cooperation, feedback control, humanitarian engineering, model predictive control (MPC), relative reputation, social dilemmas.**

I. Introduction

MANAGING a financial life is one of the most difficult decision tasks of a human being. For individuals who live in underprivileged communities, any obstacle or negative "shock" could have a severe adverse impact on their lives due to the lack of health and financial services [1]. Informal financial cooperation strategies have emerged in several parts of the world as a solution to this problem, where cooperation between members plays an important role as a mechanism to improve the resiliency of the community. Typically, these strategies involve the community members meeting during a defined period of time to save and borrow together. Examples of this type of strategy include the accumulating savings and credit associations (ASCAs) and rotating savings and credit associations (ROSCAs) that are the most commonly implemented ones around the world [2]. Even though these strategies have proven to be a useful way to promote socioeconomic development, they still have some limitations that are usually related to the stability of the cooperation process in the presence of unexpected events and the skills that are required to implement these strategies [3]. There is an on-going need to explore new strategies where such disadvantages might be minimized.

Since observing and evaluating the performance of a community that implements a financial cooperation strategy take significant time and effort, in recent research work, computer tools have been proposed to model human dynamics in scenarios where investment and cooperation strategies are implemented [4]. The main goal of that research is to understand the potential effect of such strategies to improve the living conditions of underprivileged communities. In those studies, concepts from optimal control theory are used to characterize person-to-person financial interactions [5], [6]. Optimal control theory is commonly used in engineering to derive intervention policies on a dynamical system based on an optimality criterion. First, the work in [5] proposes a financial advisor system where an individual human being is modeled as a dynamical system with several state variables that evolve depending on her/his interaction with the environment. Then, based on this work, the study in [6] modeled community cooperation strategies where each individual's behavior is characterized by money, health, and education state variables. The authors introduced a dynamic model of ASCAs in which each individual is connected to and defined measures to evaluate the performance of the community when its members are subjected to uncertain events. Also, based on some initial simulation results, the authors showed that a strategy that depends on the altruistic behavior of the individuals can produce a better resilience of the community than strategies based on ASCAs. Now, we extend their formulation to study strategies based on donations in more realistic scenarios.

Manuscript received May 7, 2020; revised September 23, 2020 and October 21, 2020; accepted November 7, 2020. This work was supported in part by Google through the Google Research Award for Latin America 2018 and 2019 and in part by the University of Los Andes and the University of Ibagué through the Convocatoria Conjunta Unibagué-Uniandes. *(Corresponding author: Andres Felipe Zambrano.)*

Andres Felipe Zambrano, Gilberto Díaz-García, Santiago Ramírez, and Luis Felipe Giraldo are with the Department of Electrical and Electronic Engineering, University of Los Andes, Bogotá 111711, Colombia (e-mail: af.zambrano10@uniandes.edu.co; gj.diaz10@uniandes.edu.co; s.ramirez429@uniandes.edu.co; lf.giraldo404@uniandes.edu.co).

Hugo Gonzalez-Villasanti is with the Crane Center for Early Childhood Research and Policy, The Ohio State University, Columbus, OH 43210 USA (e-mail: gonzalezvillasanti.1@osu.edu).

Monica Tatiana Perdomo, Iván Darío Hernández, and Jesús María Godoy are with the Department of Economics, Universidad de Ibagué, Ibagué 730001, Colombia, and also with the Department of Psychology, Universidad de Ibagué, Ibagué 730001, Colombia (e-mail: tatiana.perdomo@unibague.edu.co; ivan.hernandez@unibague.edu.co; jesus.godoy@unibague.edu.co).

Digital Object Identifier 10.1109/TCSS.2020.3037148

Our contribution in this article is a model of a community that cooperates through donations based on the theory that underlies the already proposed computational tools [5], [6] and theoretical concepts of cooperation, social capital, and group dynamics from social psychology [7]. The interaction patterns between the members of a community are defined by the topology of a social communication network [8], where each member's behavior depends on his/her financial dynamics and disposition to cooperate. This disposition is modeled by a generosity parameter that depends on well-known mechanisms to promote cooperation, such as direct and indirect reciprocity [9], [10]. That is, we adapted social concepts and models studied in [7]–[10] to the financial advisor system and donations strategy proposed in [6]. In this way, this work is able to characterize more realistic communities. We show the results of implementing a cooperation strategy based on altruistic acts and the scenarios that potentially improve the resiliency of the community. This work provides initial guidelines on how to conduct an exploratory study in real underprivileged communities.

We start this article by introducing a mathematical model of an individual's financial life (see Section II). Then, we present the donation strategy that a community uses to cooperate along with simulations (see Section III). We later incorporate the mechanisms to promote cooperation known as direct and indirect reciprocity based on reputation (see Section IV). Through simulation and mathematical analyses of the proposed model, we study different scenarios in which members of a community cooperate and how these scenarios affect the resilience of the community. We finalize this article by analyzing the feasibility of implementation of the donations strategy (see Section V) and providing some conclusions and future directions in this recently proposed research area (see Section VI).

II. Individual Financial Decision-Making

Modeling human being behavior in a variety of contexts has been widely used to provide a better understanding of the resulting dynamics and to provide a tool to explore useful interventions [11]–[15]. Recently, the work in [6] proposed a model of a human financial dynamics considering four state variables that change over time: working capital $x_1(k) \in \mathbb{R}$, accumulated savings $x_2(k) \in \mathbb{R}$, health capital $x_3(k) \in \mathbb{R}$, and education capital $x_4(k) \in \mathbb{R}$, with $k \in \mathbb{N}$ being time (in months). Working capital and savings are measured in U.S. dollars using the purchasing power parity rate. Health uses the body mass index (BMI) as an indicator. Finally, education is measured in months of school (MS). Moreover, the individual's decision variables, $u_1(k)$ and $u_m(k)$, for $m = 2$, 3, and 4, correspond to consumption of noncapital goods and transfers between working capital and the different assets, respectively. According to this, the mathematical model of an individual's behavior is given by

$$x_1(k+1) = \left(x_1(k)(1-s(k)) - \sum_{m=1}^{4} u_m(k)\right)(1+r(k)) \quad (1)$$

$$x_2(k+1) = (x_2(k) + u_2(k))(1+d_2)) \quad (2)$$

$$x_3(k+1) = x_3(k)(1-d_3) + c_3 u_3(k) \quad (3)$$

$$x_4(k+1) = x_4(k)(1-d_4) + c_4 u_4(k) \quad (4)$$

where $r(k)$ represents the return on investments (ROI) on education and health at time k, d_2 is a net return of savings, d_3 and d_4 are the depreciation rates of health and education, and c_3 and c_4 are the conversion terms between monetary units and kg/m^2 and MS, respectively. The main purpose of this model is to provide a simple characterization of the behavior of an individual when he/she manages his/her financial life through decision variables u_m and how his/her decisions affect him/her. Given this model and decision variables, a feedback controller can provide advice to increase wealth, health, and education while insuring against uncertainty. In [5] and [6], it was shown that a formulation commonly used in optimal control theory can simulate the decision-making process to define the control variables u_m. At each time k, it is assumed that the individual solves a decision-making problem such that he/she tries to reach a goal, represented as the minimization of a cost function, subjected to his/her dynamics in (1)–(4). The cost function is defined as $\sum_{i=1}^{T-1} g(x(k+i), u(k+i))$, where $T \geq 0$ is the time horizon of prediction and function g is defined as

$$g(x, u) = \sum_{i=1}^{4} q_i \left(x_i - x_i^*\right)^2 + \sum_{j=1}^{4} r_m \left(u_m - u_m^*\right)^2.$$

Parameters x_i^* and u_m^* are the desired state and expenditure for each category. According to this function, individuals make their decisions in a way that they try to eventually reach their goals. Weights $q_i \geq 0$ and $r_m \geq 0$ are used to consider the importance of reaching each desired state or consumption and ensuring subsistence. The implementation details of this model can be found in [6].

III. Cooperation Based on Donation Networks

One concept that allows us to understand the richness that community contexts can have is the sense of community. According to [16], this sense of community corresponds to the degree of identification and commitment with the values and objectives of a group, in which support networks are woven [17]. This sense of community is, hence, the result of social integration, identification with the group, and frequency of contact, among others [18]. The consolidation of this type of sense is strategic to increase the level of participation in the initiatives that the group wishes to undertake, promoting greater development of communities [19]. Interestingly, the work in [6] showed that a strategy based on a sense of community and altruistic acts can potentially have better performance than the commonly used accumulating saving and credit associations in communities. Here, we extend the work in [6] to model more realistic scenarios and analyze, in depth, the behavior of a community that implements a strategy based on donations. We present two indicators to evaluate the performance of the community and simulations to show the dynamics of cooperation under the proposed strategy.

A. Donation Strategy

We assume that: 1) the members of the community interact with each other according to the topology of a network and 2) an individual is willing to help his/her neighbor based on the neighbor's current basic needs. In the communication network, each member i is represented by a node and can only transfer money to an individual j that belongs to his/her set of neighbors N_i. Let $G_{ij} \geq 0$ be a generosity parameter that indicates how much the member i is willing to help the neighbor j. The desired amount of money $u_{ij}(k)$ that individual i will donate to a member $j \in N_i$ is given by

$$u_{ij}^{*}(k) = G_{ij}\max\left[u_3^{j*} - u_3^{j}(k-1), 0\right] \tag{5}$$

where $u_3^j(k)$ represents the amount of money that individual j spends on health at time k and u_3^{j*} is the preferred amount of money used to spend on health. Here, an individual cooperates with their neighbors based on their need for money to continue alive. This new decision variable $u_{ij}^*(k)$ is included into the dynamics of working capital of an individual i in (1) as

$$x_1^i(k+1) = (1+r(k)) \times \left(x_1^i(k)(1-s(k)) - \sum_{m=1}^{4} u_m^i(k) + \sum_{j\in N_i}(u_{ji}(k) - u_{ij}(k)) \right)$$

where the difference $u_{ji}(k) - u_{ij}(k)$ defines the utility received by individual i from donations given by j at time k. It is assumed that there is a maximum amount of money that an individual i can use for donations. We set it as $D_i \geq \sum_{j\in N_i} u_{ij}$. Also, the maximum number of donations that each individual can make at each time step is N_d. According to this model, an individual has to manage his/her financial life in a way that he/she tries to survive and help others in the community.

B. Measuring Performance of the Community

To measure the performance of the community, we use two indicators that were introduced in [6]. The management failure rate (MFR) indicates the percentage of people who cross the minimum level of BMI at the end of the simulation. The community development index (CDI), which is based on the human development index used by the United Nations [20], takes inequalities in the distribution of health, wealth, and education among the members of the community to estimate community development [6]. CDI for a community is calculated as

$$\mathrm{CDI} = \left((1-A_w)\left(\frac{\bar{\bar{w}}}{\hat{w}}\right)(1-A_h)\left(\frac{\bar{\bar{h}}}{\hat{h}}\right)(1-A_e)\left(\frac{\bar{\bar{e}}}{\hat{e}}\right) \right)^{\frac{1}{3}}$$

where A_w, A_h, and A_e are the Atkinson inequality index of wealth, health, and education [21]. Variables $\bar{\bar{w}}$, $\bar{\bar{h}}$, and $\bar{\bar{e}}$ are the mean of the state variables on all individuals, and $\hat{w}$, $\hat{h}$, and $\hat{e}$ are the maximum for each welfare characteristic reached by a specific individual in the community.

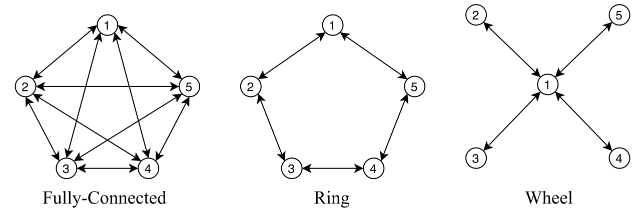


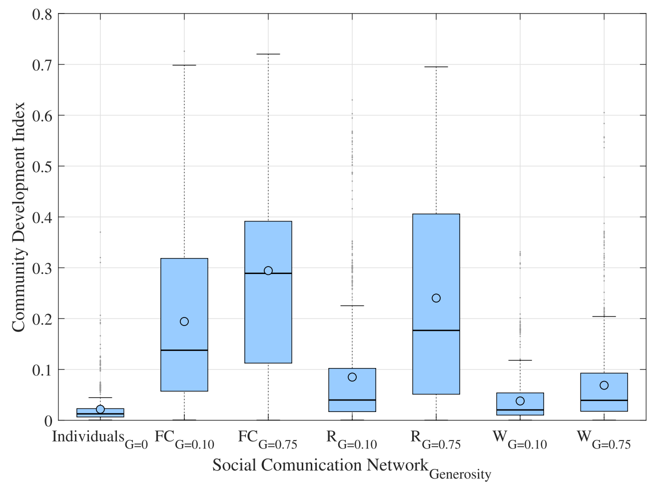
Fig. 1. Communication networks: perfectly decentralized (fully connected), partially decentralized (ring), and centralized (wheel) networks.

C. Simulation Results With Fixed Generosity

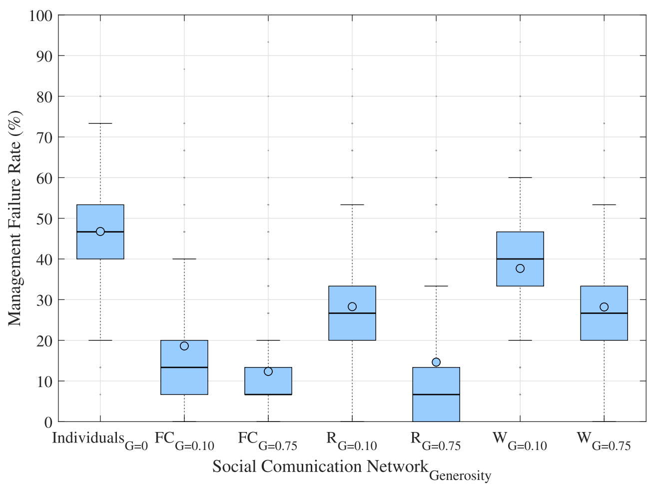
In [6], it was assumed that every member of the community interacts with each other (fully connected network), and the generosity parameter does not change over time. However, according to studies of group dynamics [8], community members interact following social communication networks. It is usual to categorize these networks into two types: centralized and noncentralized networks, as illustrated in Fig. 1. A centralized network is defined by the existence of one or a group of individuals who are highly connected in the community. A noncentralized (or decentralized) network does not have a specific leader, and every individual interacts with a similar number of neighbors. In this study, we use the wheel and ring networks as examples of the centralized and noncentralized cases of networks in small groups.

To simulate the community dynamics for each type of communication network, we select $N = 20$ members, a simulation period of 36 months, and a controller horizon of 12 months. We employ model parameters that represent a low-income individual with low initial financial and human capital. To add heterogeneity to the community, each member is randomly initialized with a return parameter, related to initial health and education level, and an initial working capital according to a Pareto probability distribution, which has been employed in modeling human groups wealth heterogeneity [22]. We conduct the Monte Carlo simulations with 1000 runs to achieve convergence of mean and standard deviation of MFR and CDI. The detailed simulation parameters can be found in [6].

Fig. 2(a) and (b) shows CDI and MFR indicators for each type of social communication network with no generosity (i.e., a community with members that do not cooperate) and low- and high-generosity scenarios. We determined these two scenarios based on the results presented in [6], which shows that the maximum development of their model of community corresponds to $G = 0.75$ (our high-generosity scenario) and the lowest development happening at $G = 0.1$ (our low-generosity scenario). According to these figures, a community whose members do not cooperate performs significantly worse than any other community that implements a donation strategy. On the other hand, in communities that cooperate, noncentralized networks tend to have a lower MFR and greater CDI than centralized networks. This improved resilience occurs because, in decentralized networks, an individual in need could potentially have more people who are able and willing to help him. However, in centralized networks, each individual who is



(a)



(b)

Fig. 2. (a) CDI and (b) MFR for a community with members that do not cooperate (leftmost boxplot) and a community that cooperates for three types of social communication networks. FC: fully connected; R: ring; and W: wheel. Low-generosity scenario is G = 0.1. High-generosity scenario: G = 0.75. Individuals: a community without cooperation, where each individual has not any neighbor.

not a center or leader of the community depends exclusively on his/her only neighbor decreasing his/her probability to receive help when necessary. This is consistent with the theory of group dynamics [8], [13], which states that groups tend to perform better at completing complex tasks in scenarios with decentralized communication patterns. As it was expected, in every social communication network, the community shows greater development when its members are more generous.

IV. Generosity Based on Reputation

In Section III, it was assumed that the sense of community, and therefore the generosity parameter between two individuals, was consolidated and does not change over time. However, it has been shown that the dynamics of cooperation change depending on interactions and the behavior that members of the community have [9]. When such a sense of community builds up in social capital, it works based on reciprocity [7]. Reciprocity refers to the mechanisms that promote cooperation and consolidate the sense of community based on reciprocal actions [9], [22]. Direct reciprocity occurs when individuals help those who have helped them [23]–[25]. On the other hand, indirect reciprocity occurs when individuals help those who help others. Both direct and indirect reciprocities can be favored by the reputation that is gained by the repeated acts of helping others. This conformation of trusting relations can mobilize future participation in the social task of the community, which is its own development. In effect, this developed reputation or trust has been recognized as the main driver of savings and finance accumulation in informal settings [26]. On the other hand, an absence or a weakening of these trusting relations, based on a lack of generosity of some individuals and a decrease in reputation, can harm the community. Thus, economic behavior is determined and rooted in the dynamics of social interaction within networks. In the context of a community that cooperates based on donations, an individual who donates to others can have an increased reputation, making others help him in situations of need. Following, we incorporate direct and indirect reciprocity into the model as mechanisms to promote cooperation in communities. We study the dynamics of generosity using a mathematical analysis of the model and simulation results.

A. Relative Generosity Model

Individual i's generosity to individual j depends on j's reputation. According to the concept of reciprocity, the dynamics of an individual's reputation depend on his/her contribution to the community's development. Remember that $G_{ij} \in [0, 1]$ is the generosity parameter that quantifies the motivation of i to help j. Let $D_j^{\text{com}}(k), D_{ji}^{\text{ind}}(k) \in [0, 1]$ be the variables that quantify the altruistic behavior of individual j based on his/her donations to the community and individual i at time k, respectively. The values of these variables closer to 1 indicate a higher amount of donations. Then, we redefine the generosity parameter G_{ij}, used to calculate the desired amount of money $u_{ij}^*(k)$ that an individual i will donate to a neighbor j in (5), as

$$\begin{aligned} G_{ij}(k+1) &\\ = G_{ij}(k) &+ \underbrace{\tau_1\left(D_j^{\text{com}}(k) - G_{ij}(k)\right)}_{\text{Community Reputation}} + \underbrace{\tau_2\left(D_{ji}^{\text{ind}}(k) - G_{ij}(k)\right)}_{\text{Individual Reputation}}. \end{aligned}$$

According to this equation, the generosity of individual i toward member j at time $k+1$ depends on two elements that depend on the contribution of j to the social task. The first element corresponds to the reputation of an individual j based on his/her contribution to the whole community (indirect reciprocity). This reputation increases when j's contribution to the community $D_j^{\text{com}}(k)$ is larger than i's current perceived generosity toward j (i.e., G_{ij}) and decreases otherwise. The second element corresponds to the direct contribution of j to individual i (direct reciprocity). If j's contribution to i ($D_{ji}^{\text{ind}}(k)$) is larger than the current perceived generosity G_{ij}, then the reputation from i to j will increase. In this way, the generosity of individual i toward j, that is, his/her motivation to help him/her, increases as i notices a contribution to the community development but decreases as the contribution is lower than the expected one. Note that both elements are weighted by constants τ_1 and τ_2, with $\tau_1, \tau_2 \geq 0$ and $\tau_1 + \tau_2 \leq 1$. These constants can be seen as forgetting parameters of the previous actions that feed the reputation of an individual, or an indicator of the reluctance of an individual to forgive his/her neighbor's actions that do not contribute to the social task.

Let $d_{jp}(k)$ be the amount of donations from j to p, and let $\sigma(a) = \min\{\max\{a, 0\}, 1\}$ be a function that saturates values that are below 0 and above 1. The altruistic contribution of an individual to the community and to his/her neighbor i are given by $D_j^{\text{com}}(k) = \sigma(\tilde{D}_j^{\text{com}}(k))$ and $D_{ji}^{\text{ind}}(k) = \sigma(\tilde{D}_{ji}^{\text{ind}}(k))$, where

$$\tilde{D}_j^{\text{com}}(k) = \frac{\sum_{p=1}^{N_d}(u_{jp}(k) - u_{pj}(k))}{\frac{1}{N}\left(\sum_{q=1}^{N}\sum_{p=1}^{N_d}|u_{qp}(k) - u_{pq}(k)|\right)}$$

$$\tilde{D}_{ji}^{\text{ind}}(k) = \frac{u_{ji}(k) - u_{ij}(k)}{\frac{1}{N}\sum_{q=1}^{N}\left(\frac{1}{N_d}\sum_{p=1}^{N_d}|u_{qp}(k) - u_{pq}(k)|\right)}$$

N is the total members of the community, and N_d is the number of donations that a specific individual can make. The variable $\tilde{D}_j^{\text{com}}(k)$ measures the amount of donations that individual j has contributed to the community relative to the mean of total donations of each individual. Variable $\tilde{D}_{ji}^{\text{ind}}(k)$ measures the amount of donations that individual j has contributed to individual i relative to the mean of the amount received by any individual donation. The denominator of these expressions acts as a weighting factor that scales the relative amount of donations to the average amount of donations given by the community. The saturation function guarantees that $D_j^{\text{com}}(k)$ is 0 when individual j has not contributed enough to the community and 1 when it has contributed more than the average contributions in the community at time k. Similarly, $D_{ji}^{\text{ind}}(k)$ is 0 when individual j has not contributed enough to individual i and 1 when it has contributed to individual i more than the average individual contributions in the community at time k.

Once the concept of sense of community is explained and the relative generosity model is defined, it is possible to characterize the behavior of the generosity dynamics between two individuals in the community given the parameters of the model τ_1 and τ_2. As a complement to the simulations provided in Section IV-C, we introduce Theorem 1 to show that the generosity of individual i toward j will tend to the overall contribution of j to community development. We expect that this behavior and consolidation of the sense of community can be seen in the modeled dynamics.

Theorem 1: Assume that the contribution of an individual between two consecutive times is bounded, i.e., $\left|D_j^{\text{com}}(k+1) - D_j^{\text{com}}(k)\right| \leq K_j^{\text{com}}$ and $\left|D_{ji}^{\text{ind}}(k+1) - D_{ji}^{\text{ind}}(k)\right| \leq K_{ji}^{\text{ind}}$. Also, let us define the expected generosity as

$$\hat{G}_{ij}(k) = \frac{\tau_1 D_j^{\text{com}}(k) + \tau_2 D_{ji}^{\text{ind}}(k)}{\tau}$$

with $\tau = \tau_1 + \tau_2$. The expected generosity $\hat{G}_{ij}(k)$ quantifies the contribution of individual j to community development at time k. Let $E_{ij}(k) = \hat{G}_{ij}(k) - G_{ij}(k)$. Then, $\left|E_{ij}(k)\right|$ is bounded for all k's. Particularly

$$\left|E_{ij}(k)\right| \leq (1-\tau)^k\left|E_{ij}(0)\right| + K_{ij}\frac{1-(1-\tau)^k}{\tau} \tag{6}$$

with

$$K_{ij} = \frac{\tau_1 K_j^{\text{com}} + \tau_2 K_{ji}^{\text{ind}}}{\tau_1 + \tau_2}.$$

Proof: In the first place, note that, if normalized donations jumps are bounded, then the expected generosity $\hat{G}_{ij}(k)$ jumps are also bounded as follows:

$$\begin{aligned}
\left|\hat{G}_{ij}(k+1) - \hat{G}_{ij}(k)\right| &= \left|\frac{\tau_1 D_j^{\text{com}}(k+1) + \tau_2 D_{ji}^{\text{ind}}(k+1)}{\tau_1+\tau_2}\right.\\
&\qquad \left. - \frac{\tau_1 D_j^{\text{com}}(k) + \tau_2 D_{ji}^{\text{ind}}(k)}{\tau_1+\tau_2}\right|\\
&= \left|\frac{\tau_1\left[D_j^{\text{com}}(k+1) - D_j^{\text{com}}(k)\right]}{\tau_1+\tau_2}\right.\\
&\qquad \left. + \frac{\tau_2\left[D_{ji}^{\text{ind}}(k+1) - D_{ji}^{\text{ind}}(k)\right]}{\tau_1+\tau_2}\right|\\
&\leq \frac{\tau_1 K_j^{\text{com}} + \tau_2 K_{ji}^{\text{ind}}}{\tau_1+\tau_2} = K_{ij}.
\end{aligned}$$

Then, using the definition of $\hat{G}_{ij}(k)$, the dynamics of $G_{ij}(k)$ can be rewritten as

$$\begin{aligned}
G_{ij}(k+1) &= G_{ij}(k) + \tau_1\left(D_j^{\text{com}}(k) - G_{ij}(k)\right)\\
&\quad + \tau_2\left(D_{ji}^{\text{ind}}(k) - G_{ij}(k)\right)\\
&= (1-\tau_1-\tau_2)G_{ij}(k) + \tau_1 D_j^{\text{com}}(k) + \tau_2 D_{ji}^{\text{ind}}(k)\\
&= (1-\tau)G_{ij}(k) + \tau\hat{G}_{ij}(k).
\end{aligned}$$

Then, the dynamics of the error $E_{ij}(k) = \hat{G}_{ij}(k) - G_{ij}(k)$ are described by

$$\begin{aligned}
E_{ij}(k+1) &= \hat{G}_{ij}(k+1) - G_{ij}(k+1)\\
&= \hat{G}_{ij}(k+1) - (1-\tau)G_{ij}(k) - \tau\hat{G}_{ij}(k)\\
&= (1-\tau)\left[\hat{G}_{ij}(k) - G_{ij}(k)\right]\\
&\quad + \left[\hat{G}_{ij}(k+1) - \hat{G}_{ij}(k)\right]\\
&= (1-\tau)E_{ij}(k) + \left[\hat{G}_{ij}(k+1) - \hat{G}_{ij}(k)\right].
\end{aligned}$$

The solution to the aforementioned system is

$$\begin{aligned}
E_{ij}(k) &= (1-\tau)^k E_{ij}(0)\\
&\quad + \sum_{m=0}^{k-1}(1-\tau)^{k-m-1}\left[\hat{G}_{ij}(m+1) - \hat{G}_{ij}(m)\right].
\end{aligned}$$

Using the triangle inequality, the error norm is

$$\begin{aligned}
\left|E_{ij}(k)\right| &\leq (1-\tau)^k\left|E_{ij}(0)\right| + K_{ij}\sum_{m=0}^{k-1}(1-\tau)^{k-m-1}\\
&\leq (1-\tau)^k\left|E_{ij}(0)\right| + K_{ij}\frac{1-(1-\tau)^k}{\tau}
\end{aligned}$$

which corresponds to the expression in (6). ■

By inspecting Theorem 1, some remarks about the community generosities $G_{ij}(k)$ can be obtained.

Remark 1: All generosities $G_{ij}(k)$ will tend to follow the contribution of individual j represented by the expected generosity $\hat{G}_{ij}(k)$.

Remark 2: Increasing the forgetting parameters, τ_1 and τ_2 will accelerate the convergence of generosities $G_{ij}(k)$ to the expected generosity $\hat{G}_{ij}(k)$.

Remark 3: The steady-state error, $E_{ij}(k)$, is bounded by K_{ij}/τ as $k \to \infty$. This means that the width of the interval that contains all generosities increases if the changes of contribution of individual j increases and decreases as the sum of the forgetting parameters $\tau = \tau_1 + \tau_2$ increases.

B. Simulation Algorithm

To simulate each community and identify the impact of a cooperation strategy based on generosity, we implemented Algorithm 1. This algorithm uses the Monte Carlo simulations with L runs, a horizon of simulation of K months, and N members of each community. For each run l and month k, after initializing the state variables of all members, each individual must actualize his or her state by calculating first the decision variables $u_m^i(k)$ and $u_{ij}(k)$ of all members using a model predictive control (MPC) with a prediction horizon of T months (lines 13–16). Note that the states cannot be updated before calculating donations of all members of the community. This process is repeated until all members failed or the simulation horizon is reached.

Algorithm 1 Pseudocode for the Simulation of a Community

1: Initialize Topology, G, τ_1, τ_2, N
2: Initialize Constants
3: **for** each Run l **do**
4:   **for** each member i **do**
5:     Initialize x_1^i, x_2^i, x_3^i, x_4^i, N_i, and $Failure^i$
6:     **for** each neighbor j **do**
7:       Initialize G_{ij}
8:     **end for**
9:   **end for**
10:   **for** each month k **do**
11:     **for** each member i **do**
12:       Calculate $s^i(k)$ and $r^i(k)$
13:       Calculate $u_1^i(k)$, $u_2^i(k)$, $u_3^i(k)$, and $u_4^i(k)$
14:       **for** each neighbor j **do**
15:         Calculate $u_{ij}(k)$
16:       **end for**
17:     **end for**
18:     **for** each member i **do**
19:       Calculate $x_1^i(k+1)$, $x_2^i(k+1)$, $x_3^i(k+1)$, and
20:       $x_4^i(k+1)$
21:       **for** each neighbor j **do**
22:         Calculate $G_{ij}(k+1)$
23:       **end for**
24:       **if** $x_1^i(k+1) < 0$ **or** $x_3^i(k+1)$ **then**
25:         $Failure^i = 1$
26:         $\#Faiures++$
27:         Retire member i from community
28:       **end if**
29:     **end for**
30:     **if** $\#Failures == N$ **or** $k == K$ **then**
31:       Calculate MFR^l and CDI^l
32:       **end** Run l
33:     **end if**
34:   **end for**
35: **end for**
36: **for** each member i **do**
37:   **for** each neighbor j **do**
38:     Calculate mean and standard deviation of:
39:     u_{ij} and G_{ij}
40:   **end for**
41:   Calculate mean and standard deviation of:
42:   x_1^i, x_2^i, x_3^i, x_4^i, u_1^i, u_2^i, u_3^i, u_4^i and G_i
43: **end for**
44: Calculate mean and standard deviation of:
45: MFR, CDI and G
46: **end** Simulation

The MPC requires solving a problem of convex quadratic programming. For this convex quadratic program, an interior point convex algorithm can be used, which has a complexity of $O(n^3)$, where n is the number of variables, as it was shown by Ye and Tse [27]. In this case, the maximum growth of the number of decision variables for each individual for N community members occurs in the fully connected network topology, where each new member is a new neighbor and a new decision variable. The maximum number of variables for the optimization algorithm is $T(3+N)$, where $3+N$ reflects the decision variables related to the four investments on each asset plus $N-1$ donations that each individual can make. For this reason, we can assume that decision and state variables for each member i of the community can be computed in $O(T^3N^3)$ arithmetic operations. Computing decisions and updating the states for each individual is the most expensive action in our algorithm. This process grows in direct proportion to the size of L, K, and N. Therefore, it can be assumed that our algorithm can be computed in $O(LKT^3N^4)$ arithmetic operations. Further details about the MPC used in this problem can be seen in [6].

C. Simulation Results

In this set of simulations, we use the same simulation parameters described in Section III-C. We explore a heterogeneous community that communicates according to three types of networks (fully connected, ring, and wheel) in two scenarios: low initial generosity and high initial generosity. It is assumed that individual and community reputations have the same importance to determine the relative generosity between two individuals ($\tau = \tau_1 = \tau_2$).

Before analyzing the CDI and MFR in communities with a variable generosity, we first study the evolution of the generosity parameter along time. Fig. 3(a)–(f) shows the mean and standard deviation of the generosity of each individual toward his/her neighbors at each time step for the three social communication networks and the two scenarios of initial

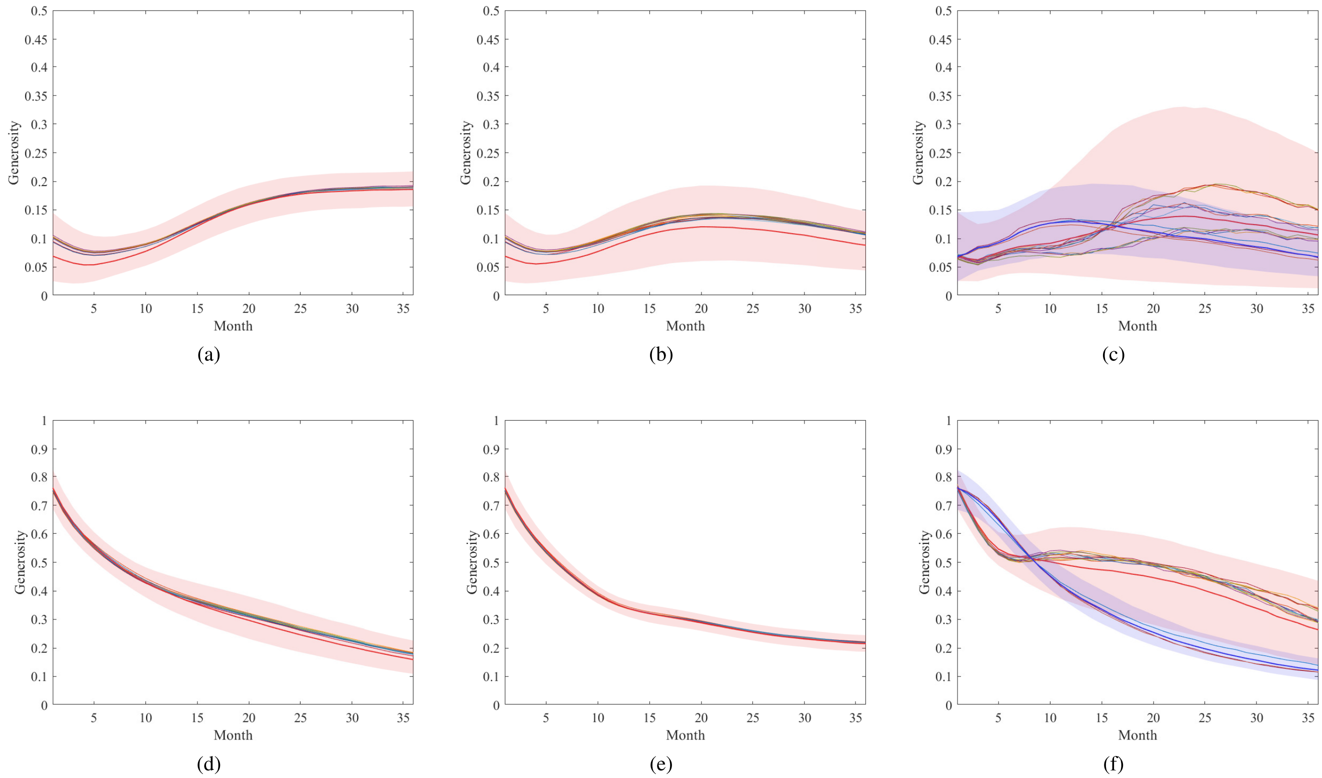


Fig. 3. Evolution of the generosity parameter for several communication network topologies with low- and high-initial-generosity scenarios. (a) Fully connected with low generosity. (b) Ring network with low generosity. (c) Wheel network with low generosity. (d) Fully connected network with high generosity. (e) Ring network with high generosity. (f) Wheel network with low generosity. The solid lines represent the mean generosity parameter for each individual, and the shaded regions represent the results between the first and third quartiles. For the wheel topology, a blue shade corresponds to the centers of the community.

generosity. Individuals in scenarios with low initial generosity tend to decrease, even more, their generosity at initial time steps because they prefer to save instead of cooperating to ensure their survival. However, as time goes by, they increase their generosity to help others because some donations occur. These donations would be the result of the requests of some members who really need the help of anyone. Probably, only close friends offer that help, but that minimal contribution can inspire and motivate others to be more generous. This can be seen as the Köhler effect that explains the situation where an individual who does not contribute to the social task could be motivated to participate if some of his/her neighbors contribute to the task completion [28].

On the other hand, in scenarios with high initial generosity, all networks show a decrease in generosity at a higher rate because some individuals fear a future defection of their neighbors due to lower donations during the first months of simulation. Those lower donations are explained by the lower needs of several members during those first months. When generous individuals do not see that others are donating to their neighbors, they become less inclined to help others. This loss of motivation to cooperate is known as the Sucker effect in a context where community development is the task to be completed [29], [30]. At most time steps, the magnitude of the slope is negative, implying that individuals become less inclined to help others when time goes by. In both cases, the generosity parameter does not reach zero values and ends in a similar interval. This is the result of loss and gain of motivation due to donations and needs of others that can decrease the generosity of an altruistic person or increase the generosity of a selfish one, reaching similar, but not necessarily equal, levels of generosity. As discussed in [23], in every community, even in nongenerous one, an individual tends to cooperate because he/she needs a reputation to get a minimum contribution from his/her neighbors to increase his/her development. Therefore, a selfish perspective can also create an altruistic behavior to improve the probability of survival and reach greater well-being even though that nonaltruistic behavior tends to imply a lower level of contribution than the one in a high-generosity scenario. This can be seen in the slightly lower level of generosity at the end of simulations for the low-initial-generosity scenario despite the increase in this parameter and its decrease in the high-initial-generosity case.

It is possible to see a clear difference between the generosity dynamics in centralized (wheel network) and noncentralized topologies (fully connected and ring networks). Noncentralized communities show a lower variance and a faster convergence in generosity because the dynamics of a reputation for each individual can be spread faster to all the members of the community. This is not the case in centralized communities, as shown in Fig. 3(c) and (f), where the spread of reputation and generosity depends mainly on the behavior

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

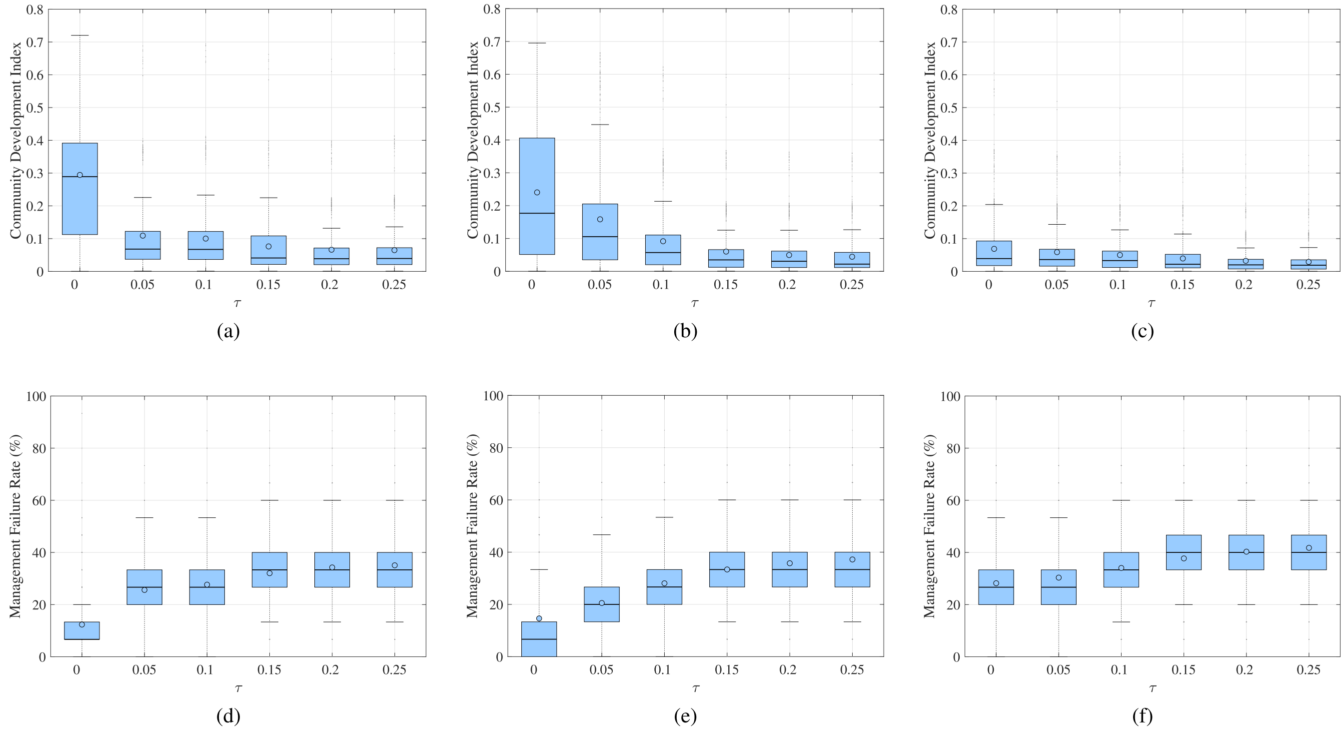


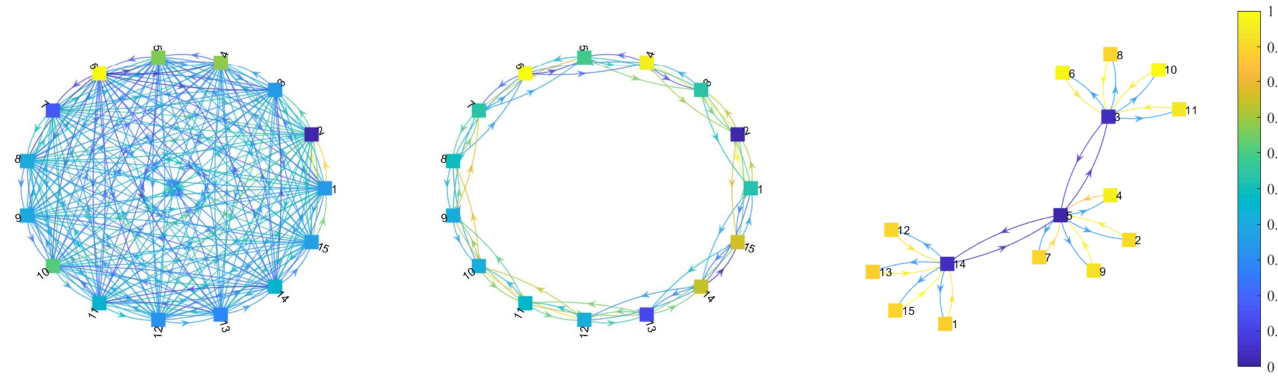
Fig. 4. CDI versus forgetting parameter τ for (a) fully connected, (b) ring, and (c) wheel networks. Also, MFR versus forgetting parameter τ for (d) fully connected, (e) ring, and (f) wheel networks.

of the central nodes (blue shadow). Focusing on the centralized network, noncentral individuals (red shadow) are forced to cooperate with their unique neighbor to aspire to be helped by the central individual (nonaltruistic generosity), increasing particularly their generosity, at least on those moments when they require a donation. Unlike these members, the centers of the network tend to help others during the first months, but their generosity decreases when they realize that noncenter members will help them even if they are less altruistic as central individuals. This absence of convergence of the generosity parameter, at least until three years of simulation, shows that centralized communication networks are difficult in the consolidation of the sense of community by having a hierarchical structure.

Knowing the generosity evolution of each type of network, it is necessary to observe its impact on the development of the community through the CDI and MFR indicators. We study the effect of the forgetting parameter τ on the CDI and MFR. Fig. 4(a)–(f) shows that the CDI increases and the MFR decreases for lower values of the forgetting parameter τ. In general, when individuals of a community do not forget easily a relationship that has been constructed with their neighbors and do not take into account a recent decrease in contributions, the community tends to develop more than a community where its members are highly sensitive to recent acts. This behavior is similar to the one observed in the generous tit-for-tat, a game that is commonly used to study the evolution of cooperation [31], [32]. The low performance of the community with a high forgetting parameter is the result of a sharp reduction in generosity and, therefore, in social capital and sense of community, when individuals do not see that all their neighbors are contributing to the social task at every moment, exemplifying again the Sucker effect [29], [30].

Fig. 4(a)–(f) shows that there is a lower development for each type of community (i.e., worse MFR and CDI) than the indicators seen in Section III where the generosity was constant. In fact, the development of communities tends to decrease when the forgetting parameter τ increases. A similar result is shown in Fig. 3(a)–(f) where that generosity tends to decrease when the forgetting parameter τ is not null. This decrease in altruism is the cause of lower development in the simulated communities [6]. Also, note that the communities that cooperate following a decentralized communication topology show better performance than centralized ones although the generosity of most of the members on centralized communities is high. This shows that communication networks and the number of trusting relations are critical for the consolidation of the sense of community and the development of the individuals beyond the desire that people have to complete any type of social task. The differences between decentralized and centralized networks narrow as the forgetting parameter increases because the generosity parameter decreases faster, and therefore, the trusting relations become almost nonexistent between any types of social communication networks.

To deepen the importance of each type of social communication network, Fig. 5(a)–(c) shows the average MFRs for each member of the community and the average amount of donations that they exchanged between them. In these figures, a lighter colored arrow indicates that donations from one individual to another were higher than donations from individuals with darker arrows. Similarly, lighter colored nodes represent the members of the community who had a lower



(a) (b) (c)

Fig. 5. Average of donations and development of individuals for (a) fully connected, (b) ring, and (c) wheel network topologies. Each node is associated with an individual in the community and his/her failure rate. Darker nodes represent the individuals with average failure rates close to 0, and light-colored nodes represent the individuals with average failure rates close to 1. Arcs and their directions represent donations from an individual to another, where dark and light colors indicate low and high amounts of donations.

development and higher failure rates than the members represented by darker nodes. According to these figures, in decentralized networks, every individual tends to have a similar development and contribution to the community, which was expected because each individual in this type of community tends to have the same generosity evolution. On the other hand, in centralized networks, there is an accentuated inequality between members. Members with more connections (centers of the topology) have a greater development than the other members of the community although these members at the center of the network tend to have lower generosity.

According to these results, it is possible to conclude that the key aspect of community development is the number of trusting relations. Communities with connections between members and a high desire to be generous with them have an improved resiliency to shocks. Therefore, it is necessary to look for strategies that promote a desire to help and interact with more people, looking for a consolidation of decentralized communication networks, even using mechanisms that promote members to increase their own reputation because it showed beneficial results to the community.

V. Feasibility of Implementation

In contexts stricken with poverty, having groups with a sense of community, which is reflected for us in an increased generosity parameter, could facilitate the incorporation of mutual aid behaviors, based on a genuine concern for the well-being of others [16], and the increase in personal well-being levels [33], replacing that need for external financial aid. The community is seen as a source of social, instrumental, and emotional supports, which facilitates the development of better living conditions. Social integration, reciprocal influence, and the perception of being able to satisfy the needs of each member of the group through the group itself are part of the elements that strengthen the sense of community [34]. In this way, the capitalization of this sense of community in interpersonal links ensues social capital [35].

When such a sense of community builds up in social capital, it works based on trust, which is a commonly used resource against malfeasance. In effect, shared expectations of penalization of fraudulent actions by the exclusion of the violator from communities' future transactions discourage fraud [36]. Such communities do not enforce penalties of exclusion by using sanctions that involve third institutional parties (judiciary courts, controllerships, and so on). A person's status or reputation in a community does not need for such hierarchical supervision because the threat of a weakening in the trusting relations is enough to motivate the desire to continue participating in community activities and to discourage opportunism [36]. This behavior tells us that a cooperation strategy based on trust, altruism, and donations can be potentially feasible for implementation and helpful in communities as long as friendship, generosity, and reciprocity are enough to discourage that opportunism that can make this type of strategy unfeasible. Even if opportunism is not avoidable, strategies that do not depend on altruism, such as savings and credits associations, can be also helpful and feasible for them.

Finally, if technological resources are available for members of these communities, such resources could be used to enforce reciprocity and avoid opportunism. Here, social media strategies and technologies, including gamification, game theory, information propagation, and public opinion control [37]–[39], can be used to incline people to increase their participation in the social task and use the peer pressure on that environment to discourage the opportunism. Of course, there are several ethical and legal considerations to be clarified before implementing these approaches, but the recent implementation of adaptive behavioral interventions in similar contexts [40] is promising for future implementation of financial behavioral intervention strategies in real communities.

VI. Conclusion

This article presented a model for financial investment decision-making in low-income communities that implement cooperative strategies based on altruistic acts. We constructed a dynamic system that models the financial behavior of individuals who have the motivation to donate money to some of their neighbors. The modeled mechanism that drives cooperation in a community is reciprocity based on reputation, in which generosity evolves depending on the perceived contribution of each individual on centralized and noncentralized communities. It is important to note that the proposed model is based on concepts that have been observed in real communities. We showed that a community whose members do not cooperate performs significantly worse than any community that implements a donation strategy. Also, we showed that, in a community that implements a donation strategy, for all type of networks, trusting relations and the consolidation of a sense of community are the key aspects to look for the development of the community and overcome any shock that an individual can suffer, being decentralized communication networks, i.e., the ones that provided higher resiliency.

Future research directions involve studying the effect of the number of members in a community and other topology patterns, such as scale-free networks, dense networks, and sparse networks [41]. Also, since informal cooperation strategies have emerged in communities around the world, altruistic behavior and potential implementations of schemes based on donations can be proposed.

References

[1] A. V. Banerjee, A. Banerjee, and E. Duflo, *Poor Economics: A Radical Rethinking Way to Fight Global Poverty*. New York, NY, USA: Public Affairs, 2011.

[2] D. Collins, J. Morduch, S. Rutherford, and O. Ruthven, *Portfolios of the Poor: How the World's Poor Live on $2 a Day*. Princeton, NJ, USA: Princeton Univ. Press, 2010.

[3] S. Rutherford, *The Poor and Their Money*. New Delhi, India: Oxford Univ. Press, 2000.

[4] F. Allahi, S. Taheri, R. Kian, and E. Sabet, "Cash-based interventions to enhance dignity in persistent humanitarian refugee crises: A system dynamics approach," *IEEE Trans. Eng. Manag.*, early access, Jul. 3, 2020, doi: 10.1109/TEM.2020.2982583.

[5] H. Gonzalez Villasanti and K. M. Passino, "Feedback controllers as financial advisors for low-income individuals," *IEEE Trans. Control Syst. Technol.*, vol. 25, no. 6, pp. 2194–2201, Nov. 2017.

[6] H. Gonzalez Villasanti, L. F. Giraldo, and K. M. Passino, "Feedback control engineering for cooperative community development: Tools for financial management advice for low-income individuals," *IEEE Control Syst. Mag.*, vol. 38, no. 3, pp. 87–101, Jun. 2018.

[7] P. S. Adler and S.-W. Kwon, "Social capital: Prospects for a new concept," *Acad. Manage. Rev.*, vol. 27, no. 1, pp. 17–40, Jan. 2002.

[8] D. R. Forsyth, *Group Dynamics*. Boston, MA, USA: Cengage Learning, 2018.

[9] M. A. Nowak, "Five rules for the evolution of cooperation," *Science*, vol. 314, no. 5805, pp. 1560–1563, Dec. 2006.

[10] L. F. Giraldo and K. M. Passino, "Dynamics of cooperation in a task completion social dilemma," *PLoS ONE*, vol. 12, no. 1, pp. 1–25, Jan. 2017.

[11] T. Zhou, X.-P. Han, and B.-H. Wang, "Towards the understanding of human dynamics," in *Science Matters: Humanities as Complex Systems*. Singapore: World Scientific, 2008, pp. 207–233.

[12] L. F. Giraldo, K. M. Passino, and J. D. Clapp, "Modeling and analysis of group dynamics in alcohol-consumption environments," *IEEE Trans. Cybern.*, vol. 47, no. 1, pp. 165–176, Jan. 2017.

[13] L. F. Giraldo and K. M. Passino, "Dynamic task performance, cohesion, and communications in human groups," *IEEE Trans. Cybern.*, vol. 46, no. 10, pp. 2207–2219, Oct. 2016.

[14] L. F. Giraldo, K. M. Passino, J. D. Clapp, and D. Ruderman, "Dynamics of metabolism and decision making during alcohol consumption: Modeling and analysis," *IEEE Trans. Cybern.*, vol. 47, no. 11, pp. 3955–3966, Nov. 2017.

[15] Q. Li, Z. Wang, B. Wu, and Y. Xiao, "Competition and cooperation: Dynamical interplay diffusion between social topic multiple messages in multiplex networks," *IEEE Trans. Comput. Social Syst.*, vol. 6, no. 3, pp. 467–478, Jun. 2019.

[16] M. Meyer, M. M. Hyde, and C. Jenkins, "Measuring sense of community: A view from the streets," *J. Health Social Policy*, vol. 20, no. 4, pp. 31–50, Oct. 2005.

[17] D. Mcmilan and D. Chavis, "Sense of community: A definition and theory," *J. Community Psychol.*, vol. 14, no. 1, pp. 6–23, 1986.

[18] R. P. Lorion and J. R. Newbrough, "Psychological sense of community: The pursuit of a field's spirit," *J. Community Psychol.*, vol. 24, no. 4, pp. 311–314, Oct. 1996.

[19] M. L. Ohmer, "How theory and research inform citizen participation in poor communities: The ecological perspective and theories on self- and collective efficacy and sense of community," *J. Hum. Behav. Social Environ.*, vol. 20, no. 1, pp. 1–19, Feb. 2010.

[20] K. M. Passino, *Humanitarian Engineering: Advancing Technologies for Sustainable Development*, 3rd ed. Columbus, OH, USA: Bede Publishing, 2016.

[21] A. B. Atkinson, "On the measurement of inequality," *J. Econ. Theory*, vol. 2, no. 3, pp. 244–263, 1970.

[22] J. Cuesta and M. Negre. (2016). Poverty and shared prosperity 2016: Taking on inequality, World Bank Group, Washington, DC, USA. [Online]. Available: http://web.archive.org/web/20080207010024/ and http://www.808multimedia.com/winnt/kernel.htm

[23] M. Nowak and R. Highfield, *SuperCooperators: Altruism, Evolution, and Why We Need Each Other to Succeed*. New York, NY, USA: Simon and Schuster, 2011.

[24] R. Axelrod and W. D. Hamilton, "The evolution of cooperation," *Science*, vol. 211, no. 4489, pp. 1390–1396, 1981.

[25] G. Roberts, "Evolution of direct and indirect reciprocity," *Proc. Roy. Soc. B, Biol. Sci.*, vol. 275, no. 1631, pp. 173–179, Jan. 2008.

[26] J. Goedecke, I. Guérin, B. D'Espallier, and G. Venkatasubramanian, "Why do financial inclusion policies fail in mobilizing savings from the poor? Lessons from rural South India," *Develop. Policy Rev.*, vol. 36, pp. O201–O219, Mar. 2018.

[27] Y. Ye and E. Tse, "An extension of Karmarkar's projective algorithm for convex quadratic programming," *Math. Program.*, vol. 44, nos. 1–3, pp. 157–179, May 1989.

[28] N. L. Kerr, L. A. Messé, D.-H. Seok, E. J. Sambolec, R. B. Lount, Jr., and E. S. Park, "Psychological mechanisms underlying the köhler motivation gain," *Personality Social Psychol. Bull.*, vol. 33, no. 6, pp. 828–841, 2007.

[29] N. L. Kerr, "Motivation losses in small groups: A social dilemma analysis," *J. Personality Social Psychol.*, vol. 45, no. 4, p. 819, 1983.

[30] J. A. Shepperd, "Productivity loss in performance groups: A motivation analysis," *Psychol. Bull.*, vol. 113, no. 1, p. 67, 1993.

[31] D. Fundenberg and E. Maskin, "Evolution and cooperation in noisy repeated games," *Amer. Econ. Rev.*, vol. 80, no. 2, pp. 274–279, 1990.

[32] M. Nowak and K. Sigmund, "A strategy of win-stay, lose-shift that outperforms tit-for-tat in the prisoner's dilemma game," *Nature*, vol. 364, no. 6432, p. 56, 1993.

[33] J. Francis, B. Giles-Corti, L. Wood, and M. Knuiman, "Creating sense of community: The role of public space," *J. Environ. Psychol.*, vol. 32, no. 4, pp. 401–409, Dec. 2012.

[34] I. Hombrados-Mendieta and T. López-Espigares, "Dimensiones del sentido de comunidad que predicen la calidad de vida residencial en barrios con diferentes posiciones socioeconómicas," *Psychosocial Intervent*, vol. 23, no. 3, pp. 159–167, Sep. 2014.

[35] J. Anchorena and F. Anjos, "Social ties and economic development," *J. Macroeconomics*, vol. 45, pp. 63–84, Sep. 2015.

[36] M. Granovetter, "Economic action and social structure: The problem of embeddedness," *Amer. J. Sociology*, vol. 91, no. 3, pp. 481–510, Nov. 1985.

[37] Y. Xiao, Q. Yang, C. Sang, and Y. Liu, "Rumor diffusion model based on representation learning and anti-rumor," *IEEE Trans. Netw. Service Manage.*, vol. 17, no. 3, pp. 1910–1923, Sep. 2020.

[38] Y. Xiao, C. Song, and Y. Liu, "Social hotspot propagation dynamics model based on multidimensional attributes and evolutionary games," *Commun. Nonlinear Sci. Numer. Simul.*, vol. 67, pp. 13–25, Feb. 2019. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1007570418301941

[39] Y. Xiao, D. Chen, S. Wei, Q. Li, H. Wang, and M. Xu, "Rumor propagation dynamic model based on evolutionary game and anti-rumor," *Nonlinear Dyn.*, vol. 95, no. 1, pp. 523–539, Jan. 2019.

[40] E. B. Hekler *et al.*, "Tutorial for using control systems engineering to optimize adaptive mobile health interventions," *J. Med. Internet Res.*, vol. 20, no. 6, p. e214, Jun. 2018.

[41] F.-Y. Wang, P. Ye, and J. Li, "Social intelligence: The way we interact, the way we go," *IEEE Trans. Comput. Social Syst.*, vol. 6, no. 6, pp. 1139–1146, Dec. 2019.