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Stress vs. slenderness ratio in compressed laminated acacia wood elements

Tensão vs. taxa de esbeltez em elementos de madeira de acácia laminada comprimida

Esfuerzos frente a la relación de esbeltez en elementos de madera de acacia laminada comprimida

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ABSTRACT
The following paper presents the relationship between stress and slenderness ratio in compressed laminated wood elements of *Acacia mangium Willd*. For this research, tests were conducted on 240 specimens of varying lengths: 14 short specimens were used to determine the modulus of elasticity based on stress vs. strain graphs, while 226 specimens were tested in three groups with varying lengths but constant rectangular cross-sectional area to determine the stress vs. slenderness ratio graph. As a result of this investigation, a compressive modulus of elasticity of 14280 MPa was determined, along with a graph of maximum stress vs. slenderness ratio and a graph of characteristic stress vs. slenderness ratio. Finally, functions were proposed to determine the characteristic stress values in compressed elements of different lengths, given their rectangular cross-sectional area. Based on these results, it is possible to design compressed laminated wood elements of *Acacia* with different slenderness ratios.

Keywords: Columns. Buckling. Euler's Critical Load. Slenderness Ratio. Laminated *Acacia Mangium Willd* Wood.

RESUMO
O artigo a seguir apresenta a relação entre a tensão e a relação de esbeltez em elementos de madeira laminada comprimida de Acacia mangium Willd. Para essa pesquisa, foram realizados testes em 240 corpos de prova de comprimentos variados: 14 corpos de prova curtos foram usados para determinar o módulo de elasticidade com base em gráficos de tensão vs. deformação, enquanto 226 corpos de prova foram testados em três grupos com comprimentos variados, mas com área de seção transversal retangular constante para determinar o gráfico de tensão vs. relação de esbeltez. Como resultado dessa investigação, foi determinado um módulo de elasticidade compressivo de 1.280 MPa, juntamente com um gráfico de tensão máxima vs. relação de esbeltez e um gráfico de tensão característica vs. relação de esbeltez. Por fim, foram propostas funções para determinar os valores de tensão característica em elementos comprimidos de diferentes comprimentos, considerando sua área de seção transversal retangular. Com base nesses resultados, é possível projetar elementos de madeira laminada comprimida de Acacia com diferentes índices de esbeltez.

Palavras-chave: Colunas. Flambagem. Carga Crítica de Euler. Índice de Esbeltez. Madeira Laminada de *Acacia Mangium Willd*.

RESUMEN
El siguiente trabajo presenta la relación entre la tensión y la relación de esbeltez en elementos de madera laminada comprimida de Acacia mangium Willd. Para esta investigación, se realizaron ensayos en 240 probetas de diferentes longitudes: 14 probetas cortas se utilizaron para determinar el módulo de elasticidad basado en gráficos de tensión vs. deformación, mientras que 226 probetas se ensayaron en tres grupos con longitudes variables pero área de sección transversal rectangular constante para determinar el gráfico de tensión vs. relación de esbeltez. Como resultado de esta investigación, se determinó un módulo de elasticidad de compresión de 14280 MPa, junto con un gráfico de tensión máxima frente a la relación de esbeltez y un gráfico de tensión característica frente a la relación de esbeltez. Por último, se propusieron funciones para determinar los valores de la tensión característica en elementos comprimidos de diferentes longitudes, dada su sección transversal rectangular. En base a estos resultados, es posible diseñar elementos comprimidos de madera laminada de Acacia con diferentes relaciones de esbeltez.

Palabras clave: Columnas. Pandeo. Carga Crítica de Euler. Relación de Esbeltez. Madera Laminada de *Acacia Mangium Willd*.

1 INTRODUCTION

The study of wood for construction purposes has been developed for several years, but lately, it has intensified to search for more environmentally friendly materials to reduce the carbon footprint produced by the production of conventional cementitious materials. The Reglamento Colombiano de Construcciones sismo resistentes (Colombian Seismic Resistant Construction Regulation), NSR-10, presents various types of wood for structural construction. However, the laminated *Acacia mangium Willd* species is not classified within this regulation. There are different studies related to the benefits of using *Acacia mangium* wood (Koutika & Richardson, 2019), (Awang & Taylor, 1993), (Jusoh, Zaharin & Adam, 2014), its anatomical characteristics (Savero *et al*, 2022), (Sahri, Ibrahim & Shukor, 1993), and the physical-mechanical properties of this wood (Sahri, Ashaari, Kader & Mohmod, 1998) (Rokeya, Hossain, Ali & Paul, 2010), including engineered wood products such as oriented strand board (Febrianto *et al*, 2009), cross-laminated timber (Mohd Yusof, Tahir, Lee, Khan & Mohammad Suffian James, 2019), and particle boards (Liew & Samin, 2021).

Some studies have also been conducted on compressive stress as a function of slenderness ratio (Li *et al*, 2019), (Nie, Wei, Huang, Liu & Dong, 2021), (Tan *et al*, 2021), (Li, He, Tao & Li, 2016). However, there are not many studies on the behavior of compression elements made with laminated *Acacia mangium Willd* of different lengths.

The research question is: What is the relationship between stress and slenderness ratio in laminated elements of Acacia mangium Willd wood subjected to axial compression load?

The objective is to determine the compressive strength of laminated elements of Acacia mangium Willd wood as a function of slenderness ratio and proposing functions to determine the characteristic compression values based on their cross-section and length.

To achieve these objectives, the modulus of elasticity in compression parallel to the grain was determined. Experimental values of maximum axial stress were determined for three groups of columns with different slenderness ratios, and the graph of characteristic stress vs. slenderness ratio was developed, differentiating the range of short, intermediate, and long columns.

This article describes the characteristics of the materials used, the theoretical bases of the tests, the methodology applied during the execution of the experiments, and the description of the tests and the specimens, as well as the calculation procedure and analysis of the results. Finally, the results obtained are presented, along with a discussion about them.

2 MATERIALS AND METHODS

2.1 MATERIALS

Laminated wood of the *Acacia Mangium Willd* species was used, sourced from the town of Tierralta in the Department of Córdoba, Colombia. The wood was received in blocks or logs, as shown in Figure 1(a). Subsequently, the blocks were cut into boards or lamellas to be taken to the drying chamber, as depicted in Figure 1(b). Through the drying process, the wood achieves a relative moisture content ranging between 10% and 12%.

Figure 1. Specimen manufacturing process.

(a) Wood blocks. (b) Drying chamber.

Source: Authors' own work.

Once the drying process was completed, the lamellas were connected at their ends using finger joint connections glued with Polyvinyl Acetate (PVA) adhesive. Then, Melamine Urea Formaldehyde adhesive was applied at room temperature, to the planed faces before pressing the process of obtaining the laminates. Subsequently, the assembled elements were pressed for a period of 24 hours, resulting in the material shown in Figure 3(a). From the laminated wood, specimens for compression testing were prepared as shown in Figure 2(b).

Figure 2. Specimens.

(a) Sheets obtained from the manufacturing process. (b) Tested specimens, only one group.

Source: Authors' own work.

2.2 METHODS

In this section, a description will be provided of the theoretical framework used for calculations, as well as an explanation of the compression tests performed on the laminates and the determination of the modulus of elasticity. Additionally, the devices used, as well as the quantity and dimensions of the tested specimens, will be discussed.

2.2.1 Analytical part

The tested laminated wood columns of *Acacia Mangium Willd* were subjected to axial compressive load. The long columns failed due to buckling, as described by Gere, Goodno, Rosales, Villarreal, and Guzmán (2016).

The critical buckling load (P_{cr}) for a column with hinged ends, according to Gere *et al*. (2016, p. 882), is given by,

$$P_{cr} = \frac{\pi^2 EI}{kL^2}, \quad (1)$$

where:

E is the modulus of elasticity of the material, I is the moment of inertia of the element, k is an effective length factor which depends on the type of supports of the column. In this case L is the length of the element. After determining the critical load for a column, the critical stress σ_{cr} is calculated as (Gere *et al*., 2016, p. 884),

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{kL^2 A}, \tag{2}$$

where:

P_{cr} is the maximum theoretical load for long columns and A is the cross-sectional area of the laminate. Equation (2) can be rewritten in a simpler form using the notation for the radius of gyration r,

$$r = \sqrt{\frac{I}{A}}. \tag{3}$$

Therefore, Equation (2) becomes,

$$\sigma_{cr} = \frac{\pi^2 EI}{(kL/r)^2}, \tag{4}$$

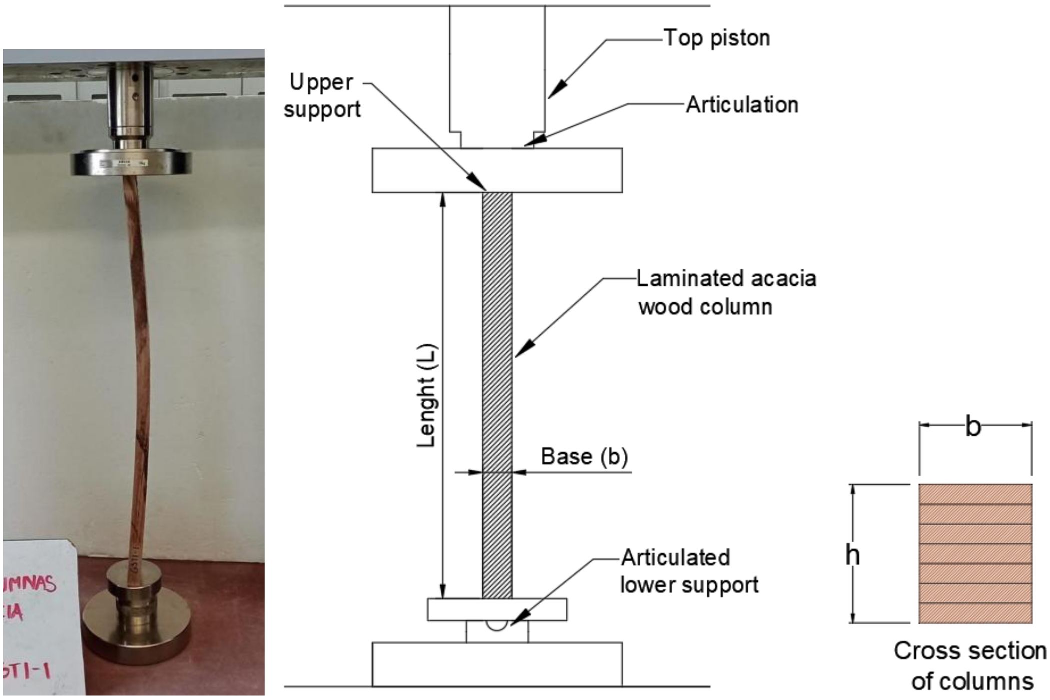
According to Gere *et al*. (2016), "kL/r is a dimensionless ratio called the slenderness ratio”. A column with a high slenderness ratio is a long column with a low critical stress, while a column with a low slenderness ratio is a short column with a high critical stress.$k = 1$ since there is a hinged support at the top and also a hinged support at the bottom, as shown in Figure 3(b).

2.2.2 Experimental part

2.2.2.1 Compression test of laminates

The test was conducted based on the parameters established by the NTC 5950 (2012) standard. A universal testing machine was used to apply axial load to the laminates, as shown in Figure 3(a). Figure 3(b) illustrates a schematic of the test, highlighting the support conditions and the cross-sectional shape of the tested specimens.

Figure 3. Experimental setup.



(a) Laboratory setup. (b) Test setup diagram.

Source: Authors' own work.

The support condition for the laminates consisted of hinged joints both at the bottom and top. The bottom support was ensured with a metal ball and socket support, while the top support featured an internal hinge in the load application piston.

The loading rate was determined by the product of a constant value of 0.00005 mm/s and the length of the laminate.

To determine the elasticity modulus, the specimens were instrumented with a digital extensometer which measured the shortening of the column due to the axial load application for determining the unit strain.

In addition, the calculation of normal stress was conducted to develop the graph for comparison among values and determine the slope of the curve that represents the modulus of elasticity (E).

The load was applied until the maximum load was reached for each column, at which point buckling or crush occurred depending on the length of the column tested. The applied axial load data was recorded for the 226 tested specimens, divided into three groups with different rectangular cross-sectional areas.

Within each group, there were different types of specimens based on their length. Tables 1, 2, and 3 provide a summary of the quantity of tested specimens and their dimensions measured using a caliper.

Table 1. Number of specimens and average measurements Group A

Group A				
Type	**Nº Specimens**	**b avg (mm)**	**h avg (mm)**	**L avg (mm)**
T1	10	19,14	78,25	639,69
T2	10	16,50	78,67	478,53
T3	10	17,23	78,10	318,27
T4	9	15,91	78,83	238,81
T5	10	15,77	78,90	157,27
T6	8	16,15	78,47	88,41
T7	8	16,66	78,57	75,50
T8	9	15,95	78,18	64,99
Total	74			

Source: Authors' own work

Table 2. Number of specimens and average measurements Group B

Grupo B				
Type	**Nº Specimens**	**b avg (mm)**	**h avg (mm)**	**L avg (mm)**
T1	11	20,64	87,13	707,39
T2	9	20,24	87,63	540,11
T3	8	20,17	87,07	356,41
T4	12	19,40	87,15	268,65
T5	10	17,73	87,28	178,54
T6	9	19,45	86,83	127,19
T7	9	19,65	86,74	77,03
T8	10	19,77	86,65	57,28
Total	78			

Source: Authors' own work

Table 3. Number of specimens and average measurements Group C

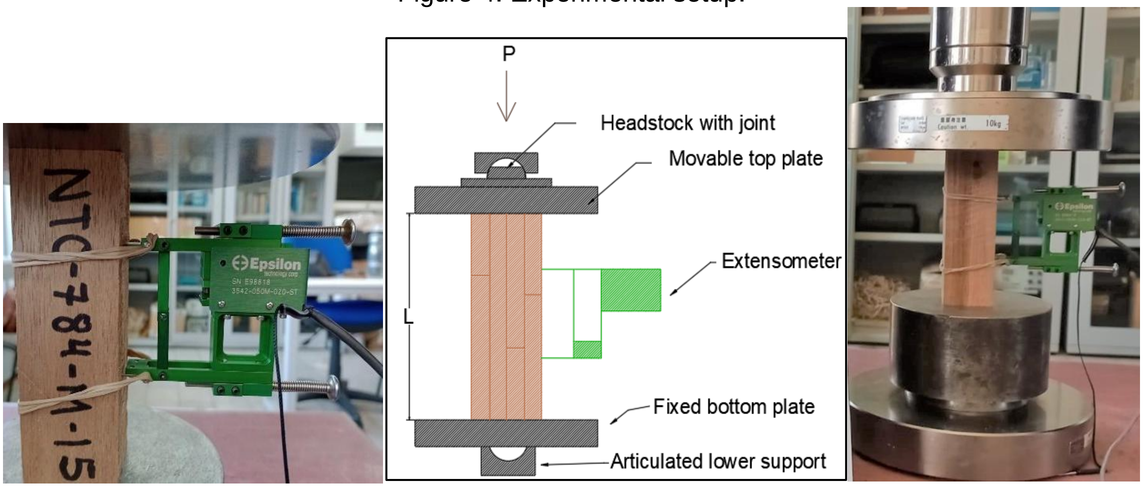
Group C				
Type	**Nº Specimens**	**b avg (mm)**	**h avg (mm)**	**L avg (mm)**
T1	12	13,43	67,15	558,61
T2	12	13,14	67,38	419,22
T3	12	13,57	67,33	275,89
T4	12	13,44	67,44	206,50
T5	12	13,01	67,2	122,17
T6	14	13,33	67,39	101,32
Total	74			

Source: Authors' own work

2.2.2.2 Determination of the modulus of elasticity

To calculate the critical buckling load in long columns, it is essential to know the modulus of elasticity of the material. For this purpose, axial compression tests were performed adapting the NTC 784 (1974) standard for solid small wood specimens, following the procedure described earlier and the assembly shown below in the figure 4 . Additionally, in these tests, an extensometer with a measurement distance of 5 cm was used to obtain data on the strain of the specimens, as shown in Figure 4(a). The experimental setup performed in the laboratory is depicted in Figure 4(b).

Figure 4. Experimental setup.



(a) Extensometer. (b) Test setup diagram. (c) Laboratory setup.
Source: Authors' own work.

For this test, 14 specimens were taken, and measurements of their cross-sectional area and length were recorded. The data is presented in Table 4. The support conditions for the specimens consisted of hinged joints at the bottom and top.

Table 4. Number of specimens and average measurements

	Average dimensions		
Specimen	**b avg (mm)**	**h avg (mm)**	**L avg (mm)**
1	38,20	38,89	121,15
2	40,98	39,39	121,18
3	41,21	38,42	121,22
4	39,07	41,84	121,07
5	38,16	41,37	121,22
6	38,60	39,00	121,26
7	38,85	40,09	121,24

8	39,76	38,11	120,94
9	40,33	38,61	121,15
10	40,33	38,61	121,15
11	38,57	41,29	121,08
12	38,28	39,68	121,08
13	38,40	40,97	121,30
14	38,56	40,14	121,11

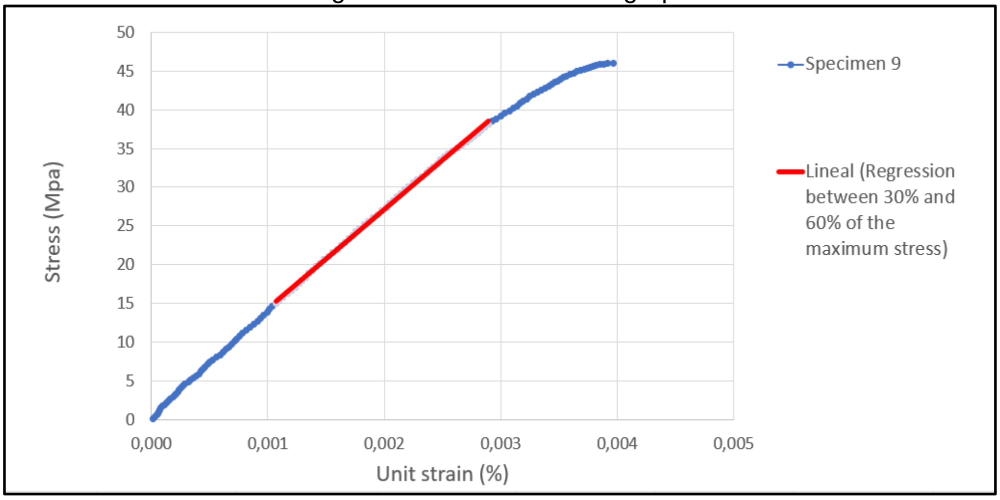
Source: Authors' own work.

3 RESULTS AND DISCUSSION

3.1 DETERMINATION OF THE MODULUS OF ELASTICITY

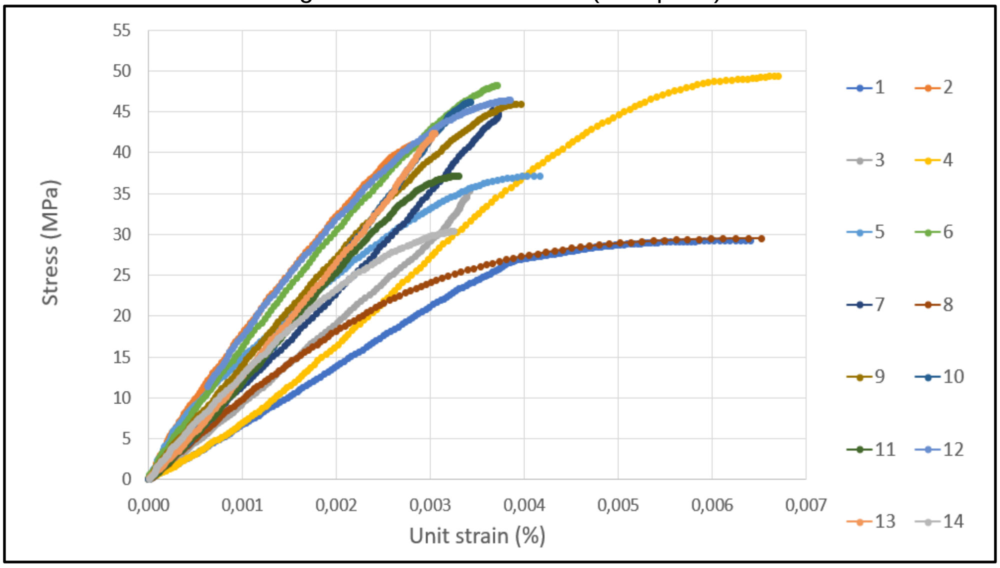
Once the test results were obtained, the normal stress $\sigma = P/A$ was calculated based on the maximum load that each specimen could withstand. Using σ and the strain calculated based on the displacement and reference length, stress vs. strain graphs were plotted. The modulus of elasticity (E) was determined as the slope of the linear regression between 30% and 60% of the maximum stress. Figure 5 shows the graph for specimen 9, where the range adopted for the determination of the modulus of elasticity is highlighted in red. The same procedure was carried out for all 14 specimens. Figure 6 displays stress vs. strain graphs for the 14 specimens.

Figure 5. Stress vs. Strain graph



Source: Authors' own work.

Figure 6. Stress vs. Strain (Complete)



Source: Authors' own work.

Once the modulus of elasticity for each specimen is obtained, the average and standard deviation are calculated. Outliers, marked with an (*) in Table 5, are removed, and the average, standard deviation, and coefficient of variation are recalculated.

Table 5. Modulus of Elasticity Results

Specimen	1*	2	3	4*	5	6	7
E (MPa)	6210	15948	9860	8493	11450	14436	12234

Specimen	8	9	10	11	12*	13*	14
E (MPa)	9964	18923	17766	18720	28806	22201	13506

Average E (MPa)	**Standard deviation**	**Coefficient of variation**
14281	3451	24%

Source: Authors' own work.

Finally, the obtained modulus of elasticity (E) is 14281 MPa.

3.2 COMPRESSION TEST OF LAMINATES

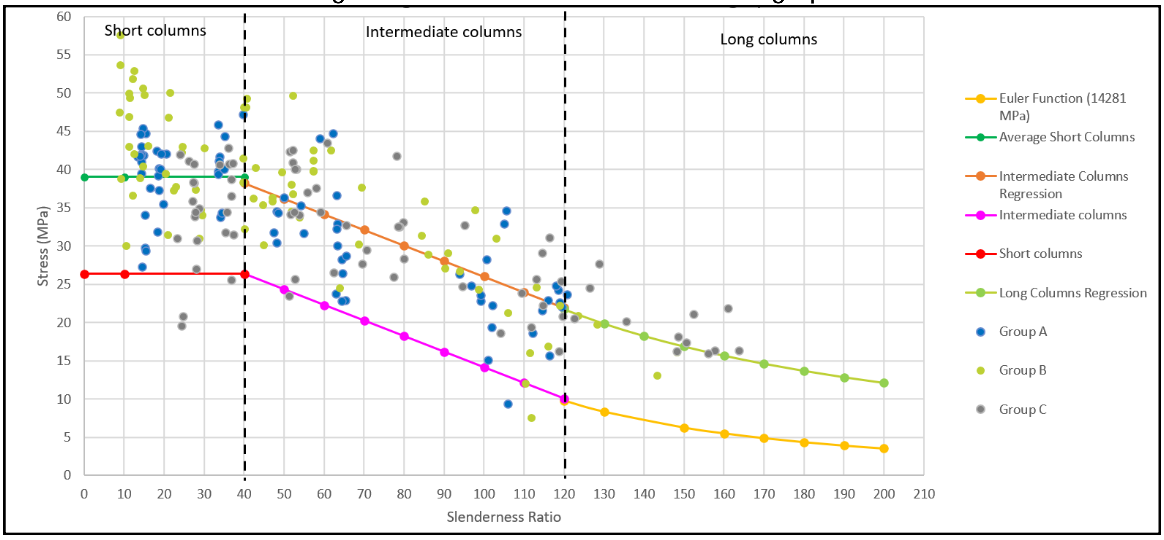
Once the modulus of elasticity was obtained, the slenderness ratio of each specimen was calculated to create the Stress vs. Slenderness Ratio graph. The data points form a cluster of points, as seen in Figure 7. Upon analyzing this cluster, three ranges were defined: short columns with a slenderness ratio between 0 and 40, intermediate columns with a slenderness ratio between 40 and 120, and long columns with a slenderness ratio greater than 120.

It was observed that the points with a slenderness ratio between 0 and 40 could be adjusted as a horizontal line representing the average values in this range. Beyond that point, the points started to decrease linearly.

In Figure 7, the results of the tests are shown, with blue points corresponding to the data obtained for Group A specimens, green points for Group B specimens, and grey points for Group C specimens. Based on the trend of the results, three zones were defined. For the first zone, the average of the points that formed the blue line in Figure 7 was calculated. For the second zone, a linear regression was performed on the points, represented by the orange line in Figure 7. Finally, for the third zone, a quadratic regression was performed, resulting in the green curve.

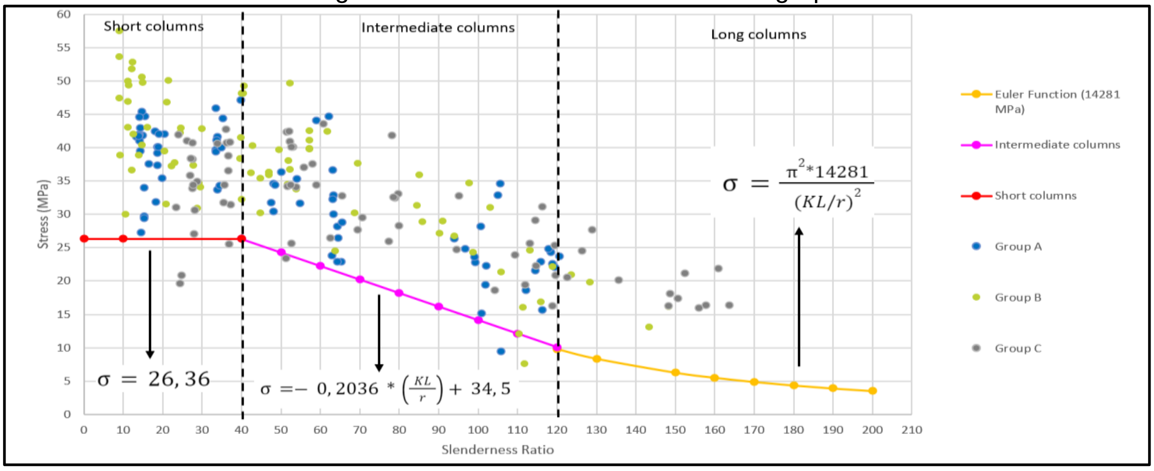
To define the curve of characteristic values, in all zones, it was ensured that 95% of the data points were above the defined lines (red and pink colors). The yellow line represents the Euler function calculated using the material's modulus of elasticity and the dimensions of each specimen (length, width b, and thickness h). Thus, short columns correspond to the first zone, intermediate columns correspond to the second zone, and long columns correspond to the Euler function zone, as shown in the graph in Figure 8. It is worth noting that 95% of the data obtained lies above the defined functions in each zone.

Figure 7. Stress vs. Slenderness Ratio graph.



Source: Authors' own work.

Figure 8. Stress vs. Slenderness Ratio graph



Source: Authors' own work.

3.3 PROPOSAL OF CHARACTERISTIC VALUES

The functions corresponding to each column type determined in this study are presented in Table 6, which also indicates the corresponding range that defines the column type.

Table 6. Proposal of Characteristic Values.

	Legend	Equation	Rage (KL/r)
Short column		$\sigma = 26{,}36$	$0 \leq KL/r \leq 40$
Intermediate column		$\sigma = -0{,}2036 * \left(\frac{KL}{r}\right) + 34{,}5$	$40 \leq KL/r \leq 120$
Long column		$\sigma = \frac{\pi^2 * 14281}{(KL/r)^2}$	$KL/r > 120$

Source: Authors' own work.

4 CONCLUSIONS

- Based on the test results, it was determined that the modulus of elasticity (E) in compression parallel to the grain of the laminated wood species *Acacia Mangium Willd* is 14281 MPa;
- Based on the results reported in Figure 8, the specimens were classified as short columns for slenderness ratios between 0 and 40, intermediate columns for slenderness ratios between 40 and 120, and long columns for slenderness ratios greater than 120;

- The relationship between stress and slenderness ratio in laminated Acacia wood members under axial compression load was defined using the equations proposed in Table 6. For this purpose, Euler's column theory and a piecewise function based on laboratory test results and the theoretical concepts of constant function, linear regression, and exponential regression were employed;
- The relationship between compression maximum stress and slenderness ratio was determined;
- Functions to determine the characteristic stress values in compressed *Acacia Mangium Willd* laminated elements in function of slenderness ratio were defined and shown in Table 6;
- According to the comparison between the average theoretical critical buckling stress (7.20 MPa) and the average experimental normal stress (19.41 MPa), it was determined that the difference between the two values is 37%;
- The results obtained in this study provide structural engineering with a tool for the analysis and design of laminated Acacia wood members under compression, offering applications for civil structures using this renewable non-conventional material.

4.1 LIMITATIONS OF THE RESEARCH AND RECOMMENDATIONS FOR FUTURE STUDIES

The theoretical results obtained are limited to the analysis of small laminated Acacia wood members. Therefore, it is recommended that future research conduct tests with full-scale structural elements.

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