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A finite element approach for modeling dynamic effects of prestressing in concrete beams

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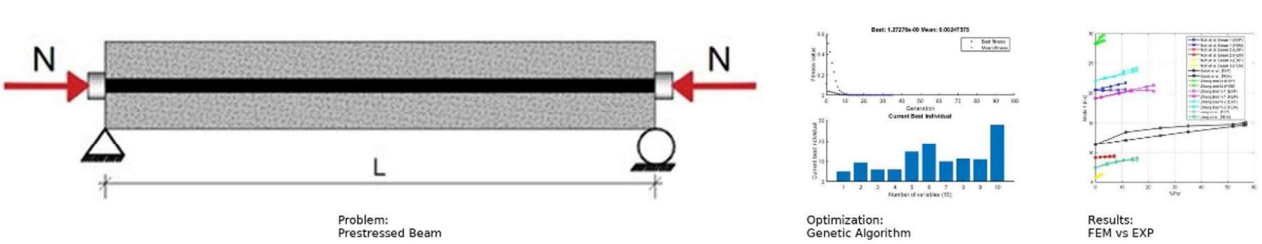
Abstract

Prestressed concrete structures are widely utilized in civil engineering due to their superior structural efficiency and load-carrying capacity. However, long-term losses in prestressing forces can compromise both structural integrity and service performance. This study develops a finite element model to assess the impact of prestressing force on the natural frequencies of simply supported beams. The model accounts for the combined effects of axial compression and the physical characteristics of the tendon, including variations in eccentricity and geometric profile. Validation against experimental results from the literature demonstrates strong correlation, particularly for the first and second vibration modes, with average discrepancies of 2.48 and 2.68%, respectively. Numerical simulations also reveal that the influence of prestressing is most pronounced during the early stages of loading, with frequency shifts tending to stabilize as the applied load nears critical levels. The proposed framework offers a valuable complementary approach for the indirect estimation of losses in prestressing forces.

Keywords

Prestressed concrete, Modal analysis, Finite Element Method, Structural dynamics, Monitoring

Graphical Abstract



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List of Symbols

A	Cross-sectional area of the beam (m^2)
Ap	Cross-sectional area of the prestressing tendon (m^2)
E	Elastic modulus of the beam material (Pa)
Ep	Elastic modulus of the prestressing tendon (Pa)
Ec	Elastic modulus of the concrete (Pa)
EI	Flexural rigidity (N·m^2)
f_n	Natural frequency at mode n (Hz)
f_{ck}	Characteristic compressive strength of concrete (MPa)
I	Moment of inertia of the cross section (m^4)
L	Length of the beam (m)
M	Bending moment (N·m)
m	Mass per unit length (kg/m)
Me	Mass matrix
N	Axial compressive force (N)
$P, P(x)$	Prestressing force as a function of position (N)
Pcr	Critical buckling load (N)
Pn	Neutral prestressing force (N)
Pp	Applied prestressing force (N)
$q(x,t)$	Transverse distributed load (N/m)
ρ	Material density (kg/m^2)
ρ_t	Total (combined) density of the system (kg/m^3)
ω_n	Angular frequency at mode n (rad/s)
$u(x,t)$	Transverse displacement at position x and time t
$\phi_i(x)$	Shape function associated with node i
α_p	Modular ratio (E_p/E_c)
e	Eccentricity of the prestressing tendon (m)
ΔP_p	Additional force to restore tendon strain (N)
$Kg1$	Geometric stiffness matrix due to axial prestress
$Kg2$	Geometric stiffness matrix due to tendon profile effect
Ke	Elastic stiffness matrix
V	Shear force (N)

1 INTRODUCTION

Since its introduction in the 1930s, prestressed concrete has revolutionized structural engineering, becoming one of the most versatile and efficient construction methods for large-scale structures (Zhao and Fang 2012). This technique involves inducing pre-compressive stresses in concrete through tensioned steel tendons, thus counteracting the tensile stresses resulting from external loads (Shi, He, and Yan 2014). This fundamental principle offers significant advantages over conventional reinforced concrete, which suffers from stress from tension. In that sense, its application extends from precast elements, such as beams and slabs, to complex structures, including long-span bridges, high-rise buildings, and reservoirs (Breccolotti 2018). Among the main benefits of prestressing are effective crack control, reduced long-term deformations, and increased load-bearing capacity of structural elements (H.-T. Kim, Seo, and Yang 2019). In addition, it enables the development of more slender and lightweight structures, with significant material savings, without compromising safety or durability. In addition, prestressing contributes to the rationalization of construction, allowing longer spans and more ambitious architectural solutions (Choudhary and Sanghai 2019). From a sustainability perspective, prestressed and precast systems contribute to increased energy efficiency and reduced embodied energy (Fleischman and Seeber 2016). Further performance improvements can be achieved through the application of optimization techniques in the design of prestressed elements, as demonstrated in recent studies (Amir and Shakour 2018; Jahjouh and Erhan 2022; Shakur, Shaked, and Amir 2024).

One of the critical challenges in the use of prestressed concrete is the progressive loss of prestressing force over time. Phenomena such as creep and shrinkage of concrete, steel relaxation, reinforcement corrosion, and thermal variations can significantly reduce system efficiency (Zanini, Faleschini, and Pellegrino 2022). In this context, the non-destructive assessment of the prestress state has become critical for the maintenance and safety of prestressed structures. To this respect, techniques based on dynamic parameters, particularly the analysis of natural vibration frequencies, have gained attention as promising alternatives for estimating prestress losses (Ribeiro et al. 2021; L. Wang, Li, and Xu 2020). Natural frequencies are highly sensitive to changes in global stiffness, which in turn is influenced by variations in the prestressing force (Saiidi, Douglas, and Feng 1994; Zhou, Li, and Zhang 2020). An example is the work by Kovalovs (Kovalovs et al. 2017), who proposes a method based on dynamic vibration monitoring, implemented through finite element analysis, and validated by comparison with experimental data. Recent advances, such as machine learning techniques (Ribeiro et al. 2021), enhanced modal identification methods (L. Wang, Li, and Xu 2020), and smart monitoring systems (Liu et al. 2022; Bao et al. 2021), have further expanded the potential for dynamic-based prestress evaluation. It should be noted that monitoring of dynamic parameters has also been used as a tool to verify the safety and integrity of structures, since these parameters are directly related to stiffness, mass, boundary conditions, and, consequently, to structural deterioration and/or cracking (Saidin et al. 2022). It is worth mentioning that Abdel-Jaber and Glisic (2019) present a review on monitoring of prestressing forces, which includes vibration methods, impedance-based methods, elasto-magnetic methods, acoustoelastic methods, and strain-based methods. In addition, Bonopera et al. (Marco Bonopera, Chang, and Lee 2020) carried out a state-of-the-art on the subject with application to prestressed concrete girders and static-based methods.

However, it is important to note that the understanding on how prestressed force affects the dynamic behavior – and consequently on natural frequencies – of prestressed beams is yet a topic of disagreement among various authors in the literature (Noble et al. 2014). In theoretical studies grounded in classical mechanics, the vibration analysis of beams subjected to axial loads is a well-established topic in technical publications. Several authors (Shaker 1975; Bokai 1988; Mamandi, Kargarnovin, and Farsi 2012) have assumed that the prestressing force in the tendon is equivalent to an axial compressive load. In this approach, natural frequencies tend to decrease with increasing compressive force due to the so-called "compression softening" effect, as predicted in Euler–Bernoulli beams (M. Bonopera et al. 2019). On the other hand, Hamed and Frostig (2006) argue that the natural frequencies of the structural elements are not affected by the prestressing force. This argument is based on a nonlinear kinematic model which concludes that the final dynamical equation of motion for a prestressed beam is independent of the magnitude of the prestressed force. An alternative methodology that has been used is the development of physical models in laboratory settings. Various experimental studies conducted on prestressed concrete beams, such as those of references (Saiidi, Douglas, and Feng 1994; Noh et al. 2015; Y. Zhang and Li 2007; Y. T. Zhang, Zheng, and Li 2011; Jang et al. 2011), have shown an increasing trend in natural frequencies with higher levels of prestressed force, contradicting theoretical studies that claim otherwise. Moreover, Risan and Zaki (2022) elaborated a meta-analysis on the subject, finding that the statistical results support the feasibility of using the change in the natural frequency for the fundamental mode in the process of determining the prestress losses.

Given the inherent complexity of modeling the dynamic behavior of prestressed structures, this study proposes a finite element modeling approach for analyzing prestressed concrete beams. This approach considers both experimental results from literature and a metaheuristic algorithm for model updating. The main contributions of this paper are as follows:

The development of a finite element modeling approach that is capable of simultaneously considering the effects of axial compression and the structural contribution of the physical presence of tendons to the global stiffness of prestressed beams. The proposed model allows for the analysis of beams with curved tendons and eccentricities, which are typical of real-world structures such as bridges and viaducts and are often neglected in previous studies.

A comprehensive review of the main theoretical and experimental studies available in the literature to support the formulation and validation of the proposed model. This validation is conducted using a genetic algorithm, where the objective function minimizes the error between experimental and analytical natural frequencies of the beams prior to the application of prestressing.

The quantification of the relationship between the natural frequencies of the beam and the level of prestressing load. Visual representations and tables are provided to facilitate the interpretation of the results.

2 LITERATURE BACKGROUND

Despite significant research efforts, the influence of prestressing force on the natural frequencies of concrete beams remains a controversial topic. Discrepancies between theoretical predictions and experimental observations have been consistently reported, leading to the development of a variety of analytical, numerical, and experimental studies to better understand this phenomenon. A brief review of the main contributions on this topic is presented below, including theoretical investigations and experimental tests.

2.1 Theoretical Research

The presence of an axial force in a homogeneous beam alters its natural vibration frequencies, a phenomenon resulting from the effect known as "compression softening," which is described by the classical Euler-Bernoulli model (Noh et al. 2015). For a simply supported prismatic beam, as illustrated in Figure 1, the equilibrium equation, considering the boundary conditions, leads to the following equation for the natural frequency:

$$\omega_n^2 = -\left(\frac{n\pi}{L}\right)^2 \frac{N}{m} + \left(\frac{n\pi}{L}\right)^4 \frac{EI}{m} \tag{1}$$

where ω_n is the natural frequency, n is the mode number, L is the length of the beam, N is the axial compressive force, E is the modulus of elasticity, I is the moment of inertia and m is the mass per unit length. As can be seen, the natural frequencies depend on the magnitude of the prestressed load. However, such an expression cannot be straightforwardly obtained for other conditions. For more complex structures, it is necessary to employ the finite element method (FEM) to describe the relationship between the characteristics of the prestressed beam and its dynamic characteristics. There are several aspects to consider, including the boundary conditions, the prestressed force magnitude, geometric non-linear behavior, and the load eccentricity.

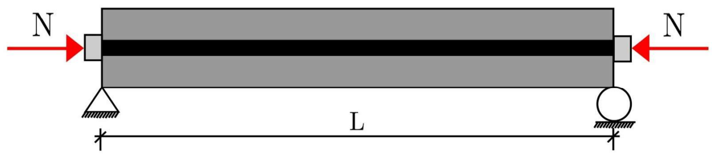

Figure 1 Model of a prestressed beam

In the literature, several studies on the subject have focused on the impact of the prestressed load on the dynamics characteristics of beam-type concrete elements. The first studies in the decades of the 70s (Shaker 1975) and 80's (Bokaian 1988; Raju and Rao 1986) pointed out the effect of increasing the axial prestressed load (prestress) on the decrement of natural frequencies of beams with different boundary conditions, especially for the fundamental and lower modes. Law and Lu (2005) analyzed the time domain response of an Euler-Bernoulli prestressed concrete beam subjected to axial compression, concluding that the lower natural frequencies were most affected by axial compression due to the compression softening effect. In addition, references (Miyamoto et al. 2000; J.-T. Kim, Ryu, and Yun 2003; Jaiswal 2008; Vibhute et al. 2016) found numerically that all natural frequencies in beams exhibited a decreasing trend. Bai-Jian, Fei, and Song (2018) studied the influence of the prestressing force, eccentricity, and cross-sectional area on simply supported steel beams and concluded that prestressing does not affect the even-order frequencies, but the odd-order frequencies decrease with increasing prestressing force. Furthermore, frequencies increase with eccentricity and cross-sectional area. In contrast, Kim proposed an alternative model that showed an increase in natural frequency with the prestressing force, particularly in the first modes, which was confirmed by experimental tests performed by Saiidi, Douglas, and Feng (1994). Hamed and Frostig (2006) in a non-linear geometric model, proposed that the prestressing force does not affect the natural frequency, justified by the ideal condition of the adhesion of the cable to the concrete. However, this model was not experimentally verified. Gan et al. (2019) conducted numerical simulations using the Finite Element Method to investigate the effect of the prestressing force on natural frequencies, considering microcracks caused by concrete shrinkage. The results showed that the prestressing force tends to close these cracks, increasing rigidity and consequently increasing natural frequency.

As can be seen in the previous paragraphs, theoretical studies suggest that the prestressing force, when considered as an external axial load, decreases the natural frequencies. However, some experimental studies suggest that prestressed beams do not have a significant effect on the natural frequencies of prestressed beams. The discrepancy between theories and experimental results suggests the need for more in-depth investigations, particularly in the experimental field, to confirm the observed trends. Additionally, the physical presence of the prestressed cable may generate internal forces that increase the beam's rigidity, resulting in a trend contrary to theoretical predictions.

2.2 Experimental Research

From an experimental point of view, several studies have tried to reproduce in the laboratory the real condition of the prestressed elements in the field to more accurately determine the relationship between presstress forces and the dynamic parameters of the beam. Hop (1991) and Saiidi, Douglas, and Feng (1994) demonstrated that, for beams with straight concentric cables, the natural frequency increases with increasing prestressing force, contradicting theoretical predictions that treat prestressing as an axial compressive force. J.-T. Kim, Ryu, and Yun (2003) proposed an analytical model, which, compared to the results of Saiidi, Douglas, and Feng (1994), also found a tendency for the natural frequency to increase. However, some explanations have been proposed to justify these discrepancies. Saiidi, Douglas, and Feng (1994) suggested that the differences between the experimental values and those predicted by classical theories could be attributed to the presence of microcracks, which close when the prestressing force is applied, thus increasing the stiffness of the beam and, consequently, its natural frequency. This hypothesis is based on Equation (2), which proposes an empirical formula for the effective stiffness $(EI)_e$, given by:

$$(EI)_e = \left(1 + 1.75\frac{N}{f_c'}\right) EI_g \tag{2}$$

where N is the applied axial force, f_c is the compressive strength of the concrete at 28 days, and EI_g is the stiffness of the beam without prestressing. Although this formula is widely used, Bai-Jian et al. (2018) highlight that it lacks complete theoretical support.

Subsequent studies, such as those of Dall'Asta and Dezi (1996) and Deak (1996), followed similar approaches but with contradictory results. Although Dall'Asta and Dezi concluded that the natural frequency was not significantly affected by the prestressing force, Deak attributed the frequency variation to the formation and closure of microcracks due to the shrinkage of the concrete prior to the application of prestressing.Jain and Goel (1996) discussed that the prestressing force does not affect the natural frequency because the cable becomes an integral part of the system. Other studies, such as those of Miyamoto et al. (2000), observed a decrease in natural frequency as the prestressing force increased in beams with external prestressing, although this trend was not observed in models with more eccentric cables. Other authors, such as Lu and Law (2006), proposed a numerical model based on Euler-Bernoulli theory and observed that for centralized straight cables, the natural frequencies increased with increasing prestressing force, again contradicting theoretical predictions. Y. Zhang and Li (2007) found the same trend in their experiments. Furthermore, Jang et al. (2011) validated a numerical model with Equation ([Eq2]) and observed a progressive increase in frequency with the increase in prestressing force.

In contrast to these results, some authors, such as (T. Wang, Huang, and Wang 2013), used the Rayleigh method to study the natural frequency in prestressed concrete beams with straight and parabolic cables. For parabolic cables, the frequency decreased with the prestressing force, while for straight cables, the frequency was unaffected. Shi, He, and Yan (2014) observed that the lower order frequencies increased with increasing prestressing force, which corroborates the findings of Saiidi, Douglas, and Feng (1994). More recent studies, such as those by Noble et al. (2016), found no clear relationship between prestressing force and natural frequency, observing that the trend was inconsistent. However, Noh et al. (2015) performed tests on models with different eccentricities and observed that for models with eccentric cables, the natural frequency increased with increasing prestressing force. Furthermore, Li and Zhang (2016) conducted a study that combined experimental tests and numerical simulation, concluding that the relationship between prestressing force and natural frequency cannot be explained by traditional theory but by a more complex model that considers cable eccentricity. Finally, M. Bonopera et al. (2019) performed laboratory tests on beams with a parabolic cable profile and found that natural frequency was not affected by the prestressing force. These results reinforce the idea that there is no consensus on the influence of the force, eccentricity, and cable profile of the prestressed concrete beams.

In summary, most studies suggest that an increase in prestressing force tends to increase the natural frequency of prestressed concrete beams, although considerations of the presence of microcracks and cable eccentricity may influence this behavior. However, as highlighted by Noh et al. (2015) and others, the relationship between these parameters still needs to be better understood.

3 PROPOSED FINITE ELEMENT MODELING

This research proposes a one-dimensional finite element modeling strategy for computing the modal properties of prestressed beams, which accounts for two simultaneous conditions: the axial compression induced by the prestressed force and the internal presence of the tendons, which generate resistance forces that increase the flexural stiffness. It is worth mentioning that the effect caused by the mentioned issues on the dynamic behavior of the prestressed beam is opposite. On the one hand, axial compression tends to reduce the natural frequencies of the beam, while the presence of the tendon contributes to their increase. Thus, the resulting effect will lie at an intermediate point between opposite trends in the dynamic parameters. Identifying the prevailing trend is essential for assessing the sensitivity of dynamic monitoring techniques to prestress losses.

Initially, the effect of axial compression due to the prestressing force will be analyzed, assuming a set of linear variations of the internal axial force along the beam's length. To this end, an equilibrium equation for a beam subjected to axial load will be formulated and numerically solved using the Galerkin method (Cook et al. 2002). Subsequently, the effect of the tendon will be modeled as an equivalent distributed load, corresponding to the resistance moment generated by the prestressing force in the cross section. The stiffness matrix will be formulated on the traditional basis of the variational approach, as described in (Zienkiewicz, Taylor, and Zhu 2013). It is highlighted that the model is computationally efficient as it was kept depending only on the main axis of the beam.