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A genetic algorithm-based methodology for the structural optimization of voronoi flat roofs

Juan Sebastián Fontalvo García [a], María Juliana Moya Olivares [a], Alfonso Gomez [b], Jesús Daniel Villalba Morales [a,*]

[a] Pontificia Universidad Javeriana, Facultad de Ingeniería, Departamento de Ingeniería Civil, Bogotá, Colombia
[b] Pontificia Universidad Javeriana, Facultad de Arquitectura y Diseño, Departamento de Arquitectura, Bogotá, Colombia

HIGHLIGHTS

- A special GA-methodology optimizes the structural topology of a Voronoi flat roof.
- An implicit redundant representation handles the unknown number of Voronoi seeds.
- Each roof shape requires of own GA parameters for guarantying better results.
- The search space shows to be multi-modal for the examples analyzed.

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ABSTRACT

Voronoi tessellations are a mathematical concept that appears in many examples in nature, such as the skin of giraffes, dry soil, and vegetable cells. In the context of biomimicry, these tessellations have been used to build impressive structures worldwide that are both aesthetically pleasing and structurally efficient. This paper proposes a methodology based on genetic algorithms (GA) to determine the structural topology of Voronoi flat roofs with tubular steel cross sections and a given boundary. The design variables correspond to the number and position of the Voronoi centers that form the tessellations within the roof, as well as the dimensions of the structural elements. This representation of the design variables creates an unstructured optimization problem. Such characteristic is addressed by an implicit redundant representation of possible solutions, which generates chromosomes with varying numbers of variables. The objective function relates to the weight of the roof, considering constraints raised in technical and constructive issues. The methodology was applied to four different roof boundaries: triangular, pentagonal, square, and rhombic. In general, the results provide optimal aesthetic solutions with a few Voronoi tessellations, based on the algorithm configuration and the multimodal nature of the search space. Convergence analysis indicates the possibility of the algorithm getting stuck in an optimum local and shows the progressive reduction of Voronoi centers. Lastly, it is observed that the maximum displacement constraint leads to the shape of the optimal roof.

1. Introduction

In the modern world, the human capacity to integrate different fields of knowledge has enabled the construction of such unique civil infrastructure projects. By leveraging expertise in engineering, architecture, and computation, it becomes possible to explore amazing possibilities while ensuring the development of structural projects with a sustainable perspective. This holistic approach has revolutionized the process of structural conception, overstimulated by the enormous opportunities of technological advances. Thus, complex structures can be envisioned, designed, and constructed by taking advantage of new engineered materials, new developments in manufacturing, a deeper comprehension of structural principles, as well as the utilization of artificial intelligence and computational power for analysis. For example, Shahbazi et al. [1] use parametric design in Grasshopper to create a freeform space structure with the support of machine learning. Another aspect concerns the

* Corresponding author.
E-mail address: jesus.villalba@javeriana.edu.co (J.D. Villalba Morales).

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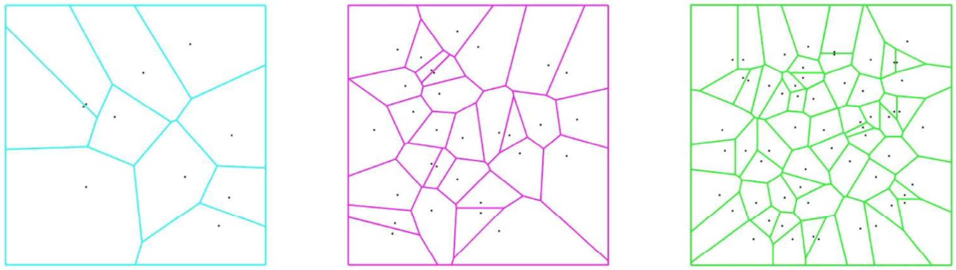

Fig. 1. Examples of Voronoi tessellations for a rectangular boundary.

relationship architects and structural engineers build as partners in the structural conception process. The success of a specific project depends on both professionals ́ ability to understand and respect each other [2].

Structural conception involves a blend of artistic inspiration, imaginative thinking, and a solid foundation in physical and mathematical principles. This amalgamation creates unique structures whose resistance and aesthetics stem from defining their shape and detailed elements. Notably, structural aesthetics is a primary concern for structural engineers due to its impact on aspects such as comfort and happiness [3], and the structure ́s influence the architectural space [4]. Thomas [5] presents a series of cases proving the importance of aesthetics in structural engineering, including the analysis of impressive structures by masters such as Oscar Niemeyer and Zaha Hadid. Furthermore, various authors delve into the correlation between aesthetics and structural efficiency, as presented for different structures in references [6–8]. The implementation of topology optimization concepts, as illustrated in [8], can further enhance structural efficiency. It is worth mentioning that sustainable principles must be considered for any structural project, with a view to minimizing environmental impact wherever possible.

Both structural engineers and architects have followed the principles of biomimicry as a source of inspiration for designing unique structures, such as the Sphinx in Giza in ancient Egypt [9] or L'hemisféric in Valencia, Spain [10]. Biomimicry (bios – life and mimesis – imitate) offers an adaptive approach to addressing life and economic challenges by emulating natural phenomena and processes [11]. Nature has inspired scientists, artists, and experts in various fields [12]. Over the last decades, scientists have mainly utilized biomimicry to provide more precise responses to natural forces because of its functional precision in the design of structures. This concept is essential for the discussion due to the interdisciplinary essence of this work, and biomimetics is a highly multidisciplinary field that involves understanding the biological functions, structures, and principles of various objects found in nature [13], [14]. One alternative to integrating natural principles into structural conception is employing Voronoi tessellations, which can be found in many natural structures, such as the wings of fireflies and turtle shells. Voronoi-based structural and architectonic configurations have demonstrated remarkable efficiency, inspiring numerous prominent projects like the Beijing National Aquatics Center. For further details, Dixit and Stefanska [15] present a review featuring a series of real cases of bio-inspired structures from the 20th and 21st centuries.

Bioinspired optimization algorithms have been utilized to generate cost-effective structural designs through one of the following strategies: optimization of size, material, shape, and topology. Examples of such applications range from concrete frame buildings [16], steel frames [17], trusses [18], transmission line towers [19], water tanks [20], and among others. Flat roofs are another interesting structural system susceptible to optimization, designed to protect people from possible rain, wind, and sun exposure. Successful examples of metaheuristic-based roof optimization can be found in references [21–28]. To the knowledge of the authors, the first reported use of optimization on the Voronoi roof was documented in [29], as Voronoi configurations can provide an adequate structural solution. Others VT application can be found in beams [30], tube structure of tall buildings [31], and quasi-3D porous structures [32]. The selection of an appropriate metaheuristic and its configuration plays a crucial role, but there is no agreement on this topic; each problem has its own possibilities.

This research proposes a methodology for the structural design of Voronoi flat roofs based on genetic algorithms. This approach aims to identify the optimal Voronoi topology and the size of the structural elements, minimizing the weight of the roof while adhering to technical constraints. A specific solution representation was devised to accommodate a changing number of variables among individuals within the population. Some knowledge-based heuristics improve the convergence of the algorithm, which uses a computational routine to generate Voronoi patterns. The key contributions of this research can be summarized as follows:

- Introduction of Voronoi tessellations for defining flat roof structural systems, combining structural efficiency with aesthetic appeal.
- Development of a tailored solution representation for the genetic algorithm to overcome the unstructured nature of topological roof optimization.
- Evaluation of genetic algorithm parameters to gauge their impact on the performance of the proposed methodology.
- Simplification of the genetic algorithm approach through an illustrative example featuring the optimization of a beam-type structure.

2. Voronoi tessellations

A Voronoi diagram partitions a planar region into smaller areas

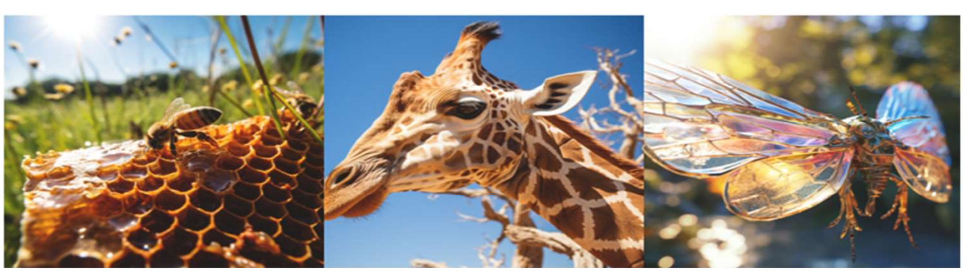

Fig. 2. Examples of Voronoi tessellations in nature. Imagined by the Midjourney IA.

Fig. 3. Examples of structures with Voronoi configurations.

Fig. 4. Voronoi structures generated by the Midjourney AI.

based on the position of a predetermined set of points known as seeds, which are distributed across the original area. These resultant regions are referred to as Voronoi Tessellations (VTs). The new vertices shaping the boundaries are calculated considering that they have the shortest distance from the seeds [33]. Fig. 1 presents cases of VTs for an original rectangular area, each created using a different number of seeds within the same area. The division of the rectangular area in VTs is pleasing and gives a distinct impression of regular distribution. However, generating these tessellations requires the definition of elaborate mathematical procedures, such as the one in [34]. There are other methods to calculate the VTs; for example, using the "frequency mesh grid", which helps create the tessellation in the regions where they are needed [35].The computational effectiveness of generating Voronoi cells plays a crucial role in optimization-based problems, as numerous potential configurations must be tested.

It is worth mentioning that VTs are present in the world around us. Nature harnesses the potential of VTs to cover surfaces such as the skin of giraffes, dry soil, vegetable cells, dragonfly wings, human bones, and honeycombs, among others (see Fig. 2). In civil engineering, Lin et al. [36] used VT for modeling heterogeneous rocks with correlated grain structure. Additionally, Voronoi patterns can be used to enhance an optimizer's ability to explore the search space while preserving population diversity [37]. In the case of analysis of human bone tissues, several applications for generating numerical models based on Voronoi patterns are observed to ease the computational process. For example, Du et al. [38] used VTs to produce an irregular porous scaffold in the industrial development of bone reconstruction. Understanding nature's wisdom about VTs could provide valuable insights for architects and engineers in envisioning innovative new infrastructure projects. At first glance, one cannot fathom VT's vast possibilities for structural and architectonic conception, but the following paragraph will give us some applications.

In the 21st century, computational systems have played a pivotal role in shaping the forms of contemporary architecture. In this context, VTs can be used to find efficiency and aesthetic forms in functional designs, as shown in Fig. 3. Some research in the literature discusses the mechanical behavior of real Voronoi structures. For instance, Agkathidis and Brown [39] document the construction process of the canopy structure tree designed for the Marriot Hotel in Frankfurt (Germany). Wang et al. [40] presented the generalities of the Beijing National Aquatics Centers for exploiting VTs presented in a water bubble. Furthermore, artificial intelligence tools have opened up new possibilities for design innovation. In this paper, Midjourney AI was employed to visualize potential applications of VTs in a structure within a natural garden setting, as depicted in Fig. 4.

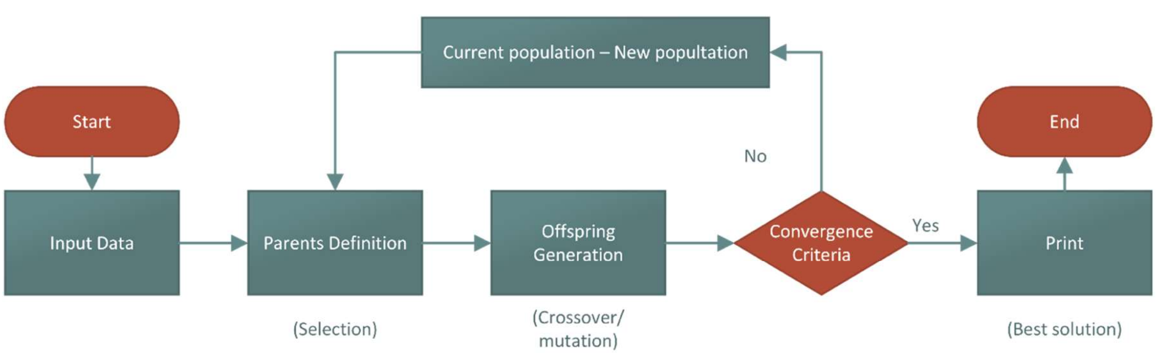

Fig. 5. General procedure for the basic genetic algorithm.

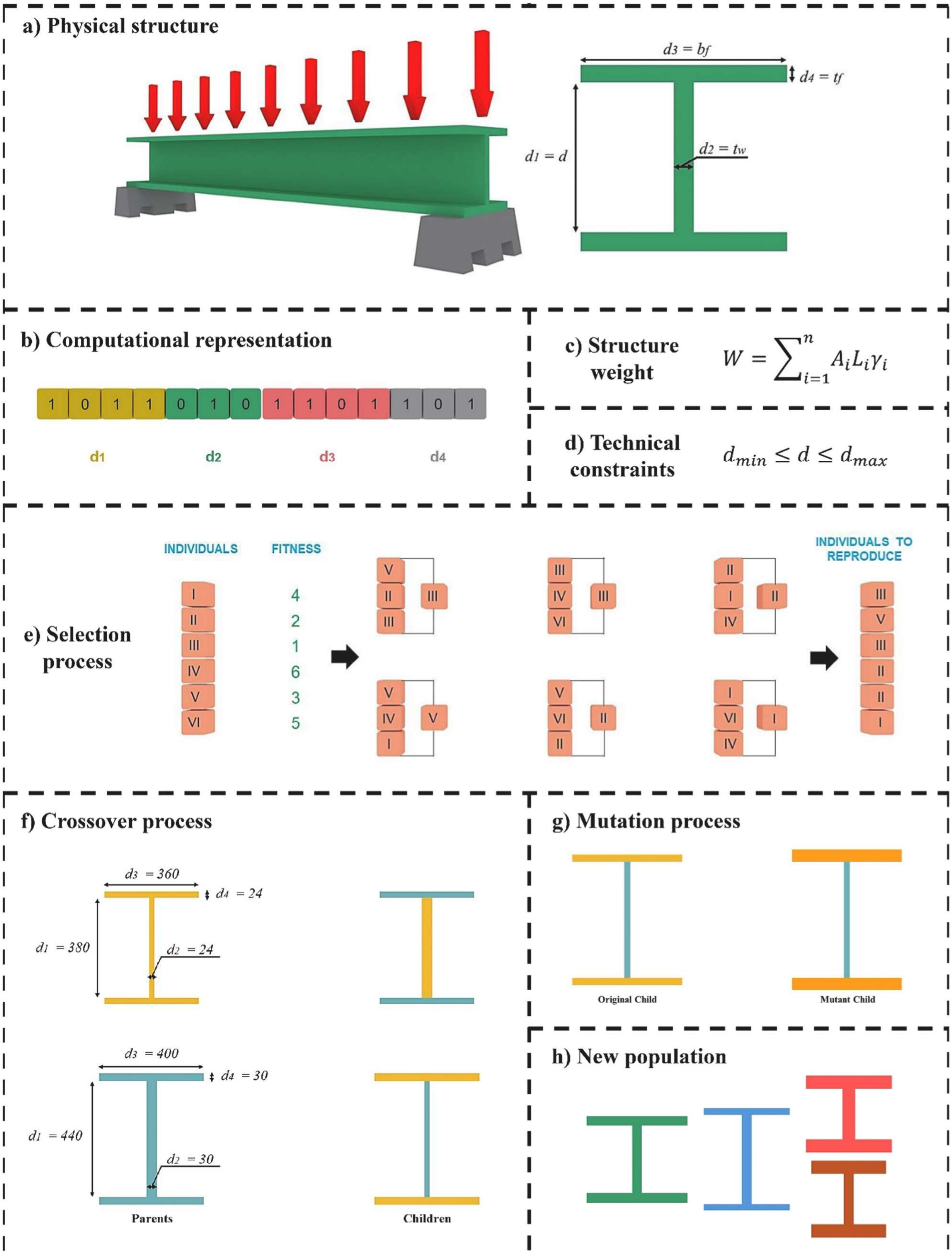

Fig. 6. GA-based proposed methodology for the optimization of a simply supported steel beam.

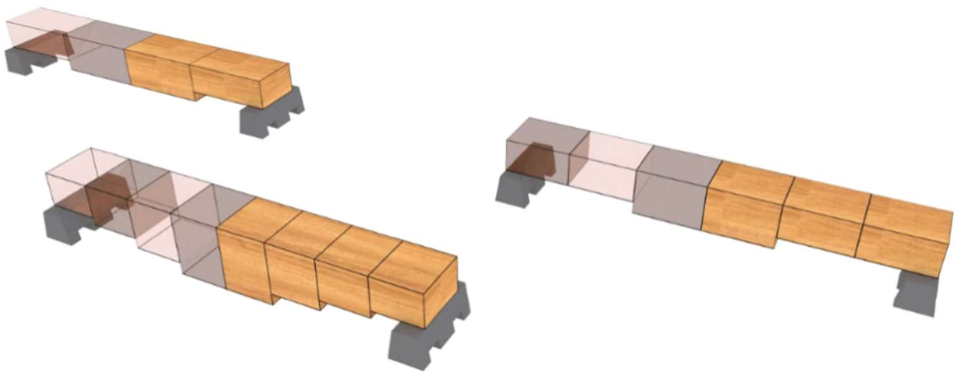

Fig. 7. Examples of wood beams with different quantities of blocks and cross-section heights.

3. Genetic algorithms

3.1. The binary coded Genetic Algorithm for structured optimization problem

Genetic algorithms (GAs) are recognized stochastic search techniques inspired by natural selection and the survival of the strongest individuals in a given environment [41]. The basic GA solves optimization problems whose design variables are already defined, utilizing binary code to represent each variable (gene). The computational concept involves evolving a population of potential solutions (individuals) through evolution-based mathematical operations. Three random number-based operators act on the current population to generate offspring: selection, crossover, and mutation. Fig. 5 presents the several iterative steps of the basic GA. While the best solution is expected to improve with each iteration, it is possible to lose it between iterations due to the genetic operators. The user must define several aspects before GA execution, including the stopping criterion and GA parameters.

Fig. 6 provides a brief overview of the GA operators within the context of a structural problem aiming to facilitate comprehension of the proposed methodology. The algorithm aims to minimize the weight of a simply supported steel beam structure (objective function), as depicted in Fig. 6a, by adjusting the dimensions of an I-shaped cross section (design variables) and considering its response to a transverse uniform load (technical constraints). The design variables encompass the height (d_1) and thickness (d_2) of the web, as well as the width (d_3) and thickness (d_4) of the flange (see Fig. 6b). It is assumed that the cross-section is symmetric about the centroidal axis of the beam. The design parameters consist of the properties of the steel (modulus of elasticity, density, and yield strength), the length of the beam, the magnitude of the uniform load, and the maximum deflection.

Coding a possible solution implies transforming the real numbers into binary ones. Thus, a binary representation requires a length of 14 bits for the four design variables, see Fig. 6b. Each variable (or 'gene') corresponds to a position in a list of allowable sizes. The search process starts with the generation of a random or heuristic initial population (set of solutions). For each variable, values are assigned within a user-defined allowable range. All individuals within the population will possess a computational representation akin to that presented in Fig. 6b.

The weight of the beam serves as the quality measure (fitness), as depicted in Fig. 6c, considering that the lightest beam is the best. Three technical constraints ensure appropriate deflection and limit normal and shear stress. Additionally, dimensions must fall within specified allowable ranges, as illustrated in Fig. 6d. Based on the dimensions of the cross sections, a solution could not meet all the design requirements. Various approaches can address these constraints, but a straightforward strategy penalizes fitness based on the number of unmet constraints.

The tournament is an effective and easy-to-implement method for selecting parents based on the solutions' quality. For instance, in Fig. 6e, the fitness ranking of a population of six individuals (beam configurations) is depicted. The method involves randomly selecting six player trios and subsequently choosing the best of each trio based on fitness ranking (middle of Fig. 6e). The beams V, II, and III are selected in the first competition, but solution III wins because its ranking is one. After the six competitions, the six parents are defined, as shown in Fig. 6e on the right. Beams 2 and 3 in the population will serve as parents twice, while beams 1 and 5 will only be parents once. Beams 4 and 6 are discarded from the offspring generation due to their lower quality.

The two-point crossover method involves generating a child by mixing the genetic information from two parents. The method entails randomly defining two crossover points, which divides the chromosome into three segments. Children are then created by exchanging segments between parents. Fig. 6f presents that child one inherits dimensions d_3 and d_4 dimensions from the father, while child two assumes the mother's information. New values are generated for both children due to a crossover point in the d_1 variable. Regarding d_2, one child takes the data from the mother and the other from the father. The crossover operation is ruled by a crossover rate, which determines the percentage of parents that do not produce offspring.

The mutation operator introduces variations in the offspring generated in the crossover operation to prevent the population from being stuck in optimal locals when the population's phenotypic information looks similar. The jump mutation introduces a swap of the randomly selected bit to mutate. Fig. 6g shows the results of applying the mutation to the solution, representing a change in dimension d_3. Typically, the user defines a mutation rate as not greater than 0.02.

After generating the new population depicted in Fig. 6h, a stopping criterion determines whether the GA operations continue. If this criterion is unmet, the genetic operators are reapplied to generate a new population. In some cases, the stopping criterion corresponds to the maximum number of generations, although a convergence-based criterion is also feasible. Ultimately, multiple runs of the GA are required to provide the process results due to the stochastic nature of the mathematical operation involved. The computational cost is primarily associated with the need for objective function evaluation, which can be critical for practical structural optimization.

3.2. The implicit redundant representation for non-structured optimization problems

Among the main characteristics of metaheuristics, the possibility of adapting them to the problem being studied becomes relevant. One class of optimization problems involves scenarios where the number of design variables is unknown, and the algorithms must determine the optimal number of variables for the problem. In structural engineering, such scenarios arise in identifying damaged zones within a structure [42], determining the placement of an unspecified number of energy