



An Automated Procedure for Continuous Dynamic Monitoring of Structures: Theory and Validation

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Received: 25 November 2022 / Revised: 11 July 2023 / Accepted: 9 August 2023 / Published online: 9 September 2023
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Abstract

Introduction Stability diagrams are a helpful tool for operational modal analysis to obtain the physical modes of a structure. These modes can be defined by visualizing stable columns formed by consistently identified modes over a range of model orders. Extracting these modes manually becomes an obstacle if continuous identification of modal parameters is required.

Materials and methods In this paper, a procedure is configured to automatically interpret the stability diagrams constructed with the identification results of the SSI-COV/ref algorithm. This procedure is based on some methodologies found in the literature, which follow three stages. First, a stability diagram cleaning stage is defined where modal validation criteria and partitioning clustering algorithms are used to detect spurious modes. Second, a mode grouping stage based on a hierarchical clustering algorithm is implemented to form sets of modes that share similar modal information. Finally, a selection stage is applied to define representative modal parameters from the set of physical modes.

Results The proposed procedure is validated by simulating a beam-type structural model with ten degrees of freedom affected by ambient temperature functions. Natural frequencies computed for the DT140 and DT220 datasets collection with the frequency-domain decomposition method agree with the computed ones with the proposed procedure, presenting MAC coefficients higher than 0.97. A total of 192 datasets are simulated, and the acceleration responses are polluted with two noise levels, SRN = 40 [dB] and SNR = 20 [dB].

Conclusions For the analyzed beam, the modal tracking results showed that the procedure could perform continuous identification automatically. The variations in the natural frequencies are correlated to the variations in the ambient temperature functions.

Keywords Modal parameters · Operational modal analysis · Dynamic monitoring · Clustering · Modal identification

Introduction

The advantages that offer operational modal analysis (OMA) have led to its increasing study and application in the mechanical [1, 2], aerospace, [3–5], and civil [6, 7] fields. As OMA has been developed to identify modal parameters (natural frequencies, damping ratios, and mode shapes) from ambient vibration measurements, no equipment is required to excite the structures, and the tests can be conducted under operational conditions. Furthermore, applying this type of test makes it possible to obtain the entire system's dynamic characteristics and effectively identify repeated and closely space modes [8]. The purpose of identifying modal parameters from OMA is to obtain reliable information that can be used in different applications, such as dynamic structural modification, numerical model updating, optimal dynamic design, vibration control, and structural health monitoring (SHM). Although

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vibration-based SHM is becoming more and more popular, the implementation of such systems is not an easy task. These must contain the dynamic characterization of structures in real time to observe and detect changes in the temporal evolution of modal parameters to be used as damage indicators. In this way, continuous or long-term identification entails a significant challenge as it requires periodic analysis of large amounts of data, ensuring the reliability of the estimated modal information. Much human input makes this process inefficient, so automation of OMA techniques is essential.

Several numerical algorithms have been developed to perform OMA. Some of them, in the frequency domain, such as the frequency-domain decomposition (FDD) [9] or the least-squares complex frequency domain (LSCF) [10], and in the time domain, such as the stochastic subspace identification algorithms (SSI-DATA, SSI-COV). The SSI-COV algorithm has been successfully used to identify modal parameters of large civil structures [11–13]. One of the benefits of this algorithm is that it allows the simultaneous identification of multiple modes, which makes it quite efficient for continuous dynamic characterization. Notwithstanding, the main issue in using the SSI-COV is to select a proper model order. It is usually recommended to set this parameter by detecting a large gap between two successive singular values, but this gap is difficult to detect when using ambient vibration measurements. Different strategies have been developed to determine an optimal model order, and some of them can be consulted in [14–16]. However, the exact value of the model order will never be known, and applying the above strategies leads to a high probability of obtaining underestimated or overestimated values of this parameter. In the first scenario, not all the physical modes of the system would be identified. On the contrary, overestimating the order would result in noise absorption and thus in modal characteristics represented by both physical and spurious modes.

Stability diagrams are a useful tool to define the true modes of a structure. Thus, the SSI-COV algorithm can be applied to a range of model orders. Then, the identification results can be displayed on a frequency vs. model order plane. The physical modes are expected to be identified consistently in each order. Since the orders are over-specified beyond a certain order, identifying spurious modes is inevitable. However, unlike the physical modes, they are not consistently identified in each order and therefore appear in the stability diagram as scattered points. Therefore, vertical alignments or stable columns can be defined as the physical modes. Manually extracting these modes can be laborious and can easily lead to errors in the modal identification process. Then, estimating modal parameters without user interaction demands the automatic interpretation of the stability diagram.

The first challenge faced when interpreting stability diagrams is the detection of spurious modes. Commonly, the degree of physicality of the identified modes is evaluated

through filters using hard validation criteria (HVC) and soft validation criteria (SVC) calculation. HVC are defined by delimiting domains with fixed thresholds and rigorous application of physical principles, while the calculation of SVC yields a range of values that require thresholds to determine the point at which modes are considered spurious. Conventional modal validation criteria include modal phase collinearity (MPC) [17], mean phase deviation (MPD) [18], modal transfer norm (MTN) [19], and modal coherence indicator (MCI) [20]. When spurious modes substantially contaminate a stability diagram, the application of these criteria may be insufficient, allowing many spurious modes to remain in the diagram, affecting the accuracy of the estimated modal parameters. Thus, some approaches for the automatic interpretation of the stability diagram have incorporated the calculation of the uncertainty in modal parameters that are used to distinguish physical and spurious modes. The modes whose modal parameters have the largest standard deviations are defined as spurious [21, 22].

Different clustering approaches based on partitioning methods and hierarchical algorithms have been applied for the automatic interpretation of the stability diagram. Partitioning methods such as K-means and Fuzzy C-means are fast in processing large amounts of data; however, one of the difficulties in conducting such algorithms is that a priori knowledge of the number of clusters is required. These algorithms have been used in different processes, such as classifying modes between physical and spurious [3, 10, 23], selecting physical modes [18], and separating close modes [24]. The hierarchical algorithm has been widely used to cluster modes with similar characteristics based on the relative differences between natural frequencies, damping rates, and MAC coefficients [25–27]. One of the main issues of this algorithm is the definition of a tree cut-off threshold, which establishes the number of clusters obtained. Furthermore, the user's definition of a static threshold in long-term monitoring can lead to identification errors, since a different stability diagram is obtained for each dataset.

In [18], a fully automated approach is proposed, which combines the capabilities of the hierarchical algorithm and k-means through the following stages: 1. Application of soft validation criteria and use of a k-means algorithm to detect spurious modes. 2. Automatic calculation of the cluster threshold based on the remaining modes and grouping the modes with similar characteristics employing a hierarchical algorithm. 3. Use a k-means algorithm to select clusters containing the physical modes and select a representative element from each cluster of physical modes. Later, in [28], an uncertainty criterion of natural frequencies is incorporated at the end of the first stage to improving the cleaning of spurious modes. In addition, an outlier detection procedure is introduced after selecting the clusters containing the physical modes to make the representative values more accurate. Both approaches were validated through automatic

operational modal analysis on bridges (the Z24 bridge and the Wetteren footbridge), showing high robustness in the estimated modal parameters.

In this work, a procedure based on the approaches proposed in [18, 28] is configured to assess its potential for long-term monitoring of structures. It is considered that ambient vibration measurements present low SNR conditions to account for operational conditions. The procedure is presented and validated by tracking the modal parameters of a beam-type structure with ten degrees of freedom. This paper is organized as follows. First, in Section “**Numerical Simulation of an Experimental Test**”, the structural model used to validate the automated procedure is introduced; Section “**Covariance-Driven Stochastic Identification (SSI-COV)**” provides the background of the SSI-COV identification method and the stability diagram. Then, in Section “**Automated Modal Parameter Estimation (MPE) Procedure**”, the automated procedure is presented, and in Section “**Tracking of Modal Parameters**”, the results of the modal parameter tracking of the simulated structural model are shown.

Numerical Simulation of an Experimental Test

A simply supported beam is simulated to determine the performance of the automated procedure for structural monitoring. The beam has a length of 10 [m] and an uniform square cross-section of side 0.2 [m]. The numerical

model of the structure is discretized into 11 finite elements of equal length. The Young’s modulus in each element is $E = 1.787 \cdot 10^9$ [N/m²] and the material density $\rho = 12,816$ [Kg/m³]. As shown in Fig. 1, this beam is also elastically restrained against translational and rotational movements at nodes 2 and 3. $k_T = 20,000 \cdot T$ [N/m] and $k_R = 40,000 \cdot T$ [N/m] are the translational and rotational spring constants, respectively, where T is a temperature function. The mass $\mathbf{M}$ and stiffness $\mathbf{K}$ matrices obtained from geometric and material properties are condensed into ten vertical degrees of freedom (DOF) to reduce the problem size. A damping matrix $\mathbf{C}$ proportional to the mass is also estimated. The ambient temperature functions used in this work are shown in Fig. 2.

The beam is excited with a white noise-type load on each vertical DOF. Then, the vibration responses are obtained using the Newmark integration method, [29] considering the matrices $\mathbf{M}$, $\mathbf{K}$ and $\mathbf{C}$, and parameters $\beta = 0.25$ and $\gamma = 0.5$ at a sampling frequency of 200 [Hz] and 15 [min] duration. Vertical acceleration measurements in the ten DOF of the beam are grouped into a dataset to be used in the modal identification process. A total of 192 measurement datasets are obtained from the simulation of the same quantity of white noise inputs. A signal with zero mean and variance equal to the noise intensity is added to each response to investigate the sensitivity of modal parameters to noise-type perturbation. A collection of 96 simulated datasets, DT140, were obtained considering the temperature function T_1 in Fig. 2a, with an SNR = 40 [dB]. The other 96 datasets depend on the temperature function T_2

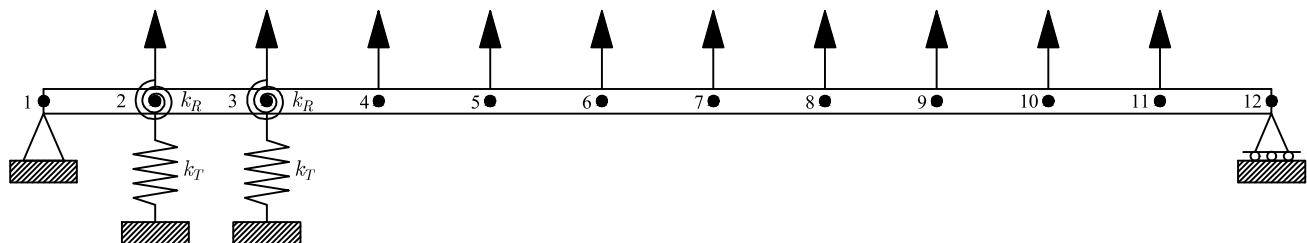


Fig. 1 Structural model

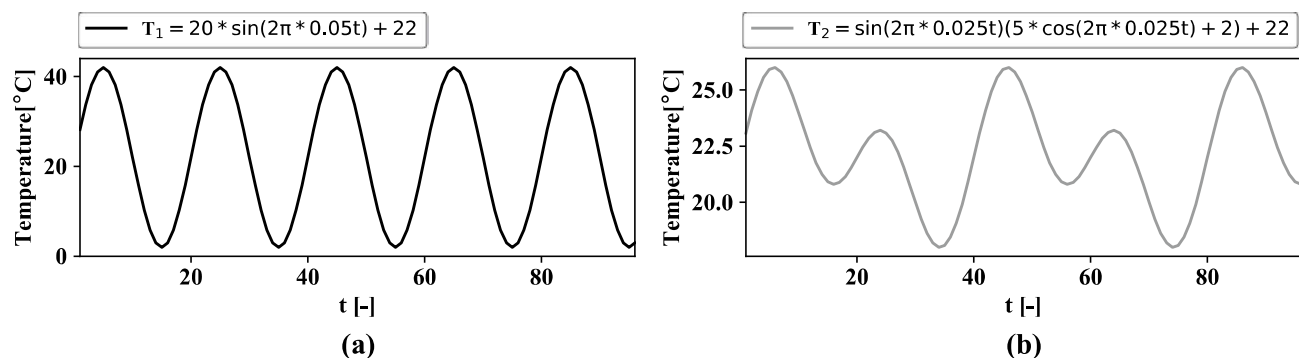


Fig. 2 Temperature functions

in Fig. 2b, with an SNR = 20 [dB]. That datasets' collection is named DT220.

The modes of the structure are in the frequency range [0–45] [Hz]. Thus, each time series was decimated by a factor of 2 to resample the data at a frequency of 100 [Hz]. To assess the performance of the automated procedure at each stage, the results are shown for the two collections of datasets, DT140 and DT220. Subsequently, the identification is conducted using the 192 simulated datasets, and the results of the modal tracking are presented.

Covariance-Driven Stochastic Identification (SSI-COV)

The SSI-COV algorithm can be used to identify modal parameters from covariance matrices of ambient vibration measurements. The reference-based generalization version of this algorithm was later proposed to reduce the influence of noise on the identification results. The SSI-COV/ref [30] algorithm establishes covariance functions Λ_l^{ref} for $l = 1, 2, \dots, 2i - 1$ between all outputs r and a limited set of reference outputs r_o to construct a Toeplitz matrix $\mathbf{T}_{l|i}^{ref} \in \mathbb{R}^{ir \times ir_o}$ and subsequently by singular value decomposition to estimate the observability matrix $\mathcal{O} \in \mathbb{R}^{ir \times n}$. From the observability matrix, it is possible to identify the state transition matrix $\mathbf{A} \in \mathbb{R}^{n \times n}$ and the observation matrix $\mathbf{C} \in \mathbb{R}^{r \times n}$ of a discrete stochastic state-space model given by

$$\begin{aligned} x_{k+1} &= \mathbf{A}x_k + w_k, \\ y_k &= \mathbf{C}x_k + v_k, \end{aligned} \quad (1)$$

where k is the sampling instant, x_k the state vector, and y_k the measurements vector. w_k and v_k are stochastic processes with zero mean representing unknown inputs and measurement noise. i is the number of block rows of the Toeplitz matrix

and n the model order with $n/2$ equal to the number of the system's degrees of freedom.

The modal parameters can be identified from the system matrices $\mathbf{A}$ and $\mathbf{C}$. Using the eigenproperties decomposition of matrix $\mathbf{A}$ given by $\mathbf{A}\psi_k = \lambda_k\psi_k$, it is obtained that

$$\psi_{ck} = \psi_k, \lambda_k = \frac{\ln(\lambda_k)}{\Delta t} \begin{cases} f_k = \frac{|\lambda_{ck}|}{2\pi} [\text{Hz}] \\ \xi_k = \frac{\Re(\lambda_{ck})}{|\lambda_{ck}|} * 100 [\%], \\ \phi_k = C\psi_k \end{cases} \quad (2)$$

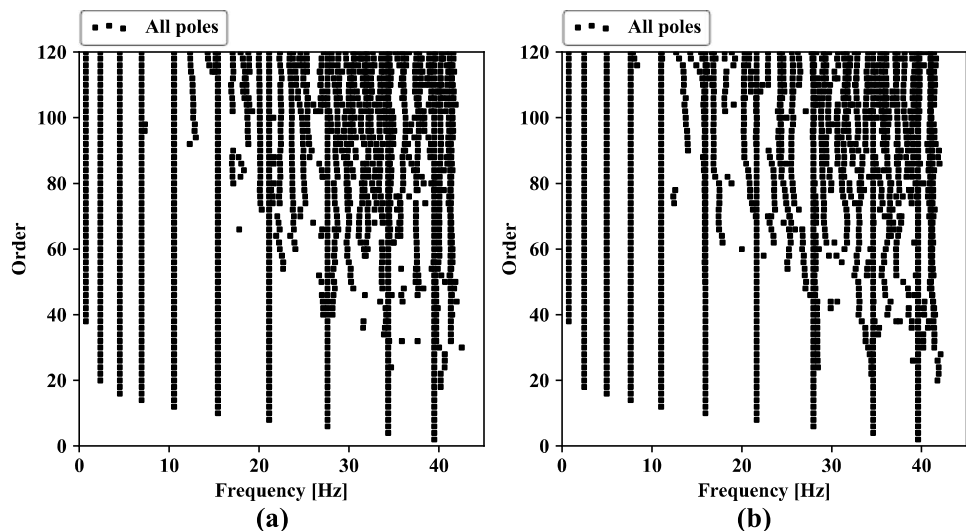
where ψ_k and λ_k are the discrete-time eigenvectors and eigenvalues of $\mathbf{A}$, respectively. ψ_{ck} and λ_{ck} are the equivalent continuous-time eigenproperties, Δt the sampling period, $\Re(\bullet)$ represents the real part of a complex, and f_k , ξ_k , and ϕ_k the identified natural frequencies, damping ratios, and mode shapes, respectively.

Stabilization Diagram

The stabilization diagram provides a graphical output of the identification results of the system. Figure 3a, b shows the poles identified for the datasets collections DT140 and DT220, respectively. A previous analysis of the identification algorithm on the DT140 showed that for model orders below $n = 40$, frequencies lower than 1 [Hz] were not being identified, so the order range for the identification was chosen from $n_{min} = 2$ to $n_{max} = 120$ in steps of 2. The number of block rows of the Toeplitz matrix was set in $i = 60$ and reference channels = (2, 3, 4, 5, 6, 7, 8, 9, 10, 11).

In both cases, well-defined vertical alignments representing physical modes are noticeable in the frequency range of 0–20 [Hz]. Manual extraction of these modes requires a lot of user interaction, as they are not given directly but require

Fig. 3 SSI-COV/ref results. **a** DT140 dataset and **b** DT220 dataset



visual recognition. As the identification is performed for a wide range of model orders and beyond a certain order, these are overestimated, resulting in many candidate modes. For the DT140 dataset, 1455 modes were identified, while for DT220, 1558 modes. These modes are composed of genuine physical and spurious modes. The stability diagrams in Fig. 3 show that contamination by spurious modes is present for frequencies higher than 20 [Hz] and model orders higher than $n = 40$, which makes stable modes with higher frequencies difficult to extract manually as their visual perception is not reliable. The following section introduces the configured procedure to automatically estimate the physical modes from the stability diagram.

Automated Modal Parameter Estimation (MPE) Procedure

Overview

The configured procedure for automatically estimating modal parameters from the stabilization diagram is presented in Fig. 4. Initially, ambient vibration measurements are received as input data. Then, these signals are pre-processed by decimation to obtain a frequency band containing the frequencies of interest. Also, the averages of the signals are subtracted, since it is necessary to compute the output covariance matrices in the identification

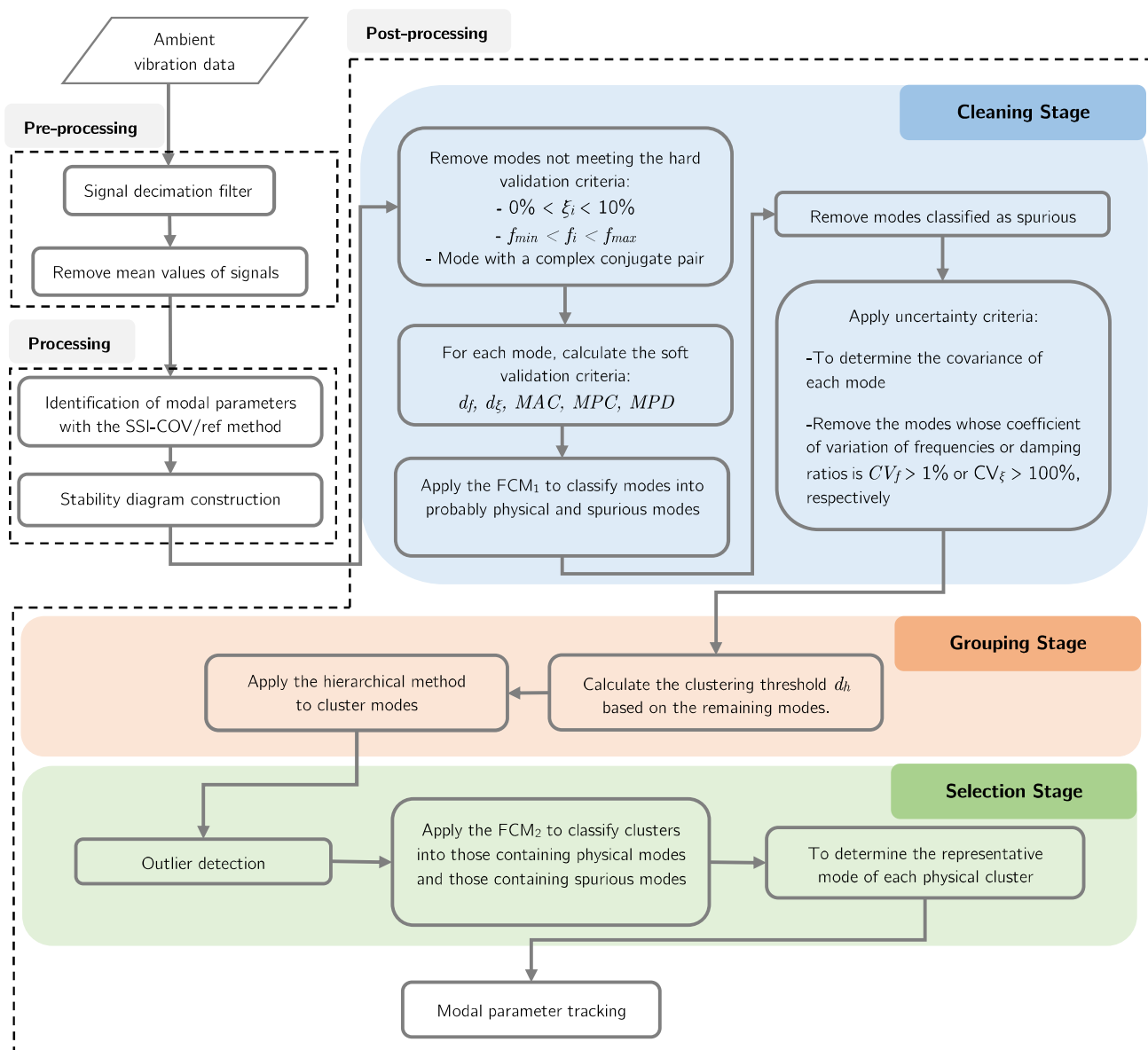


Fig. 4 Automatic MPE procedure flowchart