Computers and Structures 147 (2015) 117–125

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Ensuring reliable damage detection based on the computation of the optimal quantity of required modal data

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ARTICLE INFO

Article history:
Accepted 28 September 2014
Available online 29 October 2014

Keywords:
Damage detection
Metaheuristic
Dynamic parameters and particle swarm optimisation

ABSTRACT

This paper proposes an adaptive particle swarm optimiser to solve the optimisation problem associated with the assessment of the structural integrity of a multi-supported beam structure. Also, it presents how to determine the optimal quantity of modal data that is necessary to guarantee a correct damage detection. Adaptation is implemented to avoid defining of the PSO parameters by trial and error. The results show that a minimum quantity of modal data is necessary to guarantee the success of the damage detection methodology and that the ability to locate and quantify damage may not be improved by using excessive information.

1. Introduction

The civil, mechanical, petroleum and aeronautical industries have been interested in developing non-destructive damage detection methodologies to assess the integrity of their structures. Some of these methodologies are based on changes that occur in the dynamic parameters after damage has occurred. However, the application of this concept to damage detection could be limited by certain issues, such as low sensitivity of the dynamic parameters to the damage, a high level of complexity in the technique used to determine the damage scenario, the possibility of the dynamic parameters being affected by factors aside from the damage and incomplete measurements [1]. With respect to the last issue, if modal parameters are used, the incompleteness of the measurements is related to the fact that it is not possible to excite all of the vibration modes of the structure and measure the mode shapes in every degree of freedom (DOF) of the finite-element model that represents the undamaged structure [2].

The effect of incomplete measurements on the performance of vibration-based damage detection methodologies has been reported in the literature.

Law et al. [3] proposed an energy-based damage detection methodology. The application of this formulation required complete mode shapes, which were obtained by using a mode shape expansion technique. This type of technique was preferred instead of a model reduction because it preserves the connectivity of the structure. The authors concluded that the smaller the number of measured DOFs, the larger the set of falsely damaged elements.

Kim and Bartkowicz [4] implemented a two-level damage detection approach based on updating and design sensitivity methods. The mismatch between the analytical and experimental models was solved using a hybrid reduction/expansion technique. Numerical simulations showed that the quantity of DOFs with measured information affected the correct identification of some damage scenarios.

Araújo dos Santos et al. [5] observed that the quality of the results of a damage detection methodology was determined by the technique used to address the incomplete measurements. This effect was more pronounced as the damage extent increased.

Reza and Medhi-pour [6] proposed to locate damage using a subspace rotation method and then to quantify the damage extent using certain concepts from control theory. They concluded that an increase in the number of measurements resulted in an increase in the successful damage cases found.

Raich and Liszkai [7] used a genetic algorithm with implicit redundant representation to account for the fact that the number and position of the damaged elements were not known *a priori*. The objective function was based on changes in the frequency response function matrix of the structure. They affirmed that an increase in the number of sensors used might not facilitate damage identification in all cases but that, in general, the location and quantification of damage would be more accurate.

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http://dx.doi.org/10.1016/j.compstruc.2014.09.020
0045-7949/

Yun et al. [8] proposed implementing the damage detection process in two stages. The authors worked with the first stage, which consisted of reducing the set of elements by identifying probable damaged zones with a subset selection method based on the residual vector force. It was observed that when the measurements were complete and noise-free, then the damaged elements in a beam structure were located regardless of the number of mode shapes used. If the measurements contained noise, then more reliable results were obtained when more mode shapes were used.

Meruane and Heylen [9] presented a real-coded genetic algorithm to detect damage and tested it in a spatial truss structure. They performed an analysis of the effect of the quantity of sensors in the structure on the performance of the proposed methodology and found that the higher the number of damage locations, the higher the quantity of measured DOFs required for a successful detection.

Fan and Qiao [10] conducted a review of damage detection methodologies based on vibration and studied certain topics that affected the damage identification in a beam-type structure, including sensor spacing. They observed that if the sensor spacing was large, a damage detection methodology based on the curvature of mode shapes performed better when the curvature was measured and not computed.

Several numerical studies of vibration-based damage detection have simulated the issue of incomplete measurements by assuming that a pre-determined quantity of modal data is available for the analysed structure. For example, measurements are taken in all [11,12] or some [13,14] vertical DOFs in the case of beam structures. Conversely, some of these methodologies require complete mode shapes and therefore implement computational techniques, such as reduction techniques, mode expansion techniques or a combination of both to match the size of the numerical and experimental models [15,16]. The use of these techniques introduces numerical errors in the model [17]; consequently, it is desirable to propose damage detection methodologies that use only the measured information.

The above paragraphs establish the context in which this research was developed and show that the quantity of information available plays a very important role in the success of a damage detection methodology. The following question is posed: is the modal information available enough to detect damage reliably? Thus, the main objective of this research is to propose a damage detection methodology based on modal information and a process that determines the optimal quantity of modal data needed. The problem was formulated as an optimisation one and solved by a particle swarm optimizer (PSO). Adaptation was proposed because the performance of the original PSO [18] in solving an optimisation problem could be affected by the values chosen for the cognitive and social parameters. The version of PSO used in this research is the one proposed by Shi and Eberhart in 1998 [19], which includes a weight factor that will be deterministically computed. The objective function was formulated in terms of natural frequencies and modes shapes. The proposed function avoids utilising techniques to match the DOFs of the numerical and experimental models because it only uses the measured information. A beam structure discretised into 51 elements under different damage scenarios was analysed. A set of different quantities of modal information was proposed to study the effect of incomplete measurements, allowing the optimal quantity of modal data to be determined.

This paper is an updated and revised version of the conference paper [20] and is divided into seven sections, starting with the above introduction. Section 2 presents the basic theory of the particle swarm optimiser and the proposed modification used to set the PSO parameters. The modelling of the damage and how it affects the dynamic parameters of a structure are presented in Section 3. The proposed methodology and the assumptions are summarised in Section 4. Section 5 presents the analysed structure, the damage scenarios to be studied and the set of modal data that will be tested. Section 6 shows the results found. Finally, the main conclusions of this research are presented.

2. Adaptive particle swarm optimizer

As previously mentioned, the damage detection problem is formulated as an optimisation problem, and it is therefore necessary to define the optimisation technique used to solve it. The particle swarm optimiser (PSO) was selected for this study; the PSO is a population-based stochastic algorithm used to find an optimal or near-optimal solution to maximisation or minimisation problems. The PSO was proposed by Kennedy and Eberhart in 1995 [18] under the assumption of a simple interaction model among individuals, in which the solution to a common problem is found through the collaboration of the whole community. The PSO is characterised by the ease with which it is implemented computationally, as it will be shown in the next paragraph.

The PSO considers a swarm of particles flying through the search space that are characterised by a position, a velocity and a cost. The current position of each particle in the search space corresponds to a possible solution to the analysed problem. The velocity is a property that permits orientation of the particle's flight. The cost indicates the quality level of the solution found by the particle and is computed from the objective function previously defined for the problem. For a maximisation problem, a particle will be better than another when its cost is higher.

The velocity and position of each particle are updated iteratively using the knowledge of the swarm about the search space as shown by Eqs. (1) and (2). In this sense, each particle is able to remember the best position visited (pbest) and to transmit this information to the swarm to determine the best current position (gbest). Acceleration parameters are used to take into account the weight of the information acquired in previous iterations, which allows the particle's trust in its own knowledge and in the knowledge of the swarm to be assessed. As the iterations increase, the swarm converges to a region in the search space where the optimal solution is expected to be located. The algorithm is stopped either if the swarm converges to a solution or if a pre-specified number of iterations is reached. Due to the stochastic characteristics of the PSO, it is necessary to execute the algorithm a specific number of times to find the final solution to the problem. That solution can be specified either by combining a specific number of the best solutions or by simply choosing the solution with the highest cost among all the runs. It is worth mentioning that PSO could not achieve the solution for all runs, being the number of successful runs dependent on the complexity of the problem.

The expressions for updating the velocities, v_i, and positions, x_i, given in Ref. [19] were used in this study

$$\{v\}_i^{t+1} = w \cdot \{v\}_i^t + c_1 \cdot r_{1i}^t \cdot \left(\{p_{best}\}_i - \{x\}_i^t\right) + c_2 \cdot r_{2i}^t \cdot \left(\{g_{best}\}_i - \{x\}_i^t\right) \tag{1}$$

$$\{x\}_i^{t+1} = \{x\}_i^t + \{v\}_i^{t+1} \tag{2}$$

where i is the i-th particle in the swarm, t is the current iteration, c_1 and c_2 are the acceleration parameters (cognitive and social parameters, respectively), r_1 and r_2 are random values between 0 and 1, and w is the inertia weight that controls whether the PSO conducts a local or global search. Symbols { } and [ ] will indicate a vector and a matrix, respectively.

Various techniques have been used to set some of the PSO parameters, such as fuzzy systems [21], self-adaptation [22], deterministic adaptation based on pbest and gbest [23] and Nelder–Mead Simplex [24]. This paper presents a simple and efficient technique for adapting the PSO parameters, which assumes that each particle in the swarm has its own acceleration parameters. The cognitive parameter for the i-th particle at the iteration $t+1$, c_{1i}^{t+1}, is computed as follows

$$c_{1i}^{t+1} = c_{1i}^{t} + r_3 \cdot \left(c_{1best}^{t} - c_{1i}^{t}\right) \tag{3}$$

where c_{1best} is the value of c_1 for the best particle in the swarm in the previous iteration $t-1$ and r_3 is a random value between 0 and 1. The initial values for c_{1i} are defined as random numbers generated from a uniform distribution in the [1.5–2.5] range based on the value used in Ref. [25]. Regarding the social parameter, we followed the recommendation given in Ref. [26], which states that if the sum of the acceleration parameters is greater than four then it is necessary to use the constriction factor to control the amplitude of the velocity. Consequently, to avoid the use of a constriction factor we defined c_{2i}^{t+1} as

$$c_{2i}^{t+1} = 4 - c_{1i}^{t+1} \tag{4}$$

To compute the weight factor, it is desirable to increase the global search ability of the PSO at the beginning of the iterative process and to perform a more local search at the end. Therefore, the weight factor can be obtained as proposed in [25]:

$$w^{t} = (w_2 - w_2) \cdot \left(\frac{t_{max} - t}{t_{max}}\right) + w_1 \tag{5}$$

where t_{max} is the maximum number of iterations permitted. The values for w_2 and w_1 are 0.9 and 0.4, respectively [25].

Finally, to avoid the so-called explosion phenomenon [26] the velocity is limited by the following expression:

$$v_{max} = \frac{X^{h} - X^{l}}{5} \tag{6}$$

where h and l are the upper and lower boundary values, respectively, for the variable X.

3. Effect of damage on the dynamic parameters

As was previously mentioned, damage changes the dynamic parameters of a structure due to the alterations in the structural properties of the structure. In the following paragraphs we describe how the damage is modelled and how it affects the dynamic parameters, supposing that a finite-element model that represents the behaviour of the undamaged structure is available. The structure is assumed to be undamped and the stiffness and mass matrices of the structure, namely K_{str} and K_{str}, respectively, are computed as follows

$$[K_{est}] = \sum_{j=1}^{NElem} [k_j] \tag{7}$$

$$[M_{est}] = \sum_{j=1}^{NElem} [M_j] \tag{8}$$

where m_i and k_i are the stiffness and consistent mass matrices for the i-th element, respectively, and $NElem$ is the number of elements in the structure. The dynamic parameters of an undamped structure can be computed by the following equation

$$\left([K_{est}] - \omega_i^2 [M_{est}]\right)\{\phi_i\} = 0 \tag{9}$$

where ω is the natural frequency and ϕ is the corresponding mode shape.

To model the damage, the mass matrix was assumed to be constant after damage, and the damage could be represented as a reduction in the stiffness matrix of the damaged elements given by

$$[K_d] = \sum_{j=1}^{NElem} \left(1 - \beta_j\right)[k_j] \tag{10}$$

where K_d is the damaged stiffness matrix, d refers to the damaged condition and β_j is the stiffness reduction factor for the j-th element. The element does not present damage if the β factor assumes a value equal to 0, and the element is completely damaged if this factor is equal to 1.

Finally, the dynamic parameters of the damaged structure can be determined by

$$\left([K_d] - \omega_{di}^2 [M_{est}]\right)\{\phi_i\} = 0 \tag{11}$$

Eq. (11) shows that the dynamic parameters will be different from those obtained using Eq. (8).

4. Formulation of a damage detection methodology

In this study, the localization and quantification of the damage was performed using concepts from the dynamic behaviour of the structure. Vibration-based damage detection methodologies are based on the fact that the dynamic properties of a structure are connected to the structural properties. Any change in the matrices of the system – stiffness, mass and damping matrices – produces changes in the natural frequencies and mode shapes of the system. This concept can be used for damage detection by formulating an optimisation problem in which the difference between the dynamic parameters of a numerical model that represents the damaged structure and those obtained from a real dynamic test of the structure is minimised.

An algorithm that describes the guidelines used to solve the damage detection problem as an optimisation problem is presented in Fig. 1. It is worth mentioning that this philosophy for formulating the damage detection problem has been studied by many researchers in the last decades; one of the first studies was published by Mares and Surace in 1996 [11].

The first step in the proposed methodology consists of defining a finite element model that represents the undamaged condition of the structure. The model for the damaged condition is obtained by updating the model in Step 1, which in this case is the stiffness matrix. Herein, the optimisation variables correspond to each stiffness reduction factor in the structure, and the total is equal to the number of elements in the structure.

The parameters selected to form the objective function have to be sensitive to the damage in the structure. In this study, the objective function was based on natural frequencies and mode shape and is given by [28]:

$$G = \sum_{j=1}^{nm} \frac{a_1}{a_2 + F_j} \tag{12}$$

Begin

1. Define the finite element model, FEM, of the structure without damage.
2. Define the objective function.
3. Obtain the damaged dynamic parameters: natural frequencies and mode shapes.
4. Apply the optimiser.
5. Report the results.

End

Fig. 1. Damage detection methodology.

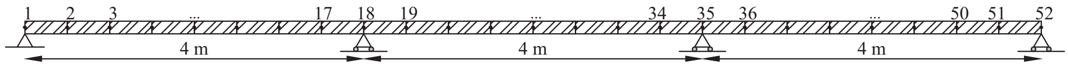

Fig. 2. Beam-type structure.

Table 1
Position of the nodes where there is information in the corresponding vertical DOF taken for each beam between supports.

Case	% of Vertical measured DOFs	Nodes (numbering starts from the left support of each beam)
I1	100	Corresponding to all of the free vertical DOF
I2	75	3, 4, 5, 6, 8, 9, 10, 11, 13, 14, 15, 16
I3	50	3, 5, 7, 9, 10, 12, 14, 16
I4	25	4, 8, 11, 15

Table 2
Simple damage scenarios used to assess the performance of the proposed methodology.

Scenario S1		Scenario S2		Scenario S3		Scenario S4	
Element	β	Element	β	Element	β	Element	β
2	0.250	6	0.350	27	0.170	27	0.320

Table 3
Multiple damage scenarios used to assess the performance of the proposed methodology.

Scenario M1		Scenario M2		Scenario M3		Scenario M4	
Element	β	Element	β	Element	β	Element	β
4	0.200	22	0.230	3	0.170	4	0.320
20	0.200	23	0.230	31	0.290	25	0.210
		24	0.230	42	0.310	46	0.250
						47	0.310

with

$$F_j = \left|\frac{w_j^{pso} - w_j^{ex}}{w_j^{ex}}\right| + W \cdot \sqrt{\frac{\sum_{i=1}^{ndf}\left(\phi_{ij}^{pso} - \phi_{ij}^{ex}\right)^2}{\sum_{i=1}^{ndf}\left(\phi_{ij}^{ex}\right)^2}} \tag{13}$$

where *nm* refers to the number of vibration modes considered, *ndf* is the number of degrees of freedom with available information, superscript *pso* refers to a value found by the particle swarm optimiser and superscript *ex* indicates the experimental results. ω_j is the j-th natural frequency and ϕ_{ij} is the value of the j-th mode shape for the i-th degree of freedom that is available. a_1 is a scale factor that does not affect the optimisation process and assumes a value of 200. $a_2 = 1$ is a constant that avoids a division by zero when the PSO and experimental dynamic parameters match exactly. $W = 2.0$ is the weight factor between the contribution of natural frequencies and mode shapes. The objective function used does not require complete modal information, avoiding the use of either model reduction techniques or mode expansion techniques. Choosing the correct objective function is very important for reliable damage detection because certain functions do not permit correct identification of the damage.

Table 4
Number of successful runs (out of ten runs) for simple damage scenario S1 depending on the quantity of modal data available.

% of Vertical DOFs	Number of measured modes				
	3	5	7	9	11
25	2	7	5	7	7
50	5	5	8	6	9
75	3	5	7	6	8
100	2	5	6	7	8

Table 5
Number of successful runs (out of ten runs) for simple damage scenario S2 depending on the quantity of modal data available.

% of Vertical DOFs	Number of measured modes				
	3	5	7	9	11
25	5	8	10	9	7
50	3	10	5	6	9
75	5	3	9	8	8
100	5	5	9	8	7

Table 6
Number of successful runs (out of ten runs) for simple damage scenario S3 depending on the quantity of modal data available.

% of Vertical DOFs	Number of measured modes				
	3	5	7	9	11
25	4	4	8	7	9
50	2	4	8	8	8
75	3	3	5	9	9
100	1	4	6	9	9

Table 7
Number of successful runs (out of ten runs) for simple damage scenario S4 depending on the quantity of modal data available.

% of Vertical DOFs	Number of measured modes				
	3	5	7	9	11
25	5	5	7	10	8
50	6	6	9	9	10
75	4	7	5	9	8
100	2	4	8	9	10

In the third step, the damaged dynamic parameters have to be experimentally determined. Because this research was numerically developed, the dynamic parameters were obtained from Eq. (10) by computing the damaged stiffness matrix for the damage scenario under investigation. A damage scenario is formed by a number of damaged elements and their corresponding damage extents. In an attempt to simulate the conditions found in a real test, the computed dynamic parameters for the damage condition were numerically perturbed by 1% for natural frequencies and 3% for mode shapes [27] to simulate the presence of noise in the measurements. Additionally, only a few incomplete mode shapes were available in a determined number of DOFs in the structure.

The selection of the algorithm for solving the optimisation problem is one of the key aspects of ensuring correct detection of the damage. Thus, an adaptive particle swarm optimiser was selected and configured as presented in Section 2. The initial position of the particles in the swarm is heuristically determined. For each variable in each particle, a random number is assigned in the [0, 1] range. If this number is smaller than 0.5, the variable assumes a new random value between 0 and 0.5; otherwise, a value equal to zero is assigned to the variable. By introducing zero values for some positions of the initial particles, the search was oriented for non-severe damage scenarios with a few damaged elements. The above procedure is also used to generate the initial velocities in the range $[-V_{Max}, V_{Max}]$, where V_{max} is given by Eq. (6) and X^h and X^l are 1 and 0, respectively. The size of the swarm and the maximum number of iterations will depend on the analysed structure. The PSO is executed and the best particle in the swarm is chosen as the solution to the problem.

The damage scenario found is shown in Step 5 and contains the elements that presented a damage extent higher than a specific threshold. Elements presenting lower values are considered undamaged. A total of 10 runs of the PSO were conducted and the solution with the highest cost was selected as the damage scenario searched for. At this point, it is important to recall that heuristic techniques do not guarantee that the correct answer will be found. Thus, the number of runs in which the algorithm finds a damage scenario close to the real scenario can be used to assess the reliability of the proposed methodology.

A two-step process can be used to determine the optimal quantity of modal data that must be used in Eq. (12) to guarantee reliable damage detection. In the first step, representative damage scenarios for the structure and different combinations of the quantity of sensors and measured modes are proposed. From the results, the combination of modal data that produces the predetermined performance of the proposed methodology is chosen. The performance is measured as the number of runs in which the algorithm detects all of the real damaged elements in the damage scenario. It is important to mention that the computational cost of this step is high because many examples will have to be analysed. In the second step, a specific quantity of random damage scenarios will be generated to verify that the combination found in step one is enough to guarantee successful detection of the damage. The scenarios will be characterised by presenting a predetermined maximum quantity of expected damaged elements and a range for the damage extent. Moreover, the maximum degree of error in the computation of the damage extent and the maximum number of misidentified elements will be determined among all the analysed cases. This information permits a further characterisation of the ability of the proposed methodology to reliably assess the integrity of a structure.

The above paragraphs describe the principal contributions of this research. The first describes a proposed vibration-based damage detection methodology and the other describes a method for determining the quantity of modal data that assures a reliable identification of the structural integrity.

5. Proposed numerical example

A beam structure (Fig. 2) with multiple supports was used to demonstrate the ability of the proposed methodology to locate and quantify damage. The beam was 12 m long and discretised into 51 elements of cross-sectional area $A = 0.001$ m^2; moment of inertia $I = 0.00005$ m^4; density $\rho = 7800$ kg/m^3 and elasticity module, $E = 200 \times 10^9$ N/m^2. An Euler–Bernoulli type finite element beam was employed to model the structure, which has 2 nodes and two DOFs per node – one rotational and one vertical DOF. Fig. 2 shows the numbering of nodes. With respect to the PSO characteristics, the swarm comprised 300 particles and a maximum of 200 iterations were permitted. As previously mentioned, the other

Table 8
Number of successful runs (out of ten runs) for simple damage scenario M1 depending on the quantity of modal data available.

% of Vertical DOFs	Number of measured modes				
	3	5	7	9	11
25	0	2	3	2	4
50	1	1	4	2	5
75	0	1	3	4	4
100	0	0	3	3	3

Table 9
Number of successful runs (out of ten runs) for simple damage scenario M2 depending on the quantity of modal data available.

% of Vertical DOFs	Number of measured modes				
	3	5	7	9	11
25	0	0	3	5	3
50	0	1	2	2	4
75	0	0	2	4	3
100	0	0	3	6	5

Table 10
Number of successful runs (out of ten runs) for simple damage scenario M3 depending on the quantity of modal data available.

% of Vertical DOFs	Number of measured modes				
	3	5	7	9	11
25	0	1	3	2	4
50	0	2	4	3	5
75	0	1	3	2	5
100	0	2	4	4	5

Table 11
Number of successful runs (out of ten runs) for simple damage scenario M4 depending on the quantity of modal data available.

% of Vertical DOFs	Number of measured modes				
	3	5	7	9	11
25	0	1	1	2	3
50	0	0	3	1	3
75	0	2	3	2	5
100	0	0	4	2	5

Table 12
Summary of the performance measurements for 50 simulated damage scenarios.

Performance measure	Damage scenario type	
	Simple	Multiple
Minimum number of successful runs	7	2
Maximum number of misidentified elements	5	6
Maximum difference in the damage extent	0.024	0.093

Table 13
Example of an incorrect damage identification.

Damaged element	Damage extent β					
	Real	1	2	3	4	5
6	–	–	–	–	–	0.107
7	–	–	–	–	–	0.073
9	–	0.063	0.080	–	–	–
10	–	–	–	–	–	0.199
11	0.258	0.273	0.246	0.258	0.247	0
31	–	–	0.068	–	–	0.081
32	0.445	0.456	0.445	0.455	0.449	0.436
33	–	–	–	–	0.075	
37	0.173	0.065	0.178	0.170	–	0.163
38	–	0.196	–	–	0.211	–
39	0.184	0	0.185	0.210	0	0.188
40	–	0.141	–	–	0.096	–
43	–	0.093	0.085	–	–	0.143
51	–	–	–	0.122	–	–
	Cost	2127.4	2126.9	2123.7	2121.6	2119.3

Table 14
Example of a correct identification for noise-free measurements.

Damaged element	Damage extent β					
	Real	1	2	3	4	5
6	–	–	–	–	–	0.080
10	–	–	–	–	–	0.201
11	0.258	0.258	0.258	0.248	0.252	0
12	–					0.163
32	0.445	0.445	0.445	0.455	0.436	0.455
33	–	–	–	–	0.099	–
37	0.173	0.173	0.173	–		0.151
38	–	–	–	0.211	0.185	–
39	0.184	0.184	0.184	0	0	0.229
40	–	–	–	0.080	–	–
43	–	–	–		–	0.079
51	–	–	–	0.133	–	–
	Cost	2200.0	2200.0	2182.4	2174.4	2160.7

acceleration parameters were allowed to adapt through the iterative process and the weight factor was computed in a deterministic way.

Table 1 shows the different cases of incomplete modal information that were analysed. These cases consist of the percentage of vertical DOFs measured and the position of the sensors in the beam. The sensors were uniformly distributed across the beam because an analysis of the effect of the sensor positions on the performance of the proposed methodology was not a goal of this study. Information about rotational DOFs is not used due to the difficulty in obtaining these data in a real dynamic test. The incompleteness of the measurements also refers to the possibility of measuring only a few modes of the structure. Thus, the performance of the proposed methodology when the number of measured modes ranges between 3 and 11 is studied.

The performance of the proposed methodology was initially assessed by the detection of the simple and multiple damage scenarios shown in Tables 2 and 3, respectively, and with the assumption that the modal information was incomplete. The simple damage scenarios were chosen in such a way that the methodology could detect damaged elements from all zones of the beam both near the supports and near the centre. The multiple damage scenarios were chosen to take into account spread and uniform damage. The algorithm was applied to each damage scenario in a total of ten runs and the number of runs in which the algorithm found the real damaged elements was determined. The optimal quantity of modal information was defined as the quantity that allows the real damage scenario to be found in at least three runs.

As shown, only a few scenarios were tested to determine the optimal quantity of modal information because the computational effort involved is very high. However, the performance of the proposed methodology using the optimal quantity of modal data will be verified by the simulation of 50 random damage scenarios. These scenarios involve between 1 and 4 damaged elements with a damage extent ranging between 0.15 and 0.5.

6. Results

6.1. Determining the minimum quantity of information for the beam structure

The proposed methodology was applied to the damage scenarios presented in Table 2 to determine the minimum quantity of modal data that is necessary to reliably detect simple damage in the beam. The obtained results are shown in Tables 4–7 and their analysis was carried out by considering all the computed damage scenarios simultaneously. The goal is to search for the combination of modal data that gives the desired performance with the minimum quantity of data, prioritizing the minimum quantity of measurement points.

In general, the methodologýs performance was slightly improved when the quantity of vertical DOFS with measurements was increased and a larger number of mode shapes was used, resulting in a higher number of successful runs of the algorithm. The desired performance was reached when the first five mode shapes are measured in 25% of the vertical DOFs. However, if the first seven mode shapes are used, then a performance greater than 50% for the number of successful runs can be achieved and when the first eleven mode shapes are used, a performance greater than 70% can be achieved. In some cases, the use of 11 modes to assess Eq. (12) can decrease in the methodology's performance in comparison to that obtained with fewer measurements, as found for scenario S4. Tables 6 and 7 illustrate the ability of the methodology to detect damage in element 27 when the damage extent of the single damaged element is varied. In general, the reliability of the proposed methodology increases when the damage extent is higher regardless of the quantity of modal information used.

The results concerning the application of the proposed methodology for detecting multiple damage scenarios are reported in Tables 8–11. For damage scenarios M1 and M3, the methodology meets the desired performance by measuring the first seven mode shapes in 25% of the vertical DOFs. For the reliable detection of scenario M2, it is necessary to measure eleven mode shapes in any of the measurement configurations shown in Table 1. The results for the damage scenario M2 show that the use of measurements in all vertical DOFs helps to more reliably locate uniform damage scenarios; however, obtaining measurements in all vertical DOFs can be impossible for technical and economic reasons. The assessment of this scenario type seems to be the most challenging for the proposed methodology. Finally, scenario M4 can be detected with the expected reliability level by using one of two combinations of measurements: (1) the first seven mode shapes in 75% of the vertical DOFs or (2) the first eleven mode shapes in 25% of the vertical DOFs.

6.2. Using the computed optimal quantity to detect damage

As previously mentioned, a total of 50 scenarios were tested to prove that the quantity of modal information used is sufficient. In this case, the least possible number of measured points was selected; therefore, we decided to use measurements in 25% of the vertical DOFs and 11 mode shapes. Table 12 summarises the results for the above damage scenarios, which were characterised by the number of damaged elements. The aforementioned minimum quantity of successful runs was almost guaranteed, as two cases presented only two successful runs. There was a low number of misidentified elements, which are defined as those undamaged elements that presented damage extents higher than 0.03. In general, the maximum difference between the real and computed damage extent was found to be low. Moreover, it is worth mentioning that better performance of the proposed methodology was achieved when only one element was damaged. These results indicate that the quantity of modal data selected was adequate to detect damage in the beam.

As shown in the above results, the methodology does not converge to the correct answer in all runs, and thus it is necessary to guarantee that the best solution corresponds to any of those successful runs. However, this fact was not verified for two of the cases analysed. Among the 10 runs, the five best solutions for one of these scenarios are reported in Table 13. Only the elements with a β value higher than 0.050 are presented. It is observed that

Table 15
Application of the proposed methodology to detect simple damage. Measurements in 25% of the vertical DOFs. The number inside the parenthesis indicates the level of error on the computation of the damage extent.

ID scenario	Element	Damage extent β			
		Real	7 modes	9 modes	11 modes
S1	1	–	–	0.082	–
	2	0.250	0 (100%)	0.276 (10%)	0.264 (6%)
	3	–	0.127	–	–
	7	–	–	–	0.062
	9	–	–	0.051	0.063
	18	–	–	0.057	–
	20	–	–	0.091	–
	30	–	–	0.056	–
	33	–	–	0.128	–
	41	–	0.054	–	–
	42	–	–	–	0.073
	44	–	–	0.052	0.056
	51	–	–	0.291	–
S2	1	–	–	0.073	–
	5	–	–	–	0.066
	6	0.350	0.369 (5%)	0.379 (8%)	0.343 (2%)
	16	–	0.109	–	–
	29	–	0.083	0.061	–
	31	–	0.069	–	–
S3	1	–	–	0.212	–
	2	–	0.062	–	–
	13	–	0.059	–	–
	25	–	0.102	–	–
	26	–	–	0.054	–
	27	0.170	0.205 (21%)	0 (100%)	0.174 (2%)
	28	–	–	0.169	–
	37	–	–	0.054	–
S4	7	–	0.111	–	–
	25	–	0.061	–	–
	27	0.320	0.282 (12%)	0.322 (1%)	0.323 (1%)
	28	–	0.078	–	–
	37	–	–	0.070	–
	45	–	0.100	–	–

final solutions with different combinations of damaged elements can have a similar cost. Elements adjacent to the real damaged elements can be identified as damaged for the analysed beam structure. For example, element 39 was not identified in the case of the solution with the highest cost, but the methodology reported the adjacent elements 38 and 40 as damaged. Table 14 shows the results for the same scenario when measurements were free of noise. In this case, the methodology was successful in two runs and achieved the maximum value for the cost ($Cost_{max}$ = 2200) corresponding to the exact solution. Thus, the presence of noise in the measurements causes the space search to become more complex, prejudicing the convergence of the proposed PSO to the real damage scenario.

Some observations about the ability of the proposed methodology to detect damage using incomplete modal information have been established from the results in Sections 6.1 and 6.2:

- There is an optimal quantity of modal information that permits the detection of most of the possible damage scenarios. If a lower quantity is used, then the methodology can fail to detect the real damage scenario.
- The computational cost involved in the determination of the optimal quantity of modal information is high because many different combinations of modal information must to be tested. It is important to mention that this set of values is limited by technical conditions, such as the number available of sensors and the quantity of excitable modes.
- Different damage scenarios can be detected with different reliability levels using the same quantity of modal information available. In general, simple damage scenarios are detected more reliably than the multiple damage scenarios.

6.3. Application to the specific damage cases varying the quantity of modes

The simulated damage scenarios described in Tables 2 and 3 were used to illustrate how the methodology works for detecting one or more damaged elements. Tables 15 and 16 show a complete description of the damage scenarios found by the proposed methodology considering different quantities of mode shapes. The number of sensors covered 25% of the vertical DOFs in the structure, and the number of mode shapes was permitted to vary. The values in parentheses correspond to the error in the computation of the damage extent for the real damaged elements. Additionally, only the elements that presented a β value higher than 0.050 are presented. For the analysed examples, if either 7 or 9 modes are used, the real damaged elements may not be found, which occurred for the damage scenarios S3, M1 and M4. A similar performance in computing the damage extent was observed when either 7 or 9 modes were measured. The best results were obtained when 11 modes were used, as the maximum errors were 6% and 21% for simple and multiple damage scenarios, respectively. It is important to mention that for some damage scenarios a significant improvement in the computation of the damage extent was not achieved by increasing the quantity of mode shapes used. Additionally, the algorithm does not tend to either underestimate or overestimate the damage extent and the values found depend on the dynamics of the execution. Finally, the results showed that only a few

Table 16
Application of the proposed methodology to detect multiple damages. Measurements in 25% of the vertical DOFs. The number inside the parenthesis indicates the level of error on the computation of the damage extent.

ID scenario	Element	Damage extent β			
		Real	7 modes	9 modes	11 modes
M1	3	–	–	0.180	–
	4	0.200	0.194 (3)	0 (100)	0.231 (16)
	12	–	–	0.079	–
	13	–	0.080	–	–
	16	–	–	–	0.059
	18	–	0.053	–	–
	20	0.200	0.236 (18)	0.215 (7)	0.178 (11)
	21	–	–	–	0.058
	27	–	–	–	0.056
	32	–	0.066	–	–
	38	–	–	0.051	–
	41	–	0.064	–	–
M2	1	–	0.113	–	–
	6	–	0.072	–	–
	17	–	–	0.081	–
	21	–	–	0.118	–
	22	0.230	0.198 (14)	0 (100)	0.218 (5)
	23	0.230	0.316 (37)	0.389 (69)	0.269 (17)
	24	0.230	0.228 (1)	0.161 (30)	0.182 (21)
	34	–	–	0.105	–
	35	–	0.060	–	–
	51	—	–	0.203	0.071
M3	1	–	0.100	–	–
	3	0.170	0.205 (21)	0.198 (16)	0.181 (6)
	11	–	–	0.076	–
	19	–	0.091	–	–
	20	–	0.100	–	0.056
	31	0.290	0.277 (4)	0.265 (9)	0.291 (0)
	32	–	0.141	–	–
	41	–	–	0.077	–
	42	0.310	0.300 (3)	0.335 (8)	0.285 (8)
M4	1	–	–	0.302	–
	3	–	0.303	–	0.061
	4	0.320	0 (100)	0.333 (4)	0.287 (10)
	5	–	0.184	–	–
	15	–	0.101	–	–
	18	–	0.082	–	–
	24	–	–	–	–
	25	0.210	0.227 (8)	0.269 (28)	0.200 (5)
	32	–	0.140	–	–
	34	–	0.097	–	–
	37		–	0.054	–
	42	–	0.065	–	–
	45	–	0.108	–	–
	46	0.250	0 (100)	0.247 (1)	0.261 (4)
	47	0.310	0.404 (30)	0.311 (0)	0.264 (15)
	49	–	–	0.073	–

elements were misidentified with low values of damage and that, in general, the methodology was more reliable in the detection of simple damage scenarios than of multiple damage scenarios.

7. Conclusions

An adaptive particle swarm optimisation has been proposed to detect damage in beam structures under the assumption that the measurements are noisy and incomplete. Each damage detection methodology requires a study to determine the minimum quantity of modal data that guarantees reliable damage detection in a specific structure. The utilisation of abundant information does not guarantee an improvement in the computation of the damage extent. The proposed methodology solved nearly all of the random damage scenarios with low error in the computation of the damage extent and a few misidentified elements. The methodology failed for only two damage scenarios, but it found an approximate damage scenario. Some improvements in the methodology must be made to diminish the number of measured points in the beam that are necessary to guarantee the correct damage detection.

Acknowledgements

The authors would like to acknowledge CNPq (Brazilian National Council for Technological and Scientific Development) for their financial support of this research.

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