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Validation of the uncertainty bounds on modal parameters identified with the SSI-COV method

Yeny V. Ardila A.[a], Iván D. Gómez Araújo[a]*, Jesús D. Villalba-Morales[b], Luis A. Aracayo[c]

[a] Engenharia Civil e Infraestrutura, Universidade Federal da Integração Latino-Americana (UNILA), Foz do Iguaçu, Brasil. Email: yva.ardila.2019@aluno.unila.edu.br, ivan.araujo@unila.edu.br
[b] Departamento de Ingenieria Civil, Pontificia Universidad Javeriana (PUJ), Bogotá, Colombia. Email: jesus.villalba@javeriana.edu.co
[c] Centro de Competência em Estruturas de Barragens (EB.DT-FPTI), Foz do Iguaçu, Brasil. Email: luis.aracayo@pti.org.br

* Corresponding author

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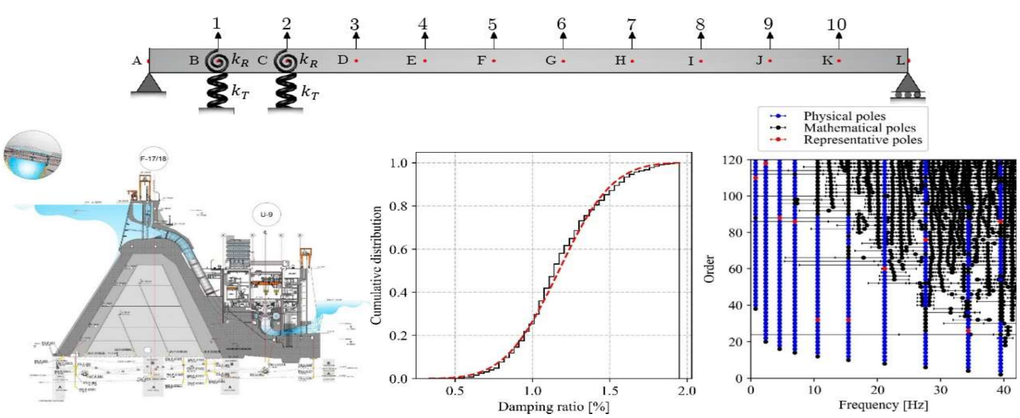
Abstract

Applying the Covariance-driven Stochastic Subspace Identification method (SSI-COV) involves uncertainties in the numerical process of identifying modal parameters in a system. The main issue is that the method does not compute the uncertainty in the results, which is required in some problems such as outlier detection. Currently, one method is available to assess the uncertainty in modal parameters obtained using the SSI-COV. This method based on the sensitivity analysis of the modal parameters to perturbations in the collected data and is efficient but highly complex for its computational implementation. Thus, this article presents a validation of the uncertainty results obtained with this procedure through the uncertainty limits obtained using the Bootstrap technique. The validation is performed on the modal parameters of a numerical beam-type structure with controlled noise levels and the modal parameters of a concrete block of the Itaipu Hydroelectric Dam. The uncertainty limits obtained using the two methodologies showed similarities in the two examples, which allowed validating the sensitivity analysis procedure.

Keywords

Uncertainty, bootstrap, sensitivity, modal parameter, identification

Graphical Abstract



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1 INTRODUCTION

Due to technological advances in measurement devices and the development of methods for automatized operational modal analysis (OMA), it is nowadays possible to extract the dynamic characteristics of large civil structures (such as buildings, bridges, and dams) under normal operating conditions from the response measurements caused by ambient vibrations. One of the main issues is selecting a reliable method to identify the structure's modal parameters (natural frequencies, damping ratios, and mode shapes). In that sense, results from applying the covariance-driven stochastic subspace identification method (SSI-COV) for built structures (Magalhães et al., 2008; Brownjohn et al., 2010; Liu et al., 2012; Pereira et al., 2018) show that the technique provides accurate results efficiently and robustly. However, the presence of several sources of error (e.g., finite number of data, stochastic nature of the ambient loading, non-linearities, instrumentation noise) induce a certain degree of uncertainty in the identified modal parameters, and the SSI-COV algorithm does not provide the information for evaluating the accuracy in the results.

Different types of uncertainties impact the SSI-COV method's stages during its application. At the beginning of the process, the uncertainty associated with the collected data affects the estimation of the covariance functions; then, the accumulated uncertainty propagates through each of the stages of system identification and finally to the modal parameters. Thus, the last stage of the method inherits the uncertainties from the previous step. Reynders et al. (2008) derived a numerical procedure to estimate the uncertainties of the modal parameters obtained at some given model order from the SSI-COV method to deal with that aspect. The approach quantifies the degree of uncertainty in the modal parameters by computing their covariances, estimated through a sensitivity analysis. The procedure was applied in the operational modal analysis of a steel transmitter mast to show its practicability. However, the direct implementation of this methodology is exceedingly expensive, mainly when the dynamic test implies an extensive sensor network and the use of high model order.

The quantification of modal parameter uncertainties plays a significant role for different purposes. For instance, in (Pereira et al., 2020), modal parameter uncertainties identified in continuous dynamic monitoring were used to remove outliers during modal tracking. Döhler et al. (2015) improved the results of a damage localization method by including the computed uncertainty levels in modal parameters. Verboven et al. (2004) performed an EMA test, finding that the mathematical modes had a much higher level of uncertainty than the physical modes. Furthermore, Wu et al. (2020) used the uncertainty in the modal parameters to remove mathematical modes from the stabilization diagram. Therefore, it is of utmost importance to demonstrate that the results of uncertainty quantification are reliable by comparing them with the results obtained from other methods used to estimate uncertainties.

Additionally, estimating the optimal model order of a structure is a difficult task and is often an overestimated parameter leading to the identification of mathematical modes. Stability diagrams are a tool to visualize the identified physical and mathematical modes at different model orders within an interval that aims to separate the physical modes from the mathematical modes. Döhler et al. (2013) modified the procedure proposed by Reynders et al. (2008) to determine more efficiently the covariances of modal parameters in multiple model orders as required by the stabilization diagram.

This paper reports the computation of confidence intervals on the modal parameters identified by using the SSI-COV method for two civil structures. Such estimations follow the uncertainty quantification approach proposed by (Reynders et al., 2008) and computationally improved by Döhler et al., (2013). Furthermore, the application of the bootstrap technique, which is a more versatile technique and not restricted to operational modal analysis through the SSI-COV method allows determining the validity of the results. As the bootstrap technique requires repetition of the identification process, a strategy for automatic estimation of modal parameters is configured. The organization of the paper is as follows: i) Section 2.1 introduces the SSI-COV method, ii) Section 2.2 presents the process of automatic estimation of the modal parameters, iii) Sections 3 explain the theory concerning the uncertainty approach from sensitivity analysis and bootstrap approach, and v) Section 4 reports the results of the validation of the two structures.

2 MODAL PARAMETER IDENTIFICATION.

This section will describe the procedure used to estimate the vibration modes from the system response. The method used to identify the modal parameters was the SSI-COV method, which is a time-domain parametric method that defines a state-space system realization from the correlation functions of the system response. A stabilization diagram is required in this methodology to identify the modal parameters. The stability diagram is an indicator of the system's poles at different model orders and it is a relatively effective way of identifying modes. However, the results of modal identification can contain spurious modes. Therefore, to extract the physical modes from the stability diagram, in this work, an automatic procedure is configured based on Reynders et al. (2012). This automatic estimation process consists of three stages: a) cleaning the stabilization diagram, b) grouping similar modes, and c) selecting the physical modes. The automatic estimation process is applied a priori in the two methodologies used in this work to define the uncertainties of the modal parameters: Uncertainty quantification from (Reynders et al., 2008; Döhler et al., 2013) and Bootstrap technique. Each of process and methodologies applied in this work are described in detail below.

2.1 COVARIANCE-DRIVEN STOCHASTIC SUBSPACE IDENTIFICATION METHOD (SSI-COV)

The problem of system realization is as follows: given an external description of a system, determine an internal state variable description that generates the given external description (Antsaklis et al., 2014). The SSI-COV method addresses the stochastic realization problem solved by (Akaike, 1974; Aoki, 1987; Arun et al., 1990), since its objective is the identification from output-only data of the state transition matrix $A \in \mathrm{R}^{n\times n}$ and the observation matrix $C \in \mathbb{R}^{r\times n}$ of the following stochastic state-space model:

$$\begin{aligned} x_{k+1} &= Ax_k + w_k \\ y_k &= Cx_k + v_k \end{aligned}, \tag{1}$$

where w_k and v_k are stochastic input processes, which are assumed to have zero mean and a constant spectrum (white noise). From the system matrices A and C the modal parameters can be estimated. The SSI-COV algorithm starts with the evaluation of the output covariance matrices Λ_l for time lags $l = 1,2,\ldots,2i-1$. By Assuming ergodicity, these covariance matrices can be formally defined as follows:

$$\Lambda_l = E[y_{k+l}y_k^T] = \lim_{N\to\infty} \frac{1}{N}\sum_{k=0}^{N-1} y_{k+l}\, y_k^T\ . \tag{2}$$

Since a finite number of samples N is available in practical applications, an estimation Λ_l can be obtained by dropping the limit. However, the direct use of this expression can be very time-consuming; hence, another alternative consists in using an approach based on the Fast Fourier Transform (FFT) (Oppenheim et al., 1975). In a second step, these output covariance matrices are organized in a block Toeplitz matrix $T_{1|i} \in \mathbb{R}^{ir\times ir}$:

$$T_{1|i} = \begin{bmatrix} [\Lambda_i] & [\Lambda_{i-1}] & \cdots & [\Lambda_1] \\ [\Lambda_{i+1}] & [\Lambda_i] & \cdots & [\Lambda_2] \\ \cdots & \cdots & \cdots & \cdots \\ [\Lambda_{2i-1}] & [\Lambda_{2i-2}] & \cdots & [\Lambda_i] \end{bmatrix}, \tag{3}$$

where i is the number of block rows of the Toeplitz matrix and r the total number of sensors. Furthermore, one of the properties for stochastic state-space models is the possibility to decompose the output covariance matrices in the form: $\Lambda_l = CA^{l-1}G$ where $G \in \mathbb{R}^{r\times n}$ is the next-state output covariance matrix given by $G = E[x_{k+1}y_k^T]$. According to these properties, $T_{1|i}$ can be decomposed as shown in Equation (4), where $\mathcal{O} \in \mathbb{R}^{ir\times n}$ is the extended observability matrix, $\Gamma \in \mathbb{R}^{n\times ir}$ the reversed extended stochastic controllability matrix, and n the model order.

$$T_{1|i} = \mathcal{O}\Gamma = \begin{bmatrix} C \\ CA \\ \cdots \\ CA^{i-1} \end{bmatrix} [A^{i-1}G \quad \cdots \quad AG \quad G]. \tag{4}$$

This Equation shows that T_{1i} is the result of multiplying a matrix with n columns by a matrix with n rows. Then, for $ir \geq n$ and an observable and controllable system, the rank of the ir by ir Toeplitz matrix is n. The application of the singular value decomposition (SVD) allows rewriting $T_{1|i}$ as:

$$T_{1|i} = USV^T = [U_1 \quad U_2] \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}^T = U_1S_1V_1^T, \tag{5}$$

where $U \in \mathbb{R}^{ir\times ir}$ and $V \in \mathbb{R}^{ir\times ir}$ are orthonormal matrices, with $U^TU = UU^T = I \in \mathbb{R}^{ir\times ir}$ and $V^TV = VV^T = I \in \mathbb{R}^{ir\times ir}$. $S \in \mathbb{R}^{ir\times ir}$ is a diagonal matrix containing the positive singular values in descending order. The observability and controllability matrices can be determined from the reduced singular value decomposition as shown in Equation (6), where $S_1 \in \mathbb{R}^{n\times n}$ contains the non-zero singular values with $U_1 \in \mathbb{R}^{ir\times n}$ and $V_1 \in \mathbb{R}^{ir\times n}$ their corresponding singular vectors.

$$\begin{aligned} \mathcal{O} &= U_1 S_1^{1/2} \\ \Gamma &= S_1^{1/2} V_1^T \end{aligned} . \tag{6}$$

Identifying system matrices A and C is simple from the previously computed observability $\mathcal{O}$ and Γ controllability matrices. C equals the first r rows of $\mathcal{O}$, and the state transition matrix A can be estimated by exploiting the shift structure of $\mathcal{O}$ (Kung, 1978), as follows:

$$\begin{bmatrix} CA \\ CA^2 \\ \cdots \\ CA^i \end{bmatrix} = \begin{bmatrix} C \\ CA \\ \cdots \\ CA^{i-1} \end{bmatrix} A \Leftrightarrow A = \begin{bmatrix} C \\ CA \\ \cdots \\ CA^{i-1} \end{bmatrix}^{\dagger} \begin{bmatrix} CA \\ CA^2 \\ \cdots \\ CA^i \end{bmatrix} = \underline{\mathcal{O}}^{\dagger}\overline{\mathcal{O}}, \tag{7}$$

where $(\cdot)^{\dagger}$ represents the Moore-Penrose pseudo-inverse of a matrix. $\underline{\mathcal{O}}$ contains the first $r(i-1)$ rows of $\mathcal{O}$, while $\overline{\mathcal{O}}$ contains the last $r(i-1)$ rows of $\mathcal{O}$. Alternatively, the matrix A could also be computed from the decomposition property of a shifted block Toeplitz matrix (Zeiger et al., 1974):

$$T_{2|i+1} = \mathcal{O} A \Gamma, \tag{8}$$

where the shifted matrix $T_{2|i+1}$ is composed of the output covariance matrices Λ_l for time lags $l = 2,3,\ldots,2i$. Introducing Equation (6) into Equation (8) and solving for A gives:

$$A = \mathcal{O}^{\dagger} T_{2|i+1} (\Gamma)^{\dagger} = S_1^{-1/2} U_1^T T_{2|i+1} V_1 S_1^{-1/2}. \tag{9}$$

From matrices A and C the modal parameters are extracted. The decomposition of the state transition matrix A is given by:

$$A = \Psi \Lambda \Psi^{-1}, \qquad A\psi_k = \lambda_k \psi_k, \tag{10}$$

where ψ_k and λ_k are the discrete-time eigenvectors and eigenvalues, respectively. As A was obtained from the discretization of the continuous-time matrix A_c , the equivalent continuous-time eigen-properties can be computed as follows:

$$\psi_{ck} = \psi_k, \quad \lambda_{ck} = \frac{ln(\lambda_k)}{\Delta t}, \tag{11}$$

Finally, natural frequencies and damping ratios are computed from:

$$f_k = \frac{|\lambda_{ck}|}{2\pi} \qquad [Hz], \tag{12}$$

$$\xi_k = -\frac{\Re(\lambda_{ck})}{|\lambda_{ck}|} * 100 \qquad [\%], \tag{13}$$

where $\Re(\bullet)$ represents the real part of a complex number. Finally, the observed mode shapes ϕ_k can be computed through the multiplication of matrix C with eigenvectors of matrix A:

$$\phi_k = C\psi_k. \tag{14}$$

2.2 AUTOMATIC MODAL PARAMETER ESTIMATION (MPE)

An automated procedure based on the methodology proposed in Reynders et al. (2012) is configured to extract the physical modes obtained with the SSI-COV method. The automation of the MPE consists of three stages: cleaning the stabilization diagram, grouping similar modes, and selecting the physical modes.

a) Cleaning the estabilization diagram.

Hard validation criteria (HVC) and soft validation criteria (SVC) are used for mode elimination in the cleaning stage. The HVC are applied to all modes. Modes that do not meet the following criteria are eliminated immediately: i) $\xi_k > 0\%$ and $\xi_k < 10\%$, ii) $f_k < f_s/2$, where f_s is the sampling frequency, and iii) mode with a complex conjugate pair. The SVC are summarized in Table 1. A mode is composed of a frequency (f), a damping ratio (ξ), and a mode shape (ϕ), then, assuming that i and j are a pair of closest modes according to the distance:

$$d(i,j) = d_f\left(f_i, f_j\right) + 1 - MAC\left(\phi_i, \phi_j\right), \tag{15}$$

d_f, d_ξ and MAC measure the similarity between these modes. On the other hand, the MPC and MPD indicators measure the complexity of the mode shapes. (Reynders et al., 2012), presents detailed information on how to compute these criteria.

Table 1 Soft validation criteria (SVC).

Criterion	Value	Ideal Physical	Ideal Mathematical
$\zeta^{(1)}$	$d_f =$ Relative frequency	0	1
$\zeta^{(2)}$	$d_\xi =$ Relative damping ratio	0	1
$\zeta^{(3)}$	$MAC =$ Modal Assurance	1	0
$\zeta^{(4)}$	$MPC =$ Modal Phase Collinearity	1	0
$\zeta^{(5)}$	$MPD =$ Mean Phase Deviation	0	1

Based on these discriminative criteria, an iterative Fuzzy C-means (FCM) clustering algorithm is employed to classify the modes into likely physical and mathematical modes. This approach divides a dataset into C clusters or classes by minimizing the weighted sum of the Euclidean distance between the dataset and centroids of each class. Thus, for the case in question, the objective function of the FCM can be written as:

$$J_m = \sum_{j=1}^{n_p} \sum_{i=1}^{2} \eta_{ij}^m \left\|\zeta_j - V_i\right\|^2, \tag{16}$$

where η_{ij} is the degree of membership of the mode j to the class i (likely physical mode or mathematical mode), V_i is the center of the classes, and $m \geq 1$ (commonly set to 2) is a weighting exponent. The objective function optimization is carried out through an iterative process, updating each class's degrees of membership and centroids until a termination criterion is met, either a maximum number of iterations or a minimum error. In fuzzy logic theory, the degree of membership represents a value in the range of 0 and 1. A value closer to 1 indicates that an element belongs to a particular class, while values closer to 0 indicate that the element does not belong to the particular class. As a result, the factor $0 < \eta_{ij} < 1$ indicates how much the modes $j = 1,2,\ldots,n_p$ belong to each class $i = 1,2$. As the summation of membership of each mode should be equal to one, modes with a degree of membership of less than 0.5 to the probably physical class are considered mathematical modes and are eliminated.

b) Grouping similar modes

After the cleaning stage, the probably physical modes are grouped according to similar features. This grouping is carried out using a hierarchical clustering approach in which the similarity information between each pair of modes computed through Equation (15) is used to construct a distance matrix D. Then, modes/clusters that are in proximity are linked together using a linkage function. Finally, using a threshold d_h, cut the hierarchical tree into clusters. In