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Nuevas Tendencias en la Construcción Sostenible

THE PROBLEM OF COMPUTING THE LATEST TIMES OF THE ACTIVITIES IN FUZZY SCHEDULING; A LITERATURE REVIEW.

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A B S T R A C T

In multiple knowledge fields is necessary to schedule projects involving unavailable or incomplete information. To address this problem multiple approach has been proposed, being most common the parameter estimation and stochastic methods. The parameter estimation can produce highly risky schedules depending on the estimation method, therefore is not safe to use it. Stochastic methods are closer to reality and produce appropriate results, but it can be very complex to apply and possibly requiring specialized staff to model the problem and apply the methods. Beside those approaches, using Fuzzy Logic Sets has been proposed as solution to schedule projects with incomplete information, being easily applicable and producing appropriate results. As the other approaches using Fuzzy Logic arise some problems, being the most serious the computation of the latest times of the activities, which frequently produce negative starting times. To fix this problem several authors have proposed distinct approaches obtaining the desired results without compromising the accessibility of Fuzzy Logic. In this research, the basics of Fuzzy Logic are introduced and six different method for computing the latest times in Fuzzy Scheduling will be exposed to propose the most appropriate model.

Keywords:

Fuzzy Logic, Fuzzy Project Scheduling, Fuzzy Latest Times.

1. Introduction.

In Architecture, engineering and construction industry, projects are planned by construction schedulers using a group of tools known as the critical path method (CPM), involving unavailable or incomplete information, which requires rough estimations in the forecast of project parameters, producing a high risk of failure. To solve the problem of risk management in projects, statistical approaches have been proposed such as the Program Evaluation Review Technique, or the Monte Carlo Simulation Models based on random distributions.

Several authors argue that the nature of the risk associated to construction projects is not associated with the presence of random variables but with the imprecision of the estimates produced by limited and incomplete information. Those proposals states that the Theory of Fuzzy Sets (TFS) is appropriate, closer to reality and simpler to use than stochastic models, facilitating the implementation of values that are not precisely known, but that can be limited within certain bounds of membership or fuzziness.

Traditional CPM allows computing the times of the activities in an easy and efficient way, but when imprecision is included to the model, applying the fuzzy logic and fuzzy arithmetic, several problems arises. The main problem is to compute the latest times of the activities, which frequently produce negative starting times when traditional algorithms are applied. In order to deal with this issue, researchers have developed several models along the analyzed literature, but it is not clear which method is the best one to compute the latest times on the activities when fuzzy arithmetic is considered.

In this research, different method for computing the latest times in Fuzzy Scheduling will be exposed to propose the most appropriate model. In order to introduce this proposal properly, the following section provides the principles of fuzzy sets and fuzzy arithmetic, whereas section 3 gives a brief statement of the problem of computing the latest times of the activities. Section 4 details the proposed models found in the literature. Later, a brief discussion and comparison of methods is be exposed in section 6. Finally, conclusions and limitations of the research are drawn.

2. Principles of fuzzy sets and fuzzy arithmetic.

Zadeh (1965) defined fuzzy sets (Ω) as sets of elements with no precise boundaries, whose elements have different degrees of membership to the set. The membership function is a matter of the degree of belonging to the set, not a matter of the duality between affirmation and denial about whether an element is part of the set. Accordingly, each element has its grade of membership to the concept represented by the fuzzy set (real numbers in the closed unit interval [0, 1]).

A fuzzy number of a set Ω (fuzzy set of real numbers) is a membership function written $\tilde{A}(x)$ which produces values in $[0, 1]$ for al x in Ω. Different shapes for a fuzzy number can be adopted, being the triangular fuzzy number (known as TFN) one of the most common membership functions. It is defined by three numbers $a_1 \leq a_2 \leq a_3$, where the base of the triangle is the interval $[a_1, a_3]$ called support (membership equals zero), and the vertex is a_2 (membership equals one), as shown in equation 1.

$$\tilde{A}(x) = \{x, \mu(x) | x \in \Omega\};\ \tilde{A}(x) = (a_1, a_2, a_3) \tag{1}$$

Alpha-cut of $\tilde{A}(x)$, written $\tilde{A}[\alpha]$, are slices through a fuzzy set that produces closed and bounded subsets in which its membership function is greater than or equal to α (Figure 1), and for a triangular fuzzy number the interval will be written and obtained applying equation 2.

$$[A]^\alpha = [a_1(\alpha), a_3(\alpha)] = [a_1 + \alpha \cdot (a_2 - a_1), a_3 - \alpha \cdot (a_3 - a_2)] \tag{2}$$

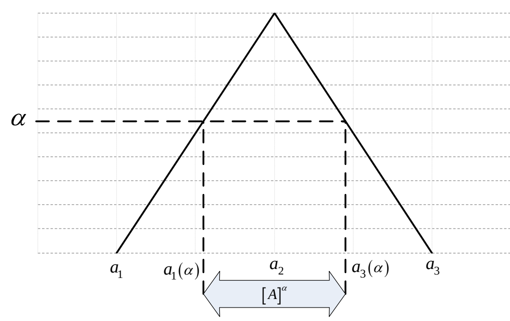


Figure 1. Triangular fuzzy number $\tilde{A}(x)$ and alpha cut $[A]^\alpha$ of $\tilde{A}(x)$

Fuzzy numbers can be operated in a similar way as the crisp ones (traditional numbers) by applying the fuzzy arithmetic rules. Fuzzy operations (represented as #) between two fuzzy numbers $\tilde{A}(x)$ and $\tilde{B}(x)$, are based on extension principle and arithmetic by intervals. By applying the interval arithmetic for alpha values between 0 and 1 $(0 \leq \alpha \leq 1)$ is defined as expressed in equation 3 and 4.

$$[C]^\alpha = [A]^\alpha \# [B]^\alpha = [c_1(\alpha), c_3(\alpha)];\ being\ \#\ an\ arithmetc\ operator \tag{3}$$

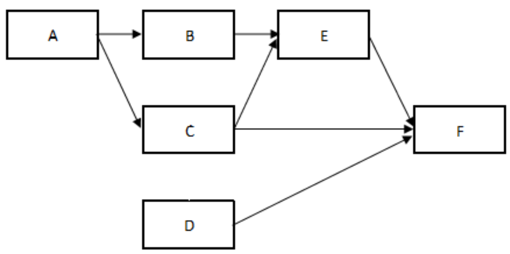
$$\begin{gathered} c_1(\alpha) = \boldsymbol{min}\big(a_1(\alpha)\#b_1(\alpha), a_3(\alpha)\#b_3(\alpha), a_3(\alpha)\#b_1(\alpha), a_3(\alpha)\#b_3(\alpha)\big) \\ c_3(\alpha) = \boldsymbol{max}\big(a_1(\alpha)\#b_1(\alpha), a_3(\alpha)\#b_3(\alpha), a_3(\alpha)\#b_1(\alpha), a_3(\alpha)\#b_3(\alpha)\big) \end{gathered} \tag{4}$$

3. Problem Statement of Computing the Latest Times of the activities.

Computing the latest times (LS_i) of the activities is an important issue which can provide incorrect values for the obtained fuzzy numbers. This happens because the backward pass of the traditional algorithm is strictly applied as with crisp values (Eq. 8), and, if a fuzzy difference is applied, negative values can be obtained for the times of the activities, providing incorrect solutions (Shi & Blomquist, 2012).

$$[LF]_i^\alpha = \boldsymbol{min}\left([LS]_j^\alpha\right) \forall\, j \in S \; successors \; of \; i$$
$$[LS]_i^\alpha = [LF]_i^\alpha \ominus [Dur]_i^\alpha \; \forall\, i \qquad (5)$$

For a better comprehension of the remainder of the article, a small project example of an AON network with six activities has been designed with the topology and fuzzy durations exposed in Figure 2. The shapes for fuzzy LS times computed applying fuzzy difference (traditional approach) for the project example are presented in Figure 3. In Table 1 are exposed the $[ES]^{\alpha=0}$ (early starting time for alpha=0), the $[EF]^{\alpha=0}$ (early finishing time for alpha=0), $[LS]^{\alpha=0}$ (latest starting time for alpha=0) and the $[LF]^{\alpha=0}$ (latest finishing time for alpha=0).



Activities	Fuzzy duration		
	Inferior	*Core*	*Superior*
A	1	2	3
B	3	4	9
C	2	5	8
D	8	9	15
E	3	4	7
F	5	7	12

Figure 2. Topology and fuzzy duration of the project example

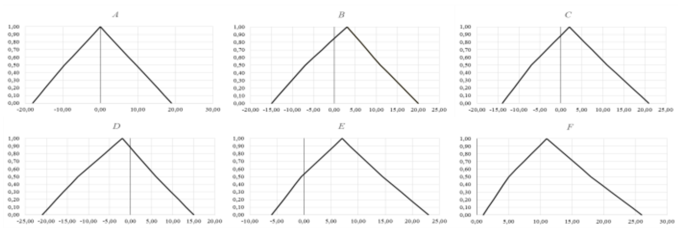


Figure 3. LS of the activities applying fuzzy difference

Activity	$[ES]^{\alpha=0}$	$[EF]^{\alpha=0}$	$[LS]^{\alpha=0}$	$[LF]^{\alpha=0}$
A	$[0.0, 0.0]$	$[1.0, 3.0]$	$[-18.0, 19.0]$	$[-15.0, 20.0]$
B	$[1.0, 3.0]$	$[4.0, 12.0]$	$[-15.0, 20.0]$	$[-6.0, 23.0]$
C	$[1.0, 3.0]$	$[3.0, 11.0]$	$[-14.0, 21.0]$	$[-6.0, 23.0]$
D	$[0.0, 0.0]$	$[8.0, 15.0]$	$[-21.0, 15.0]$	$[-6.0, 23.0]$
E	$[4.0, 12.0]$	$[7.0, 19.0]$	$[-6.0, 23.0]$	$[1.0, 26.0]$
F	$[8.0, 19.0]$	$[13.0, 31.0]$	$[1.0, 26.0]$	$[13.0, 31.0]$

Table 1. Computed fuzzy times applying fuzzy difference

As can be seen in Figure 3 and Table 1, the latest starting times of the activities A, B, C, D, E, and the latest finishing times of the activities A, B, C, D, offer negative values for the left support of the fuzzy number, which obviously is not correct.

4. Proposals for Computing the Latest Times of the activities.

Along the analyzed literature, researchers have deal with the problem of computing the times of the activities for avoiding negative values in the left support of the fuzzy number. The first three proposals are an evolution of the traditional method solving the problem of computing the LS by equation solving (Buckley & Eslami, 2002) (Ponz-Tienda, Pellicer, & Yepes, 2012) (Chrysafis & Papadopoulos, 2014), applying the Minkowski´s subtraction and reshaping (Ponz-Tienda J. L., Pellicer, Benlloch-Marco, & Andrés-Romano, 2015), or simply avoiding the negative values of the left side of the fuzzy number (McCahon & & Lee, 1988). The last three models are based on the graph topology as the Nasution (1994) model, mathematical programing (Madhuri, Siresha, & Shankar, 2012), and the interaction between the activities of the graph (Slyeptsov & Tyshchuk, 2003).

4.1. LS computing by equation solving.

Ponz-Tienda (2012), Buckley & Eslami (2002) and Chrysafis & Papadopoulos (2014) uphold that the LS of the activities is the result of solving an equation with the LS as the unknown term as can be seen in equation 6.

$$LS_i + Dur_i = LF_i \rightarrow LS_i = LF_i - Dur_i \qquad (6)$$

This fact does not have transcendental implications on scheduling with crisp values, but with fuzzy values, the equation must be solved as a crisp difference of the alpha cuts of the fuzzy numbers (equation 7), resulting in the method exposed in equation 8 that guarantee that the obtained solution is always culminate in an positive confidence interval (Gil-Aluja, 2004). This method is also known as Minkowski´s subtraction (Buckley & Eslami, 2002) (Chrysafis & Papadopoulos, 2014). In Figure 4 are presented the shapes for the LS and in Table 2 the obtained fuzzy times following the previously exposed criteria for Table 1.

$$
\begin{aligned}
[LS]_i^{\alpha} \oplus [Dur]_i^{\alpha} &= [LF]_i^{\alpha} \\
[LS]_i^{\alpha} &= [LF]_i^{\alpha} - [Dur]_i^{\alpha}
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
&[ls_1(\alpha), ls_3(\alpha)] \oplus [dur_1(\alpha), dur_3(\alpha)] = [lf_1(\alpha), lf_3(\alpha)] \\
&[ls_1(\alpha) = lf_1(\alpha) - dur_1(\alpha), ls_3(\alpha) = lf_3(\alpha) - dur_3(\alpha)]
\end{aligned} \tag{8}
$$

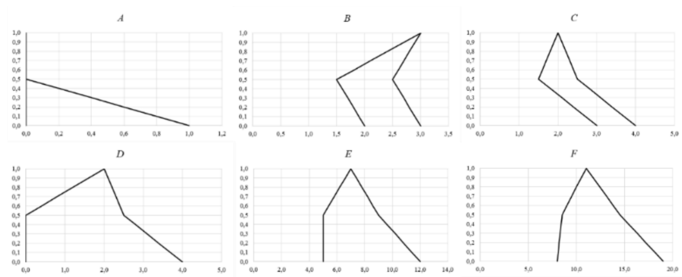


Figure 4 LS of the activities applying equation solving

Activity	$[ES]^{\alpha=0}$	$[EF]^{\alpha=0}$	$[LS]^{\alpha=0}$	$[LF]^{\alpha=0}$
A	[0.0, 1.0]	[1.0, 2.0]	[0.0, 1.0]	[2.0, 3.0]
B	[1.0, 3.0]	[4.0, 12.0]	[2.0, 3.0]	[5.0, 12.0]
C	[1.0, 3.0]	[3.0, 11.0]	[3.0,4.0]	[5.0, 12.0]
D	[0.0, 0.0]	[8.0, 15.0]	[0.0, 4.0]	[8.0, 19.0]
E	[4.0, 12.0]	[7.0, 19.0]	[5.0, 12.0]	[8.0, 19.0]
F	[8.0, 19.0]	[13.0, 31.0]	[8.0, 19.0]	[13.0, 31.0]

Table 2 Computed fuzzy times applying fuzzy equation solving

As can be seen in Figure 4 and Table 2, all the obtained fuzzy times are positive values, but the fuzzy LS times for the activities B and C are non-convex fuzzy numbers.

4.2. LS computing by Ponz-Tienda et al (2015).

Ponz-Tienda et al (2015) proposes the use of the Minkowski´s subtraction modified in such way that left side of the latest start is obtained as the maximum between the lower bound of the early start and the difference between the upper bound of the duration and the lower bound of the latest finish. The right side of the latest start is obtained as the difference between the upper bound of the latest finish and the lower bound of the duration as can be seen in equation 9.

$$
\begin{aligned}
[LF]_i^{\alpha} &= \boldsymbol{min}\big([LS]_j^{\alpha}\big) \;\forall\, j \in S \text{ successors of } i \\
[LS]_i^{\alpha} &= \big[\boldsymbol{max}\big(es_1(\alpha); lf_1(\alpha) - dur_3(\alpha)\big), lf_3(\alpha) - dur_1(\alpha)\big]
\end{aligned} \tag{9}
$$

In Figure 5 are presented the shapes for the LS and in Table 3the obtained fuzzy times following the previously exposed criteria.

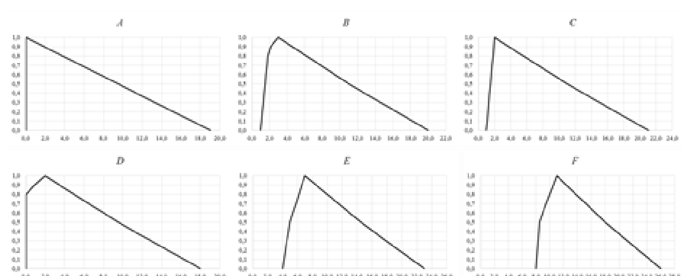


Figure 5. LS of the activities applying Ponz-Tienda et al (2015)

Activity	$[ES]^{\alpha=0}$	$[EF]^{\alpha=0}$	$[LS]^{\alpha=0}$	$[LF]^{\alpha=0}$
A	[0.0, 1.0]	[1.0, 2.0]	[0.0, 8.0]	[1.0, 9.0]
B	[1.0, 3.0]	[4.0, 12.0]	[1.0, 9.0]	[4.0, 12.0]
C	[1.0, 3.0]	[3.0, 11.0]	[1.0,10.0]	[4.0, 12.0]
D	[0.0, 0.0]	[8.0, 15.0]	[0.0, 18.0]	[8.0, 26.0]
E	[4.0, 12.0]	[7.0, 19.0]	[4.0, 12.0]	[8.0, 26.0]
F	[8.0, 19.0]	[13.0, 31.0]	[8.0, 26.0]	[13.0, 31.0]

Table 3. Computed fuzzy times applying Ponz-Tienda et al (2015)

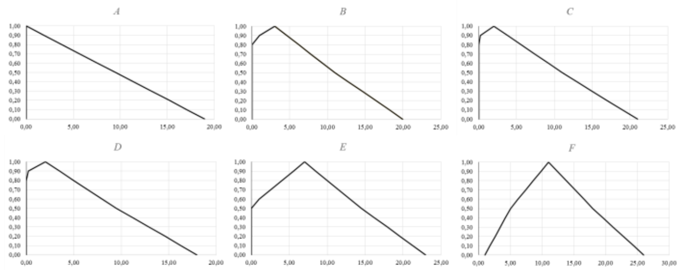
Applying the Ponz-Tienda et al (2015) method, the obtained fuzzy times are convex and positive values in an easy way without complex calculations.

4.3. LS computing by Mccahon & Lee (1988).

McCahon & Lee (1988), propose two new methods to deal with negative fuzzy values the “composite method” and the “comparative method”. The comparative methods consist of computing the earliest and latest times as with the traditional method with crisp values replacing negative values by zero (equation 10).

$$
\begin{aligned}
[LF]_i^{\alpha} &= \boldsymbol{min}\big([LS]_j^{\alpha}\big) \;\forall\, j \in S \text{ successors of } i \\
[LS]_i^{\alpha} &= \boldsymbol{max}([LF]_i^{\alpha} \ominus [Dur]_i^{\alpha}, 0\,) \;\forall\, i
\end{aligned} \tag{10}
$$

The Project example has been solved applying the composite method obtaining the shapes and fuzzy times exposed in Figure 6 and Table 4.



Activity	$[ES]^{\alpha=0}$	$[EF]^{\alpha=0}$	$[LS]^{\alpha=0}$	$[LF]^{\alpha=0}$
A	[0.0, 1.0]	[1.0, 2.0]	[0.0, 19.0]	[0.0, 20.0]
B	[1.0, 3.0]	[4.0, 12.0]	[0.0, 20.0]	[0.0, 23.0]
C	[1.0, 3.0]	[3.0, 11.0]	[0.0,21.0]	[0.0, 23.0]
D	[0.0, 0.0]	[8.0, 15.0]	[0.0, 18.0]	[1.0, 26.0]
E	[4.0, 12.0]	[7.0, 19.0]	[0.0, 23.0]	[1.0, 26.0]
F	[8.0, 19.0]	[13.0, 31.0]	[1.0, 26.0]	[13.0, 31.0]

Figure 6. LS of the activities applying Mccahon & Lee (1988)

Table 4. Computed fuzzy times applying Mccahon & Lee (1988)

The Mccahon & Lee (1988) proposal is easy to apply (similar as the traditional way) providing convex number without negative values, but provide inconsistent values with great ranges of imprecision. Additionally, paths with consecutive activities (E-B-F in the project example) present latest start times equal to zero not taking into account the finishing times of the predecessor activities.

4.4. LS computing by Nasution (1994).

Sofjan H. Nasution (1994) presented a method that consists on representing the fuzzy numbers as the core and the deviation to the left and right, corresponding to the inverse of the slope of the number. The numerical representation and transformation of a TFN into a Nasution fuzzy number ($\bar{\tilde{A}}(x)$) are exposed in equation 11, and the fuzzy arithmetic for the sum and difference operations in equation 12.

$$\tilde{A}(x) = (a_1, a_2, a_3) \rightarrow \bar{\tilde{A}}(x) = (a, a_l, a_r) \begin{cases} a = a_2 \\ a_l = a_2 - a_1 \\ a_r = a_3 - a_2 \end{cases} \tag{11}$$

$$\begin{gathered} (a, a_l, a_r) + (b, b_l, b_r) = (a + b, a_l + b_l, a_r + b_r) \\ (a, a_l, a_r) - (b, b_l, b_r) = (a - b, a_l + b_l, a_r + b_r) \end{gathered} \tag{12}$$

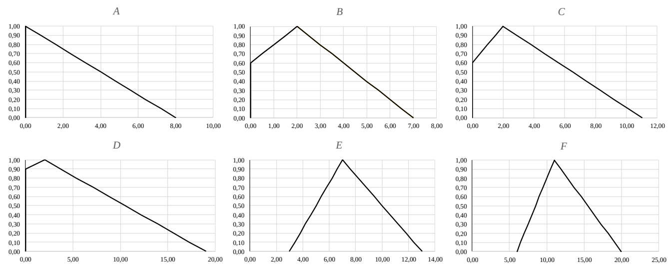
Applying the Nasution transformation to the fuzzy durations of the project example, the new durations are exposed in Table 5.

Table 5 Computed fuzzy times applying Mccahon & Lee (1988)

Activity	d	d_l	d_r
A	2	1	1
B	4	1	5
C	5	3	3
D	9	1	6
E	4	1	3
F	7	2	5

Once obtained the Nasution fuzzy durations ($\bar{\tilde{d}}(x)$), the earliest times are obtained as with the traditional way. The method first establishes all the paths of the graph from the initial activity to the finish one and the sub-paths from every activity to the finish one, obtaining several paths for the graph and at least one sub-path for each activity. The method proceeds to analyse each activity, so each sub-path of this activity is analysed. Then, the first sub-path is subtracted from each path in the graph and the maximum value of all resulting differences is select. The same process is performed for all sub-paths of the activity and the minimum of the resulting values is selected as the LS of the activity.

The Project example has been solved applying the Nasution method obtaining the shapes and fuzzy times exposed in Figure 7 and Table 6.



Activity	$[ES]^{\alpha=0}$	$[EF]^{\alpha=0}$	$[LS]^{\alpha=0}$	$[LF]^{\alpha=0}$
A	$[0.0, 1.0]$	$[1.0, 2.0]$	$[0.0, 8.0]$	$[0.0, 11.0]$
B	$[1.0, 3.0]$	$[4.0, 12.0]$	$[0.0, 7.0]$	$[0.0, 16.0]$
C	$[1.0, 3.0]$	$[3.0, 11.0]$	$[0.0, 11.0]$	$[0.0, 19.0]$
D	$[0.0, 0.0]$	$[8.0, 15.0]$	$[0.0, 19.0]$	$[0.0, 34.0]$
E	$[4.0, 12.0]$	$[7.0, 19.0]$	$[3.0, 13.0]$	$[6.0, 20.0]$
F	$[8.0, 19.0]$	$[13.0, 31.0]$	$[6.0, 20.0]$	$[11.0, 32.0]$

Figure 7. LS of the activities applying Nasution (1994)

Table 6. Computed fuzzy times applying Nasution (1994)

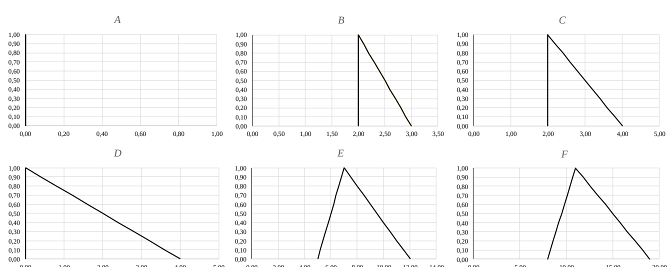
The Nasution (1994) proposal is relatively difficult and complicated to apply, obtaining results with very wide ranges where LS dates have left limits equal to zero for some activities. The results should not be used because the obtained TFN are not suitable for reliable analysis.

4.5. LS computing by Madhuri, Siresha & Shankar (2012).

Madhuri et al (2012) presented a method that consists on optimize all paths in the graph to become them critic. This optimization is performed using computational tools. The method uses graph with arrow activities. The optimization minimizes the sum of the right and left limit of the ES and maximizes the sum of the left and right limit of the date LS obeying the following restrictions:

$$\begin{gathered} ES_{Y_{low}} \geq ES_{X_{low}} + d_{i_{low}};\ ES_{Y_{up}} \geq ES_{X_{up}} + d_{i_{up}} \\ LS_{Y_{low}} \geq LS_{X_{low}} + d_{i_{low}};\ LS_{Y_{up}} \geq LS_{X_{up}} + d_{i_{up}} \\ LS_{S_{low}} = 0;\ LS_{F_{low}} = ES_{F_{low}} \\ LS_{S_{up}} = 0;\ LS_{F_{up}} = ES_{F_{up}} \end{gathered} \tag{13}$$

Where “low” refers to the left limit of the TFN, “up” refers to the right limit of the TFN, “i” refers to the activity on the arrow, “Y” refers to the node on the right of the arrow and “X” refers to the node on the left of the arrow. The shapes and fuzzy times solved applying the Nasution method are exposed in Figure 8 and Table 7.



Activity	$[ES]^{\alpha=0}$	$[EF]^{\alpha=0}$	$[LS]^{\alpha=0}$	$[LF]^{\alpha=0}$
A	$[0.0, 0.0]$	$[1.0, 2.0]$	$[0.0, 0.0]$	$[1.0, 3.0]$
B	$[1.0, 3.0]$	$[4.0, 12.0]$	$[2.0, 3.0]$	$[5.0, 12.0]$
C	$[1.0, 3.0]$	$[3.0, 11.0]$	$[2.0, 4.0]$	$[4.0, 11.0]$
D	$[0.0, 0.0]$	$[8.0, 15.0]$	$[0.0, 4.0]$	$[8.0, 19.0]$
E	$[4.0, 12.0]$	$[7.0, 19.0]$	$[5.0, 12.0]$	$[8.0, 19.0]$
F	$[8.0, 19.0]$	$[13.0, 31.0]$	$[8.0, 19.0]$	$[13.0, 31.0]$

Figure 8. LS of the activities applying Madhuriet al (2012)

Table 7. Computed fuzzy times applying Madhuri et al (2012)

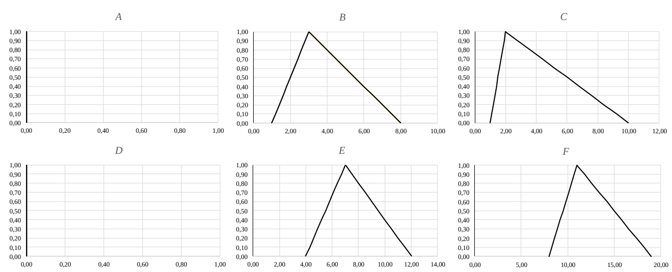
The Madhuri (2012) proposal is easy to apply with an optimization program, obtaining results with appropriate wide ranges and keep the convexity of the TFN. The constrains of the optimization are used to guarantee the convexity and the correct wide.

4.6. LS computing by Slyeptsov & Tyshchuk (2003).

Slyeptov y Tyshchuk (2003) presented a method that consists of the behavior analysis of each activity with respect to all other activities. The earliest times are obtained as with the traditional way. To calculate the LS and LF dates is required to assemble tuples for each activity, these tuples are formed with the right limit, left limit or both limits of representative fuzzy number of the duration of all project activities, taking as a selection criterion the behavior of the analyzed activity with respect the other activities, such behavior can be non-decreasing, not increasing, not monotone or may not affect the behavior of the number. To understand the behavior of each activity it is necessary to classify them in the following groups:

- Separator Vertex: Node to be removed for completely separates the graph to be the only connection between its predecessor activities and successor activities
- Nodal Vertex (s): All separator vertex, the start node or the end node of the graph of the project.
- Lobe Graph (G_j): Sub-graph which is within the graph of the project that has the largest number of connections between which a separator vertex is not found.
- P_{jr}: All roads within a graph lobe "j" go through the activity "r"
- $P_{j\hat{r}}$: All roads within a graph lobe "j" do not go through the activity "r"
- R_{jr}: Set of no nodal activities that go through the path P_{jr} that pass through them must go through “r”. Where R_{jr}^{pre} are predecessor activities of “r” and R_{jr}^{suc} are successor activities of “r”.
- $R_{j\hat{r}}$: Set of no nodal activities that go through the path $P_{j\hat{r}}$ that pass through them must not go through “r”.
- $R_{j\underline{r}}$: Set of no nodal activities that pass through them must or must not go through a path that contains “r”. Where $R_{j\underline{r}}^{pre}$ are predecessor activities of “r” and $R_{j\underline{r}}^{suc}$ are the successor activities of “r”.
- $R_{\hat{j}r}$: Set of no nodal activities that are not in the graph “G_j”, where “G_j” is the sub-graph that contains the activity “r”. Where $R_{\hat{j}r}^{pre}$ are the predecessor activities of “G_j” and $R_{\hat{j}r}^{suc}$ are the successor activities of “G_j”.

The Project example solved applying the Slyeptsov method produce the shapes and fuzzy times exposed in Figure 9 and Table 11.



Activity	$[ES]^{\alpha=0}$	$[EF]^{\alpha=0}$	$[LS]^{\alpha=0}$	$[LF]^{\alpha=0}$
A	[0.0, 0.0]	[1.0, 2.0]	[0.0, 0.0]	[1.0, 3.0]
B	[1.0, 3.0]	[4.0, 12.0]	[1.0, 8.0]	[4.0, 12.0]
C	[1.0, 3.0]	[3.0, 11.0]	[1.0, 10.0]	[4.0, 12.0]
D	[0.0, 0.0]	[8.0, 15.0]	[0.0, 0.0]	[8.0, 15.0]
E	[4.0, 12.0]	[7.0, 19.0]	[4.0, 12.0]	[7.0, 19.0]
F	[8.0, 19.0]	[13.0, 31.0]	[8.0, 19.0]	[13.0, 31.0]

Figure 9. LS of the activities applying Slyeptsov & Tyshchuk (2003) Table 8. Computed fuzzy times applying Slyeptsov & Tyshchuk (2003)

The Slyeptsov (2003) proposal is easy to apply, however, it is a long process of analysis and classification but it is possible to do a computer program that organize and characterize the activities, the tuples, and the results, obtaining TFN with appropriate wide ranges and keep the convexity.

5. Discussion and comparison of methods.

Once analyzed the proposals found in the literature, the proposals that provides convex and positive fuzzy values for computing the fuzzy LS times are by Ponz-Tienda et al. (2015), Mccahon & Lee (1988), Nasution (1994), Madhuri, Siresha & Shankar (2012) and Slyeptsov & Tyshchuk (2003).

Nevertheless, the values provided by the previously exposed methods are different. The Slyeptsov & Tyshchuk (2003) method provide crisp values for the activities A and D, fact that is not reliable because the huge difference in the duration of the acivities. In a similar way, the Madhuri, Siresha & Shankar (2012) method offers a crisp value for the LS of the activity A, suggesting that A should be always a critical activity, but the project conditions makes that the critical path could start in the activity A and/or in the activity B.

The Nasution (1994) and Madhuri, Siresha & Shankar (2012) methods provide similar values, especially for activity C that shows the possibility for its LS times a value of zero, value that is impossible because the activity A (predecessor of C) presents a left support of one temporal unit for the fuzzy duration. This implies that C cannot start at zero time.

Finally, the method proposed by Ponz-Tienda et al. (2015) seems to be the most reliable, not only providing convex and positive fuzzy values for the LS times, but also being the easiest to implement.

6. Conclusions.

Project scheduling in AEC industry involves unavailable or incomplete information producing a high risk of failure. To solve the problem of risk management in projects, statistical approaches have been proposed , but several authors argue that the nature of the risk associated to construction projects is not with the presence of random variables but with the imprecision of the estimates produced by limited and incomplete information. Theory of Fuzzy Sets (TFS) is closer to reality and simpler to use than stochastic models, facilitating the implementation of values that are not precisely known.

When fuzzy logic and fuzzy arithmetic is applied, several problems arises, especially to compute the latest times of the activities, providing negative values. In order to deal with this issue several models has been developed, but it is not clear which method is the best one to compute the latest times on the activities when fuzzy arithmetic is considered.

In this research, different method for computing the latest times in Fuzzy Scheduling has been analyzed. The Ponz-Tienda et al. (2015), Mccahon & Lee (1988), Nasution (1994), Madhuri, Siresha & Shankar (2012) and Slyeptsov & Tyshchuk (2003) provides convex and positive fuzzy values for the fuzzy LS times, but with different shapes.

- The Slyeptsov & Tyshchuk (2003) method provide crisp values for some initial activities fact that is not reliable in all cases.
- The Madhuri, Siresha & Shankar (2012) method offers a crisp value for the LS for some activities.
- The Nasution (1994) and Madhuri, Siresha & Shankar (2012) methods provide values with LS time equal to zero for activities that have predecessors with duration greater to zero.
- The Ponz-Tienda et al. (2015) method seems to be the most reliable, not only providing convex and positive fuzzy values for the LS times, but also being the easiest to implement.

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